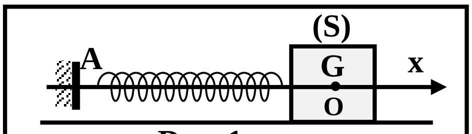
مسابقة في مادة الفيزياء	الاسم:
المدة: ساعة ونصف	الرقم:

This exam is formed of three obligatory exercises in three pages.
The use of non-programmable calculator is recommended.

Exercise 1 (7 pts)

Mechanical oscillations



Doc. 1

A mechanical oscillator consists of a block (S) of mass m and a spring of negligible mass and force constant k = 20 N/m.
The spring is connected from one of its ends to a fixed support A.
(S) is attached to the other end of the spring and it may slide without friction on a horizontal support (Doc. 1).
At equilibrium, G, the center of mass of (S), coincides with the origin O of the x-axis.
At the instant $t_0 = 0$, G is at O and we launch (S) with a velocity $\vec{v}_0 = v_0\,\vec{i}$; thus, (S) undergoes mechanical oscillations with an amplitude X_m.
At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity is $v = x' = \frac{dx}{dt}$.
The aim of this exercise is to study for this oscillator the effect of v_0 on the oscillation amplitude X_m.
Take:

- the horizontal plane passing through G as a reference level for gravitational potential energy;
- $g = 10\ m/s^2$ and $\pi^2 = 10$.

1) Theoretical study

1.1) Write the expression of the mechanical energy ME of the system (Oscillator , Earth) in terms of x, m, k and v.

1.2) Determine the second order differential equation that governs the variation of x.

1.3) Deduce the expression of the proper (natural) period T_0 of the oscillations in terms of m and k.

2) Experimental study

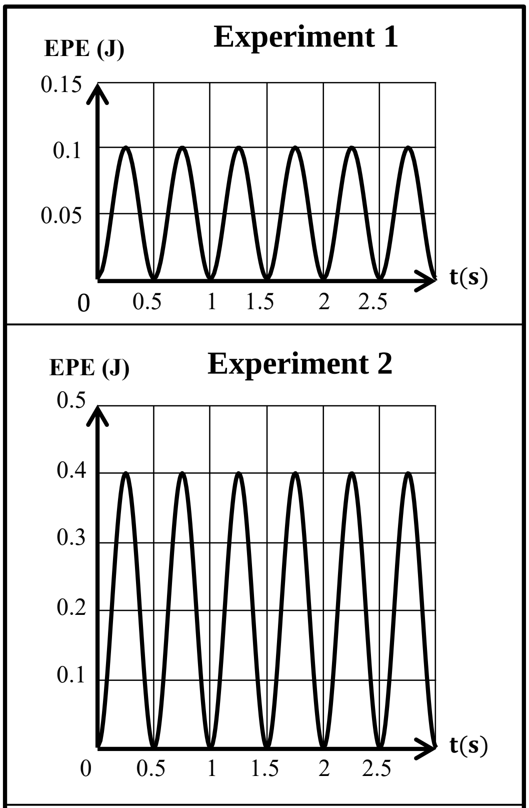
An appropriate device gives the elastic potential energy EPE of the oscillator as a function of time for two different experiments, experiment 1 and experiment 2 (Doc. 2).

2.1) Use the graphs of document 2 in order to:

2.1.1) justify that the oscillations of (S) are undamped.

2.1.2) copy and then complete the following table:

	Experiment 1	Experiment 2
The maximum value of EPE		
The value of the period T_E of EPE		



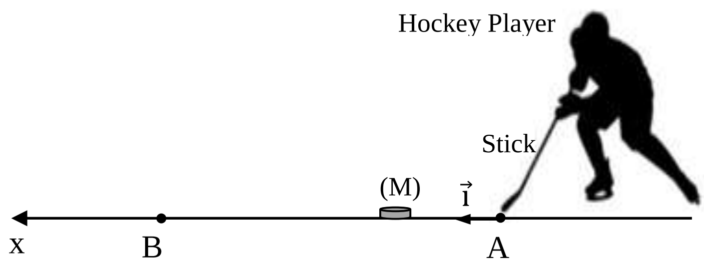
Doc. 2

2.2) Show that m = 0.5 kg knowing that $T_0 = 2T_E$.
2.3) Show that $X_{m(2)} = 2\ X_{m(1)}$, where $X_{m(1)}$ and $X_{m(2)}$ are the amplitudes of the oscillations in experiments 1 and 2 respectively.
2.4) Determine the values of v_0 for the two experiments.
2.5) Deduce whether X_m increases, decreases, or remains the same as v_0 increases.

