مسابقة في مادة الفيزياء	الاسم:
المدة: ساعة ونصف	الرقم:

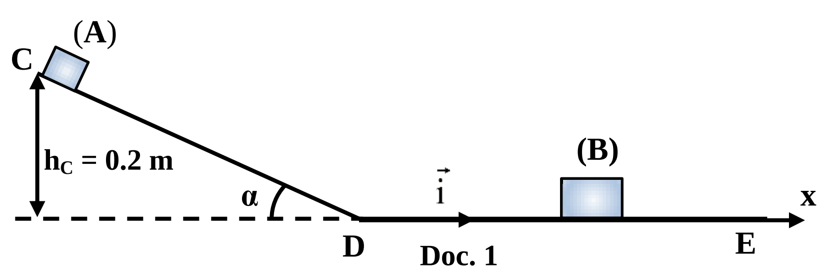
<u>This exam is formed of three obligatory exercises in three pages.</u>
<u>The use of non-programmable calculator is recommended.</u>

Exercise 1 (6 pts.)

Verification of the principle of interaction

The aim of this exercise is to verify the principle of interaction between two blocks.
For this purpose, we consider two blocks (A) and (B) considered as particles of respective masses $m_A = 200$ g and $m_B = 800$ g.
(A) and (B) can move without friction on a track CDE lying in a vertical plane.
This track is formed of two parts: the first one CD is straight and inclined by an angle α with respect to the horizontal and the second one DE is straight and horizontal.
Block (A) is released without initial velocity from point C situated at a height $h_C = 0.2$ m above a horizontal x-axis, confounded with DE, of unit vector $\vec{i}$ (Doc. 1).
Take:

- the horizontal plane containing the x-axis as a reference level for gravitational potential energy;
- $g = 10$ m/s^2.



Doc. 1

1) The mechanical energy of the system [(A), Track, Earth] is conserved between C and D. Why?

2) Deduce that the speed of (A) at point D is $V_A = 2$ m/s.

3) (A) continues its motion with a velocity $\overrightarrow{V_A} = 2\,\vec{i}$ (m/s) along track DE until it makes a head-on elastic collision with (B) initially at rest.
Show that the velocities of (A) and (B) right after the collision are $\overrightarrow{V'_A} = -1.2\,\vec{i}$ (m/s) and $\overrightarrow{V'_B} = 0.8\,\vec{i}$ (m/s) respectively.

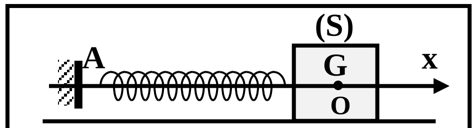
4) The duration of the collision is $\Delta t = 0.1$ s, so $\dfrac{\Delta\vec{P}}{\Delta t} \cong \dfrac{d\vec{P}}{dt}$.
Apply, during Δt, Newton's second law:
4.1) on (B) to determine the force $\vec{F}_{A/B}$ exerted by (A) on (B);
4.2) on (A) to determine the force $\vec{F}_{B/A}$ exerted by (B) on (A).

5) Deduce that the principle of interaction is verified.

Exercise 2 (7 pts.)

Mechanical oscillations

A mechanical oscillator is formed of a block (S) of mass m and a horizontal light spring of force constant $k = 100$ N/m.
The spring is connected from one of its ends to a fixed support A.
(S) is attached to the other end of the spring and it may slide without friction on a horizontal surface (Doc. 2).



Doc. 2

At equilibrium, the center of mass (G) of (S) coincides with the origin O of the x-axis.

At an instant $t_0 = 0$, (G) is at O and (S) is launched, in the negative direction, with an initial velocity $\overrightarrow{V_0}$.
(G) thus performs mechanical oscillations.

At an instant t, the abscissa of (G) is $x = \overline{OG}$ and the algebraic value of its velocity is $v = x' = \frac{dx}{dt}$.

The curve of document 3 represents, as a function of time, one of these three forms of energy:
the kinetic energy, the elastic potential energy, or the mechanical energy of the system (Oscillator, Earth).
The aim of this exercise is to determine the value of m and the time equation of (G).
Take the horizontal plane containing (G) as a reference level for gravitational potential energy.

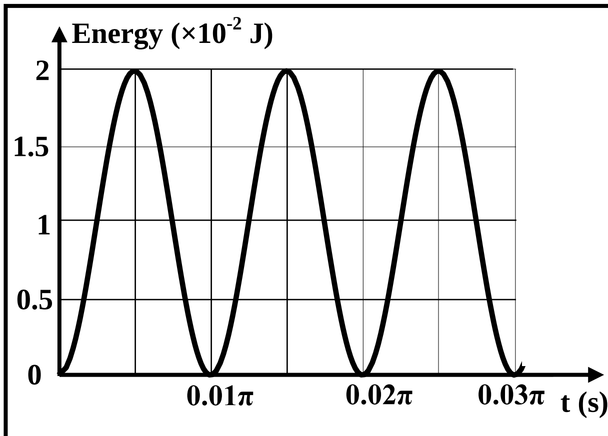
1) Specify the type of the oscillations of (G).

