

الاسم:  
الرقم:

مسابقة في مادة الفيزياء  
المدة: ساعة ونصف

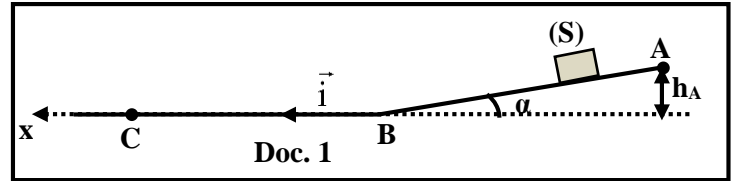
**This exam is formed of three obligatory exercises in three pages.**  
**The use of non-programmable calculator is recommended.**

### Exercise 1 (6.5 pts)

#### Determination of the force of friction

A block (S), considered as a particle, of mass  $m = 100 \text{ g}$ , can slide on path ABC situated in a vertical plane. This path is formed of two parts:

- AB is straight and inclined by an angle  $\alpha$  with respect to the horizontal ( $\sin \alpha = 0.1$ );
- BC is straight and horizontal.



At instant  $t_0 = 0$ , the block (S) is released without initial velocity from point A, situated at a height  $h_A$  above the horizontal x-axis, confounded with BC, and of unit vector  $\vec{i}$  (Doc. 1).

Along part AB, the motion of (S) takes place without friction, and along part BC, (S) is subjected to a force of friction  $\vec{f}$  supposed constant and parallel to the displacement.

The aim of this exercise is to determine the magnitude  $f$  of the force of friction  $\vec{f}$ .

Take:

- the horizontal plane containing the x-axis as the reference level for gravitational potential energy;
- $g = 10 \text{ m/s}^2$ .

#### 1) Motion of (S) between A and B

The block (S) slides without friction along part AB and reaches B at  $t = 2 \text{ s}$ .

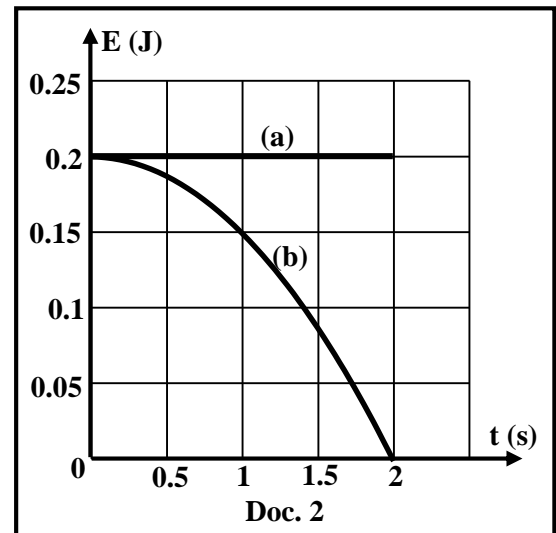
The two curves (a) and (b) shown in document 2 represent the gravitational potential energy and the mechanical energy of the system [(S), Earth] as functions of time, during the motion of (S) between A and B.

1.1) Indicate for each curve the appropriate energy.

Justify.

1.2) Using document 2:

- determine the distance AB covered by (S) along the inclined plane;
- show that the speed of (S) at B is  $V_B = 2 \text{ m/s}$ .



#### 2) Motion of (S) between B and C

At  $t = 2 \text{ s}$ , the block (S) reaches B and continues its motion along part BC and stops at C at  $t = 4 \text{ s}$ .

2.1) Determine the linear momenta of (S),  $\langle \vec{P}_B \rangle$  at B and  $\langle \vec{P}_C \rangle$  at C.

2.2) Deduce the variation  $\Delta \vec{P}$  of the linear momentum of (S) between B and C.

2.3) Show that the sum of the external forces exerted on (S) between B and C is  $\Sigma \vec{F}_{\text{ext}} = -f \vec{i}$ .

2.4) Determine the magnitude  $f$  of  $\vec{f}$ , knowing that  $\Delta \vec{P} \cong \Sigma \vec{F}_{\text{ext}} \cdot \Delta t$ , where  $\Delta t$  is the duration of the motion between B and C.

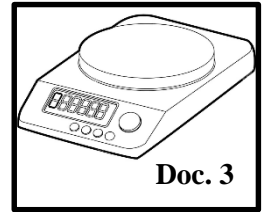
## Exercise 2 (7.5 pts)

### Capacitor in a digital balance

The aim of this exercise is to study the role of a capacitor in a digital balance (Doc. 3).

For this purpose, we set up the series circuit of document 4 that includes:

- an ideal battery (G) of electromotive force  $E$ ;
- a resistor (D) of resistance  $R$ ;
- a capacitor, initially uncharged, of capacitance  $C$ ;
- a switch  $K$ .