Exercise 2 (6.5 pts)

Motion of a hockey puck

The purpose of this exercise is to study the motion of a hockey puck (M).
(M), taken as a particle of mass m = 170 g, can slide on a horizontal ice rink. A hockey player hits puck (M) with his stick from point A (Doc. 3).
Take the horizontal plane passing through (M) as a reference level for gravitational potential energy.



Doc. 3

1) The collision between (M) and the stick occurs in a very short time. Choose the correct sentence out of the three following sentences.
Sentence 1: During this collision, the linear momentum and the kinetic energy of the system [Stick , (M)] are necessarily conserved.
Sentence 2: During this collision, the linear momentum of the system [Stick , (M)] is conserved but the kinetic energy of this system is not necessarily conserved.
Sentence 3: During this collision, the linear momentum of the system [Stick , (M)] is not necessarily conserved but the kinetic energy of this system is necessarily conserved.

2) Just after the collision, (M) is launched from point A with a velocity $\vec{v}_A = 18\,\vec{i}$ (m/s). Puck (M) moves on the ice rink along an x-axis, and it stops at point B after travelling a distance AB = 54 m during a time Δt (Doc. 3).

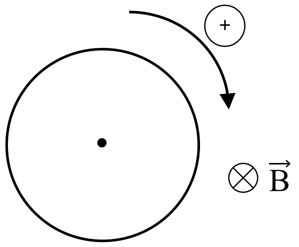
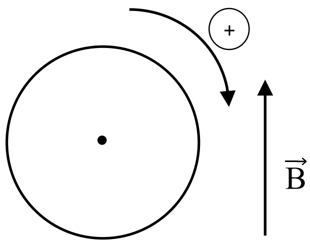
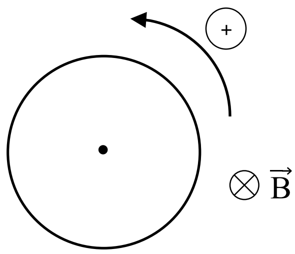
2.1) Calculate the mechanical energy of the system [(M) , Earth] at A and then at B.
2.2) Deduce that (M) is submitted to a friction force $\vec{f}$ during its motion between A and B.
2.3) Given that the value f of $\vec{f}$ is constant. Deduce that f = 0.51 N.
2.4) Name the external forces acting on (M) between A and B, and then draw, not to scale, a diagram for these forces.
2.5) Show that the sum of these forces is $\sum \vec{F}_{ext} = -\ 0.51\ \vec{i}$ (N).
2.6) Determine the linear momenta of (M), « $\vec{P}_A$ » at point A and « $\vec{P}_B$ » at point B.
2.7) Deduce the variation $\Delta\vec{P}$ of the linear momentum of (M) during Δt.
2.8) Calculate Δt knowing that $\Delta\vec{P} = (\sum \vec{F}_{ext})\ \Delta t$.

Exercise 3 (6.5 pts)

Electromagnetic induction

The purpose of this exercise is to determine the direction of the induced current in a circular loop by two different methods.
Consider a circular conducting loop of radius r = 10 cm and resistance R = 2 Ω. The loop is placed in a uniform magnetic field $\vec{B}$.

1) Document 4 shows three different cases.

1st case	2nd case	3rd case
The plane of the loop is perpendicular to the magnetic field lines of $\vec{B}$.	The plane of the loop is parallel to the magnetic field lines of $\vec{B}$.	The plane of the loop is perpendicular to the magnetic field lines of $\vec{B}$.
		

Doc. 4

Match each of the following sentences 1, 2 and 3 to its appropriate case. Justify.

Sentence 1: The magnetic flux through the loop is zero.
Sentence 2: The magnetic flux through the loop is positive.
Sentence 3: The magnetic flux through the loop is negative.