2) The curve of document 3 represents the elastic potential energy of the system (Oscillator, Earth) as a function of time. Why?

3) Use document 3 to answer the following questions:

3.1) Calculate the amplitude X_m of the oscillations of (G).

3.2) Knowing that the period T_{energy} of the elastic potential energy of the above system is half the proper (natural) period T_0 of oscillations of (G) $\left(T_{energy} = \frac{T_0}{2}\right)$, calculate T_0.



Doc. 3

4) The time equation of the motion of (G) is given by: $x = X_m \sin(\omega_0 t + \varphi)$, where φ is constant and ω_0 is the proper (natural) angular frequency of the oscillator.

4.1) Determine the value of φ.

4.2) Calculate the value of ω_0.

4.3) Deduce the expression of x as a function of time.

5) Knowing that $\omega_0 = \sqrt{\frac{k}{m}}$, calculate the value of m.

Exercise 3 (7 pts)

Brightness of a lamp

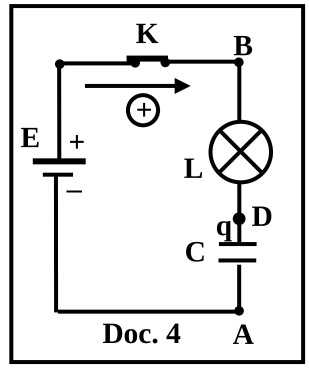
The aim of this exercise is to study the brightness of a lamp in two experiments.
For this purpose, consider:

- an ideal battery of electromotive force E = 9 V;
- a lamp L acting as a resistor of resistance R = 10 Ω;
- a capacitor of capacitance C = 0.1 F;
- a switch K.

Given that the brightness of the lamp increases with the increase of the current it carries and vice-versa.

1) First experiment: charging the capacitor

We connect the capacitor, initially uncharged, in series with the lamp and switch K across the battery (Doc. 4).



Doc. 4

Switch K is closed at $t_0 = 0$, and the capacitor starts charging.

1.1) Show that the differential equation that governs the variation of the voltage, $u_{DA} = u_C$, across the capacitor is: $E = RC\frac{du_C}{dt} + u_C$.

1.2) The solution of the obtained differential equation is of the form:

$u_C = E\left(1 - e^{\frac{-t}{\tau}}\right)$, where τ is constant.

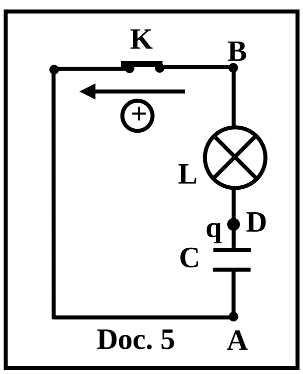
1.2.1) Determine the expression of τ in terms of R and C.

1.2.2) Calculate τ.

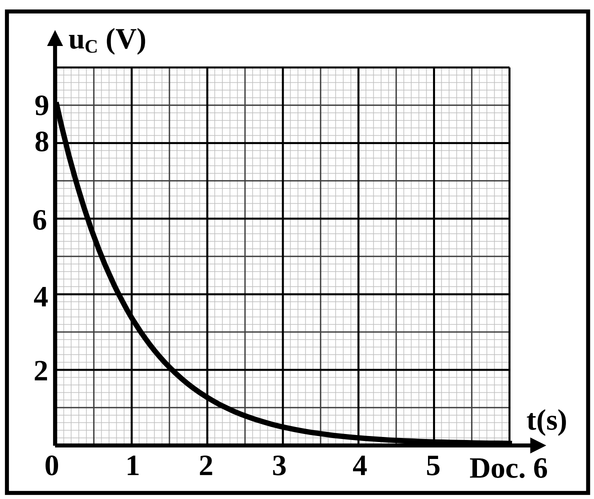
1.3) Deduce that the expression of the charge current is $i = 0.9\ e^{-t}$ (SI).

2) Second experiment: discharging the capacitor

The fully charged capacitor is connected in series with the lamp and switch K.
We close K at $t_0 = 0$ taken as a new initial time.
The capacitor discharges through the lamp (Doc. 5).
Document 6 shows the voltage $u_{DA} = u_C$ as a function of time.



Doc. 5



Doc. 6

2.1) Use document 6 to determine the value of the time constant τ' of this RC circuit.