#### 1) Theoretical study

At instant  $t_0 = 0$ ,  $K$  is closed and the charging process of the capacitor starts.

At instant  $t$ , plate H of the capacitor carries a charge  $q$  and the circuit carries a current  $i$ .

1.1) Redraw the circuit of document 4 showing on it the direction of the current  $i$ .

1.2) Show that the differential equation that governs the variation of the voltage

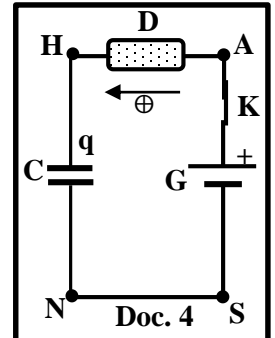
$$u_{HN} = u_C \text{ across the capacitor is : } E = RC \frac{d u_C}{d t} + u_C.$$

1.3) The solution of the obtained differential equation has the form:

$$u_C = a + b e^{-\frac{t}{\tau}}, \text{ where } a, b \text{ and } \tau \text{ are constants.}$$

Determine  $a$ ,  $b$  and  $\tau$  in terms of  $E$ ,  $R$  and  $C$ .

1.4) Calculate the ratio  $\frac{u_C}{E}$  at  $t = \tau$ .



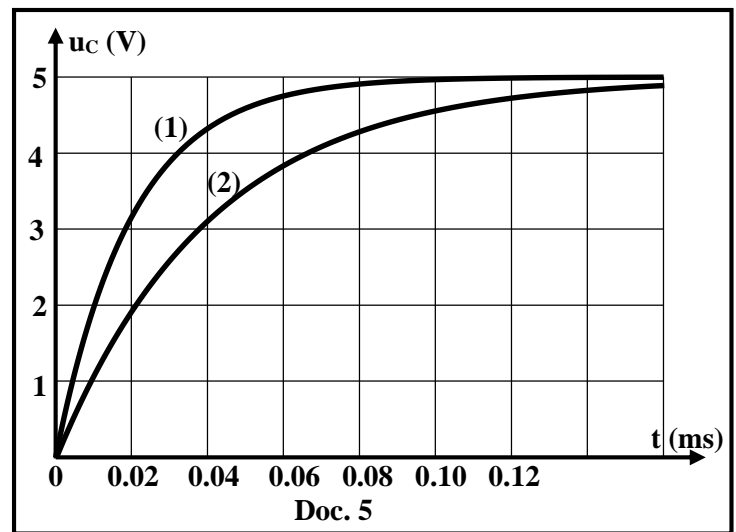
#### 2) Measurement of the mass of an object

Document 4 is a simplified circuit used in a digital balance, where the capacitance  $C$  varies with the mass of the object placed on the balance.

Two objects of respective masses  $m_1$  and  $m_2$  are placed successively on this digital balance.

For each object, the capacitor in the balance has a different value of capacitance.

Curves (1) and (2) shown in document 5, represent the voltage  $u_C$  as functions of time, corresponding to each of the masses  $m_1$  and  $m_2$  respectively. Given that  $R = 10^7 \Omega$ .



2.1) Using curve (1) of document 5:

2.1.1) indicate the value of  $E$ ;

2.1.2) determine the capacitance  $C_1$  corresponding to the object of mass  $m_1$ ;

2.2) Calculate  $m_1$ , knowing that the relation between the mass  $m$  of the object and the capacitance  $C$

$$\text{of the capacitor is: } C = \frac{1.066 \times 10^{-12}}{1 - m}; \text{ (m in kg, C in F) and } 0 < m < 1 \text{ kg.}$$

2.3) Determine whether  $m_1$  is greater or less than  $m_2$ .

### Exercise 3 (6 pts)

### Diffraction of light

The aim of this exercise is to determine the width of a thin slit using the phenomenon of diffraction.

#### Wave Behaviors

“The visible light spectrum is the segment of the electromagnetic spectrum that the human eye can view. The human eye can detect wavelengths, in air, from 380 to 700 nanometers...

Waves across the electromagnetic spectrum behave in similar ways. When light waves encounter an object, they are either transmitted, reflected, absorbed, refracted, diffracted, or scattered depending on the composition of the object and the wavelength of the light wave.

Diffraction is the bending and spreading of waves around an obstacle. It is most clear one when a light wave strikes an object with a size comparable to its own wavelength...”

[www.science.nasa.gov](http://www.science.nasa.gov)

Doc. 6

- 1) The text of document 6 mentions that visible light waves can undergo diffraction like any electromagnetic wave. Pick out from document 6:
  - 1.1) the statement that describes the phenomenon of diffraction of waves;
  - 1.2) the condition to obtain a clear diffraction pattern.