2) Consider the first case of document 4. During the time interval [0 , 2 s], the value B of the magnetic field $\vec{B}$ decreases with time according to the relation:

$$B = -0.04\,t + 0.8 \quad \text{(SI)}$$

2.1) A current is induced in the loop during the time interval [0 , 2s]. Justify.
2.2) Apply Lenz’s law in order to specify the direction of the induced current.
2.3) Determine the expression of the magnetic flux crossing the loop as a function of time.
2.4) Deduce the value of the induced electromotive force « e ».
2.5) The current carried by the loop is given by the relation $i = \frac{e}{R}$. Deduce the value and the direction of « i ».
2.6) Compare the direction of the induced current obtained in part (2.5) to that obtained in part (2.2).

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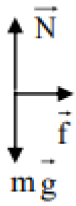
Exercise 1 : Mechanical oscillations (7 pts)

Part			Answer	Mark
1	1.1		$ME = KE + EPE = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$	0.5
	1.2		Friction is neglected, then the mechanical energy is conserved. Or: The sum of the works done by the nonconservative forces is zero, then ME is conserved. Then, $\frac{dME}{dt} = 0$, so $m v v' + k x x' = 0$ { $v = x'$ and $v' = x''$ } $v (m x'' + k x) = 0$, but $v = 0$ is rejected , so $m x'' + k x = 0$; therefore, $x'' + \frac{k}{m} x = 0$	1
	1.3		The differential equation is of the form: $x'' + \omega_0^2 x = 0$ with $\omega_0 = \sqrt{\frac{k}{m}}$ $T_0 = \frac{2\pi}{\omega_0}$, then $T_0 = 2\pi\sqrt{\frac{m}{k}}$	1
2	2.1	1	$EPE_{max} = \frac{1}{2} k X_m^2$ = constant. k is constant, then X_m is constant; therefore, the oscillations are undamped.	0.5
		2	(see table below)	0.5 0.5
	2.2		$T_0 = 2 T_E = 2 (0.5) = 1$ s $T_0 = 2\pi\sqrt{\frac{m}{k}}$, then $T_0^2 = 4\pi^2 \frac{m}{K}$, so $m = \frac{k T_0^2}{4\pi^2}$ $m = \frac{20 \times 1}{4 \times 10}$, hence $m = 0.5$ kg	0.5
	2.3		Experiment 1 : $EPE_{max} = 0.1 = \frac{1}{2} k X_{m(1)}^2$ … eq(1) Experiment 2 : $EPE_{max} = 0.4 = \frac{1}{2} k X_{m(2)}^2$ …eq(2) ; Dividing eq(2) by eq(1) gives: $\frac{0.4}{0.1} = \frac{X_{m(2)}^2}{X_{m(1)}^2}$, then $4 = (\frac{X_{m(2)}}{X_{m(1)}})^2$, hence $2 = \frac{X_{m(2)}}{X_{m(1)}}$ Therefore, $X_{m(2)} = 2 X_{m(1)}$	0.5
	2.4		ME = constant , then $ME = EPE_{max} = KE_{max}$, so $EPE_{max} = \frac{1}{2} m v_0^2$ Experiment 1 : $0.1 = \frac{1}{2} (0.5) v_{0(1)}^2$, then $v_{0(1)} = 0.63$ m/s Experiment 2 : $0.4 = \frac{1}{2} (0.5) v_{0(2)}^2$, then $v_{0(2)} = 1.26$ m/s	0.5 0.25 0.25
	2.5		v_o in experiment 2 is greater than v_o in experiment 1 ($v_{0(2)} > v_{0(1)}$) and $X_{m(2)} > 2 X_{m(1)}$; therefore, as v_0 increases X_m increases.	0.5 0.5

Table for 2.1.2:

	Experiment 1	Experiment 2
The maximum value of EPE	0.1 J	0.4 J
The value of the period T_E of EPE	0.5 s	0.5 s