2.2) Given that $u_C = E\ e^{\frac{-t}{\tau'}}$. Deduce the expression of the discharge current as a function of time.

3) Conclusion

Using parts **(1.3)** and **(2.2)**, describe the brightness of the lamp in the first and the second experiments during the time interval [0 , 5 s]. Justify your answer.

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أسس التصحيح - إنكليزي

Exercise 1 (6 pts) **Verification of the principle of interaction**

Part		Answer	Mark
1		Friction is neglected or the sum of the works done by the non-conservative forces is zero, therefore the mechanical energy is conserved.	0.5
2		ME is conserved , then $ME_C = ME_D$ $KE_C + GPE_C = KE_D + GPE_D$; ($V_C = 0$, then $KE_C = 0$ and $h_D = 0$, so $GPE_D = 0$) $0 + m_A g h_C = \frac{1}{2} m_A V_A^2 + 0$, then $V_A = \sqrt{2gh} = \sqrt{(2)(10)(0.2)} = 2$ m/s	1.5
3		During the collision, linear momentum is conserved: $\vec{P}_{before} = \vec{P}_{after}$ $m_A \vec{V}_A = m_A \vec{V}'_A + m_B \vec{V}'_B$ This is a head-on collision, then the velocities are collinear, so we can write the equation in the algebraic form: $m_A V_A = m_A V'_A + m_B V'_B$ $m_A (V_A - V'_A) = m_B V'_B$ ….**(equation 1)** The collision is elastic, then the kinetic energy is conserved: $KE_{before} = KE_{after}$ $\frac{1}{2} m_A V_A^2 = \frac{1}{2} m_A V_A'^2 + \frac{1}{2} m_B V_B'^2$, then $m_A (V_A^2 - V_A'^2) = m_B V_B'^2$ $m_A(V_A - V'_A)(V_A + V'_A) = m_B V_B'^2$ **(equation 2)** $\frac{\text{equation 2}}{\text{equaiton 1}}$: $V_A + V'_A = V'_B$ **(equation 3)** Replace V'_B in equation 1 by its expression in equation 2: $V'_A = \frac{m_A - m_B}{m_A + m_B} V_A$ $V'_A = \frac{0.2 - 0.6}{0.2 + 0.8} \times 2 = -1.2$m/s , hence $\vec{V}'_A = V'_A \vec{i} = -1.2 \vec{i}$ (m/s) Equation 3 : $V'_B = V'_A + V_A = -1.2 + 2 = 0.8$ m/s , so $\vec{V}'_B = V'_B \vec{i} = 0.8 \vec{i}$ (m/s) **<u>Or</u>** :$V'_B = \frac{2m_A}{m_A + m_B} V_A = \frac{2\ (0.2)}{0.2 + 0.8} \times 2 = 0.8$ m/s , so $\vec{V}'_B = V'_B \vec{i} = 0.8 \vec{i}$ (m/s)	2
4	4.1	Newton's 2nd law on (B): $\Sigma \vec{F}_{ext} = \frac{d\vec{P}_B}{dt}$, then $m_B \vec{g} + \vec{N}_B + \vec{F}_{A/B} = \frac{\Delta \vec{P}_B}{\Delta t}$; $m\vec{g} + \vec{N} = \vec{0}$ $\vec{F}_{A/B} = \frac{m_B \vec{V}'_B - m_B \vec{V}_B}{\Delta t} = \frac{0.8 \times 0.8\,\vec{i} - \vec{0}}{0.1} = 6.4\,\vec{i}$ (N)	1
	4.2	Newton's 2nd law on (A): $\Sigma \vec{F}_{ext} = \frac{d\vec{P}_A}{dt}$, then $m_A \vec{g} + \vec{N}_A + \vec{F}_{B/A} = \frac{\Delta \vec{P}_A}{\Delta t}$ $\vec{F}_{B/A} = \frac{m_A \vec{V}'_A - m_A \vec{V}_A}{\Delta t} = \frac{0.2\,(-1.2\,\vec{i}) - 0.2\,(-2\,\vec{i})}{0.1} = -6.4\,\vec{i}$ (N)	0.5
5		$\vec{F}_{A/B} = -\vec{F}_{B/A}$, then the principle of interaction is verified.	0.5