- 2) A source (S) emits in air an electromagnetic wave of frequency  $\nu = 4.34 \times 10^{14}$  Hz. A cylindrical beam from this source falls under normal incidence on a horizontal narrow slit F of width  $\ll a \gg$ , which is cut in an opaque screen (P). A screen (E) is placed parallel to (P) at a distance  $D = 2$  m away from it (Doc. 7).

Given:

- speed of electromagnetic waves in air is  $c = 3 \times 10^8$  m/s;
- the diffraction angles in this exercise are small;
- the diffraction angle  $\theta_n$  corresponding to a dark fringe of order  $n$  is given by:

$$\sin \theta_n = \frac{n\lambda}{a}, \text{ where } \lambda \text{ is the wavelength of the electromagnetic wave, with } n = \pm 1, \pm 2, \pm 3 \dots$$

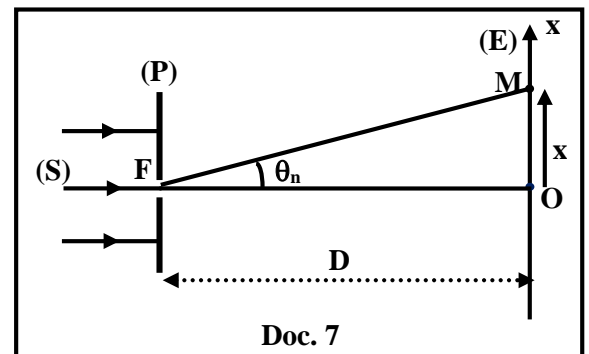
For small angles, take  $\sin \theta \approx \tan \theta \approx \theta$  in radians.

- 2.1) Show that the wavelength of the electromagnetic wave emitted by (S) is  $\lambda = 6.91 \times 10^{-7}$  m.
- 2.2) Deduce that this wave is visible by human eye.
- 2.3) Compare the direction of the diffraction pattern to the direction of the slit.
- 2.4) A point M on the screen (E) is the center of a dark fringe of order  $n$  in the diffraction pattern.

The position of M is  $x = OM$  relative to the center O of the central bright fringe.

Show that the abscissa of M is  $x = \frac{n \lambda D}{a}$ .

- 2.5) Calculate the width  $\ll a \gg$  of the slit, knowing that the distance between O and the center of the second dark fringe is  $x = 6$  mm.



مسابقة في مادة الفيزياء  
أسس التصحيح - إنكليزي

Exercise 1 (6.5 pts)		Determination of the force of friction	Note
Part	Answer		Note
1.1	Curve (a) corresponds to ME. Since no friction therefore ME = constant		0.25
	Curve (b) corresponds to GPE, since as height decreases GPE decreases		0.25
1.2.1	At A: $GPE_A = 0.2 \text{ J}$ . But $GPE_A = mgh_A = mg(AB \sin \alpha)$ So $0.2 = 0.1 \times 10 \times AB \times 0.1$ , we get: $AB = 2 \text{ m}$		1
1.2.2	$ME_B = KE_B + GPE_B$ $0.2 = \frac{1}{2} \times 0.1 \times V_B^2 + 0$ , we get $V_B = 2 \text{ m/s}$		1
2.1	$\vec{P}_B = m \vec{V}_B$ , so $\vec{P}_B = 0.2 \dot{i}$ ; $\vec{P}_C = m \vec{V}_C = \vec{0}$ (kg m/s)		1
2.2	$\Delta \vec{P} = \vec{P}_C - \vec{P}_B$ , so $\Delta \vec{P} = \vec{0} - 0.2 \dot{i} = -0.2 \dot{i}$ (kg m/s)		1
2.3	$\Sigma \vec{F}_{\text{ext}} = m \vec{g} + \vec{N} + \vec{f}$ ; $m \vec{g} + \vec{N} = \vec{0}$ So: $\Sigma \vec{F}_{\text{ext}} = \vec{f} = -f \dot{i}$		1
2.4	$\Delta \vec{P} = \Sigma \vec{F}_{\text{ext}} \cdot \Delta t$ , so $-0.2 \dot{i} = -f \dot{i} \times 2$ , we get: $f = 0.1 \text{ N}$		0.5