Exercise 2: Motion of a hockey puck (6.5 pts)			
Part		**Answer**	**Mark**
1		Sentence 2	**0.5**
2	**2.1**	$GPE_A = GPE_B = 0$ since (M) is at the reference level. $ME_A = KE_A + GPE_A = \frac{1}{2} m v_A^2 + 0 = \frac{1}{2} \times 0.17 \times 18^2$, then $ME_A = 27.54$ J $KE_B = 0$ since (M) stops at point B. $ME_B = KE_B + GPE_B = 0 + 0$, then $ME_B = 0$	**0.75** **0.25**
	2.2	$ME_B < ME_A$, then (M) is submitted to a friction force.	**0.25**
	2.3	$\Delta ME = W_{\vec{f}} = \vec{f} . \overrightarrow{AB}$, then $ME_B - ME_A = - f \times AB$ $0 - 27.54 = - f \times 54$, hence $f = 0.51$ N	**1**
	2.4	Forces acting on (M) : The weight $m\vec{g}$ The normal force $\vec{N}$ exerted by the ice rink The friction force $\vec{f}$  	**0.5** **0.5**
	2.5	$\sum \vec{F}_{ext} = m\vec{g} + \vec{N} + \vec{f}$, but $m\vec{g} + \vec{N} = \vec{0}$ Then, $\sum \vec{F}_{ext} = \vec{f} = -f\vec{i} = - 0.51\,\vec{i}$ (N)	**0.75**
	2.6	$\vec{P}_A = m\,\vec{v}_A = 0.17 \times 18\,\vec{i}$, then $\vec{P}_A = 3.06\,\vec{i}$ (kg.m/s) $\vec{P}_B = m\,\vec{v}_B = m\,(\vec{0})$, then $\vec{P}_B = \vec{0}$	**0.75** **0.25**
	2.7	$\Delta\vec{P} = \vec{P}_B - \vec{P}_A = \vec{0} - 3.06\,\vec{i}$, then $\Delta\vec{P} = - 3.06\,\vec{i}$ (kg.m/s)	**0.5**
	2.8	$\Delta t = \frac{\Delta\vec{P}}{\sum \vec{F}_{ext}} = \frac{-3.06\,\vec{i}}{-0.51\,\vec{i}}$, then $\Delta t = 6$ s	**0.5**

Exercise 3 (6.5 pts)	**Electromagnetic induction**	
Part	**Answer**	**Mark**
1	<u>Sentence 1 corresponds to the 2nd case, because:</u> • $\phi = \vec{B} \cdot \vec{n}\, S = B\, S \cos(\vec{B}, \vec{n}) = B\, S \cos 90^o = 0$ • **<u>or</u>** the plane of the loop is parallel to the field lines • **<u>or</u>** the field lines do not cross the loop <u>Sentence 2 corresponds to the 1nd case, because:</u> • the angle between the unit vector $\vec{n}$ and $\vec{B}$ is zero • **<u>or</u>** $\phi = B\, S \cos 0^o = B\, S\, (1)$, but B and S are positive ; therefore, ϕ is positive. <u>Sentence 3 corresponds to the 3rd case, because:</u> • the angle between the unit vector $\vec{n}$ and $\vec{B}$ is 180° • **<u>or</u>** $\phi = B\, S \cos 180^o = -\, B\, S$, but B and S are positive ; therefore, ϕ is negative.	**0.5** **0.5** **0.5**
2.1	During [0, 2s], the magnitude B of $\vec{B}$ changes, then the loop is crossed by a variable magnetic flux; therefore, the loop becomes the seat of induced emf. The loop forms a closed circuit, then it carries electric current.	**0.75**
2.2	During [0, 2s], B decreases, then the direction of the induced magnetic field is the same as that of $\vec{B}$ in order to oppose the decrease in B. According to the right hand rule, the induced current passes in the loop in the chosen positive sense (clockwise).	**0.75**
2.3	$\phi = \vec{B} \cdot \vec{n}\, S = B\, S \cos(\vec{B}, \vec{n}) = B\, S \cos 0^o = B\, S = B\, \pi\, r^2$ $\phi = (-\,0.04\, t + 0.8) \times \pi \times (0.1)^2$ $\phi = -\,4\pi \times 10^{-4}\, t + 8\pi \times 10^{-4}$ (SI)	**1**
2.4	$e = -\frac{d\varphi}{dt} = -\,(-\,4\pi \times 10^{-4})$, then $e = 4\pi \times 10^{-4}$ V	**1**
2.5	$i = \frac{e}{R} = \frac{4\pi \times 10^{-4}}{2} = 6.3 \times 10^{-3}$ A i > 0, then the current is in the chosen positive sense (Clockwise).	**1**
2.6	The direction is the same in the two parts.	**0.5**