Exercise 2 (7pts) **Mechanical Oscillations**

<table>
<tr><th colspan="2">Part</th><th>Answer</th><th>Mark</th></tr>
<tr><td colspan="2">1</td><td>(S) moves without friction, then the type of oscillation is free undamped mechanical oscillation.</td><td>1</td></tr>
<tr><td colspan="2">2</td><td>At $t_o = 0$, (G) is at O, then $x = 0$; hence $EPE_o = \frac{1}{2}kx^2 = 0$.
Or: At $t_o = 0$, $V_o \neq 0$, then $KE_o \neq 0$. Also, $ME \neq 0$ at all instants. So, this curve does not represent KE and ME; therefore, the curve represents EPE versus time.</td><td>0.5</td></tr>
<tr><td rowspan="2">3</td><td>3.1</td><td>EPE is maximum when $x = X_m$, then $EPE_{max} = \frac{1}{2} k X_m^2$
$2\times10^{-2} = \frac{1}{2}(100) X_m^2$, then $X_m = 0.02$ m</td><td>1</td></tr>
<tr><td>3.2</td><td>Graphically: $T_{energy} = 0.01\pi$ (s)
$T_{energy} = \frac{T_0}{2}$, then $T_0 = 2 T_{energy} = 2 \times 0.01\pi = 0.02\pi$ s</td><td>0.75</td></tr>
<tr><td rowspan="3">4</td><td>4.1</td><td>$x = X_m \sin(\omega_0 t + \varphi)$
At $t_0 = 0$: $x_0 = 0$; $0.02 \sin(\varphi) = 0$, then $\varphi = 0$ or $\varphi = \pi$ rad

$v = \omega_0 X_m \cos(\omega_0 t + \varphi)$
$V_o = \omega_0 X_m \cos(\varphi)$
At $t_0 = 0$: $V_o < 0$ [(S) is launched in the negative direction]
But, $\omega_0 X_m > 0$, then $\cos(\varphi) < 0$
Therefore, the acceptable value is $\varphi = \pi$ rad.</td><td>1.5</td></tr>
<tr><td>4.2</td><td>$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{0.02\pi} = 100$ rad/s.</td><td>0.75</td></tr>
<tr><td>4.3</td><td>$x = 0.02 \sin(100 t + \pi)$, with x in (m) and t in (s)</td><td>0.5</td></tr>
<tr><td colspan="2">5</td><td>$\omega_0 = \sqrt{\frac{k}{m}}$, then $\omega_0^2 = \frac{k}{m}$, so $m = \frac{k}{\omega_0^2} = \frac{100}{100^2} = 0.01$ kg</td><td>1</td></tr>
</table>

Exercise 3 (7 pts) **Brightness of a lamp**

Part		Answer	Mark
1	1.1	$u_{BA} = u_{BD} + u_{DA}$, then $E = Ri + u_C$ $i = \frac{dq}{dt} = C\frac{du_C}{dt}$ Then , $E = RC\frac{du_C}{dt} + u_C$	1
	1.2.1	$u_C = E(1 - e^{\frac{-t}{\tau}}) = E - E\,e^{-\frac{t}{\tau}}$, then $\frac{du_C}{dt} = \frac{E}{\tau}\,e^{\frac{-t}{\tau}}$ Replacing u_C and $\frac{du_C}{dt}$ by their expressions in the differential equation, gives: $E = RC\,\frac{E}{\tau}\,e^{\frac{-t}{\tau}} + E - E\,e^{\frac{-t}{\tau}}$, then $E\,e^{\frac{-t}{\tau}}(\frac{RC}{\tau} - 1) = 0$ $E\,e^{\frac{-t}{\tau}} = 0$ is rejected Then, $(\frac{RC}{\tau} - 1) = 0$; therefore, $\tau = RC$	1.5
	1.2.2	$\tau = RC = 10 \times 0.1$, then $\tau = 1$ s	0.5
	1.3	$i = C\frac{du_C}{dt}$, then $i = C\,\frac{E}{\tau}\,e^{\frac{-t}{\tau}} = \frac{CE}{RC}\,e^{\frac{-t}{\tau}} = \frac{E}{R}\,e^{-\frac{t}{\tau}}$ $i = \frac{9}{10}\,e^{-\frac{t}{1}}$, so $i = 0.9\,e^{-t}$ SI	1
2	2.1	At t = τ' ; $u_C = 0.37 \times u_{C_{maximum}} = 0.37 \times 9 = 3.33$ V. Graphically, $u_C = 3.33$ V at t = 1 s , then τ' = 1 s	1
	2.2	$i = -\frac{dq}{dt} = -C\,\frac{du_C}{dt} = C\,\frac{E}{\tau'}\,e^{-\frac{t}{\tau'}}$ Then, $i = 0.1 \times \frac{9}{1}e^{-\frac{t}{1}} = 0.9\,e^{-t}$	1
3		In both experiments, $i = 0.9e^{-t}$, then the current decreases with time; therefore, the brightness of the lamp decreases. **<u>Or:</u>** Experiment 1 : $i = 0.9e^{-t}$; For t = 0, i = 0.9A and for t = 5 s, i ≅ 0. Then, the current decreases with time; therefore, the brightness of the lamp decreases. Same explanation in experiment 2.	1