Exercise 2 (7.5 pts)		Capacitor in a Digital Balance
Part	Answer	Note
1.1		0.25
1.2	<p>Law of addition of voltages: <math>u_{AS} = u_{AH} + u_{HN} + u_{NS}</math></p> <p><math>E = R i + u_C</math>, but <math>i = \frac{dq}{dt}</math> and <math>q = C u_C</math> so <math>i = C \frac{du_C}{dt}</math></p> <p>This implies: <math>E = R C \frac{du_C}{dt} + u_C</math></p>	1
1.3	<p><math>u_C = a + b e^{-\frac{t}{\tau}}</math> so <math>\frac{du_C}{dt} = -\frac{1}{\tau} b e^{-\frac{t}{\tau}}</math></p> <p>Replace <math>u_C</math> and <math>\frac{du_C}{dt}</math> in the differential equation:</p> <p><math>R C [-\frac{1}{\tau} b e^{-\frac{t}{\tau}}] + a + b e^{-\frac{t}{\tau}} = E</math> then: <math>b e^{-\frac{t}{\tau}} [-\frac{RC}{\tau} + 1] + a = E</math> ;</p> <p><math>b e^{-\frac{t}{\tau}} \neq 0</math> so by comparison <math>a = E</math> and <math>-\frac{RC}{\tau} + 1 = 0</math> so <math>\tau = R C</math></p> <p><math>u_C = a + b e^{-\frac{t}{\tau}}</math>. But at <math>t = 0</math>; <math>u_C = 0</math> so <math>b = -a = -E</math></p> <p>Thus, <math>u_C = E (1 - e^{-\frac{t}{\tau}})</math> where <math>\tau = RC</math></p>	2
1.4	At $t = \tau$ : $u_C = E (1 - e^{-1}) = 0.63 E$ ; so $\frac{u_C}{E} = 0.63$	0.5
2.1.1	$E = 5 \text{ V}$	0.5
2.1.2	At $t = \tau$ : $u_C = 0.63 \times 5 = 3.15 \text{ V}$ Graphically $\tau = 0.02 \text{ ms} = 2 \times 10^{-5} \text{ s}$ $\tau = R C_1$ so $C_1 = 2 \times 10^{-12} \text{ F}$	1
2.2	$C_1 = \frac{1.066 \times 10^{-12}}{1 - m_1}$ so $2 \times 10^{-12} = \frac{1.066 \times 10^{-12}}{1 - m_1}$ $1 - m_1 = \frac{1.066 \times 10^{-12}}{2 \times 10^{-12}}$ , we get: $m_1 = 0.467 \text{ kg}$	1
2.3	<p>Curve (2): <math>u_C = 3.15 \text{ V}</math> at <math>t = \tau_2 = 0.04 \text{ ms} = 4 \times 10^{-5} \text{ s}</math> so <math>C_2 = 4 \times 10^{-12} \text{ F}</math></p> $4 \times 10^{-12} = \frac{1.066 \times 10^{-12}}{1 - m_2}$ we get $1 - m_2 = \frac{1.066 \times 10^{-12}}{4 \times 10^{-12}}$ , So: $m_2 = 0.7335 \text{ kg}$ <p>then <math>m_1 &lt; m_2</math></p> <p><b>Or</b></p> <p>The curve (1) reaches its maximum value before than (2) therefore <math>\tau_1 &lt; \tau_2</math> then <math>C_1 &lt; C_2</math>.</p> <p>But <math>C</math> and <math>(1 - m)</math> are inversely proportional, hence <math>1 - m_1 &gt; 1 - m_2</math> therefore <math>m_1 &lt; m_2</math>.</p>	1.25

<b>Exercise 3 (6 pts)</b>		<b>Diffraction of light</b>
<b>Part</b>	<b>Answer</b>	<b>Mark</b>
<b>1.1</b>	Diffraction is the bending and spreading of waves around an obstacle	<b>1</b>
<b>1.2</b>	It is most clear one when a light wave strikes an object with a size comparable to its own wavelength.	<b>1</b>
<b>2.1</b>	$\lambda = \frac{c}{\nu}$ so $\lambda = \frac{3 \times 10^8}{4.34 \times 10^{14}} = 6.91 \times 10^{-6} \text{ m} = 691 \text{ nm}$	<b>1</b>
<b>2.2</b>	It is visible since it is between 380 to 700 nm	<b>0.5</b>
<b>2.3</b>	The direction of the pattern is perpendicular to that of the slit F.	<b>0.5</b>
<b>2.4</b>	Dark fringe of order n : $\sin\theta_n = \frac{n\lambda}{a}$ , so $\theta_n = \frac{n\lambda}{a}$ Dark fringe of order n: $\tan\theta_n = \frac{x}{D}$ , so $\theta_n = \frac{x}{D}$ Thus $\frac{n\lambda}{a} = \frac{x}{D}$ this implies $x = \frac{n\lambda D}{a}$ .	<b>1</b>
<b>2.5</b>	$x = \frac{n\lambda D}{a}$ , so $6 \times 10^{-3} = \frac{2 \times 691 \times 10^{-9} \times 2}{a}$ , $a = 0.46 \times 10^{-3} \text{ m} = 0.46 \text{ mm}$	<b>1</b>