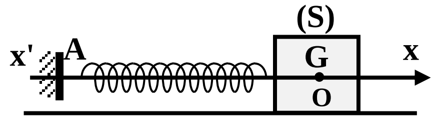
مسابقة في مادة الفيزياء	الاسم:	
المدة: ساعة ونصف	الرقم:	

This exam is formed of three obligatory exercises in three pages.
The use of non-programmable calculator is recommended.

Exercise 1 (7 pts) Mechanical oscillator

A mechanical oscillator is constituted of a block (S) of mass M and a spring of negligible mass and force constant k.
The spring, placed horizontally, is connected from one of its extremities to a fixed support A. (S) is attached to the other extremity of the spring and it may slide without friction on a horizontal surface (Doc. 1).



Doc. 1

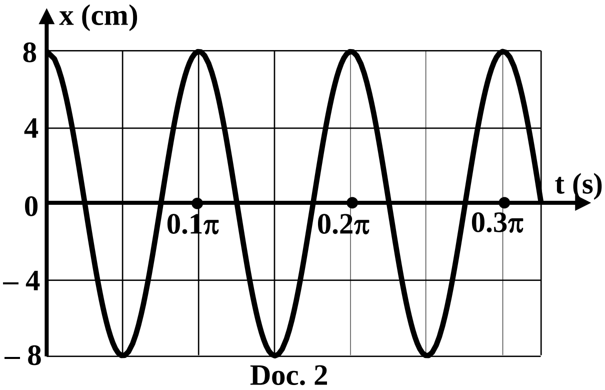
The aim of this exercise is to determine the values of M and k.
At equilibrium, the center of mass G of (S) coincides with the origin O of the x-axis.
(S) is shifted from its equilibrium position in the positive direction and then released without initial velocity at the instant $t_0 = 0$. Thus, (S) performs mechanical oscillations. At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity is $v = x' = \frac{dx}{dt}$.
The horizontal plane containing G is considered as a reference level for gravitational potential energy.

1) Write, at an instant t, the expression of the mechanical energy ME of the system (Oscillator, Earth) in terms of x, M, k and v.

2) Establish the second order differential equation in x that governs the motion of G.

3) Deduce that the expression of the proper (natural) period of the oscillations is $T_1 = 2\pi\sqrt{\frac{M}{k}}$.

4) An appropriate device traces x as a function of time (Doc. 2).
Referring to document 2, indicate:
4.1) the type of oscillations of G;
4.2) the amplitude X_m of the oscillations;
4.3) the value of T_1.



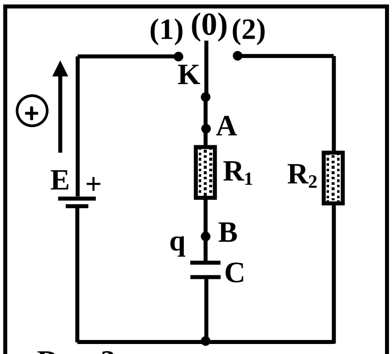
Doc. 2

5) The same experiment is repeated by putting on (S) an object, considered as a particle, of mass m = 50 g. The duration of 10 oscillations becomes $\Delta t = 3.67$ s.
5.1) Write the expression of the new proper (natural) period T_2 of the oscillations in terms of k, M and m.
5.2) Using the expressions of T_1 and T_2, show that $k = \frac{4\pi^2 m}{T_2^2 - T_1^2}$.
5.3) Determine the values of k and M.

Exercise 2 (7 pts) Charging and discharging a capacitor

The aim of this exercise is to study the charging and the discharging of a capacitor.
For this purpose, we set up the circuit of document 3 that includes:

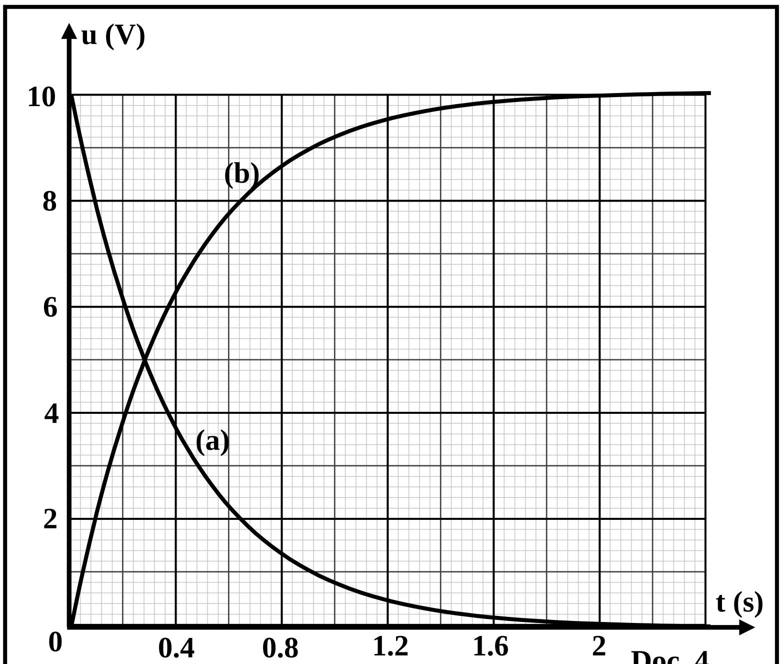
- an ideal battery of electromotive force E = 10 V;
- two resistors of resistances $R_1 = R_2 = 4\ k\Omega$;
- a capacitor of capacitance C;
- a switch K.



Doc. 3

1) Charging the capacitor

The switch K is initially at position (0) and the capacitor is uncharged.
At the instant $t_0 = 0$, K is turned to position (1) and the charging process of the capacitor starts.
At an instant t, plate B of the capacitor carries a charge q and the circuit carries a current i.
An appropriate device allows us to display the voltage $u_{AB} = u_{R_1}$ across the resistor and the voltage $u_{BD} = u_C$ across the capacitor.
Curves (a) and (b) of document 4 show these voltages as functions of time.



Doc. 4

1.1) Curve (a) represents u_{R_1} and curve (b) represents u_C. Justify.

1.2) The time constant of this circuit is given by $\tau_1 = R_1 C$.

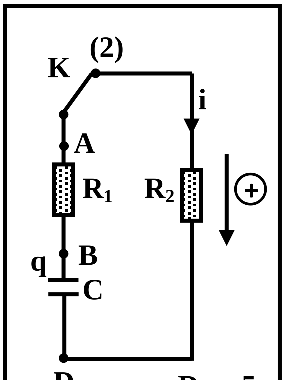
1.2.1) Using document 4, determine the value of τ_1.

1.2.2) Deduce the value of C.

1.3) Calculate the time « t_1 » needed by the capacitor to practically become completely charged.

2) Discharging the capacitor

The capacitor is completely charged. At an instant taken as a new initial time $t_0 = 0$, the switch K is turned to position (2), and the capacitor starts discharging through the resistors of resistances R_1 and R_2. At an instant t the circuit carries a current i (Doc. 5).



Doc. 5

2.1) Show, using the law of addition of voltages, that the differential equation which governs u_C is:

$$RC\frac{du_C}{dt} + u_C = 0 \text{ where } R = R_1 + R_2.$$

2.2) The solution of this differential equation is of the form: $u_C = E\ e^{\frac{-t}{\tau_2}}$ where τ_2 is the time constant of the circuit of document 5.
Determine the expression of τ_2 in terms of R and C.

2.3) Verify that the time needed by the capacitor to practically become completely discharged is $t_2 = 5\tau_2$.

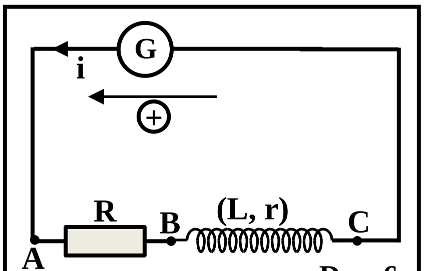
3) Duration of charging and discharging the capacitor

Show, without calculation, that « t_2 » is greater than « t_1 ».

Exercise 3 (6 pts) **Characteristics of a coil**

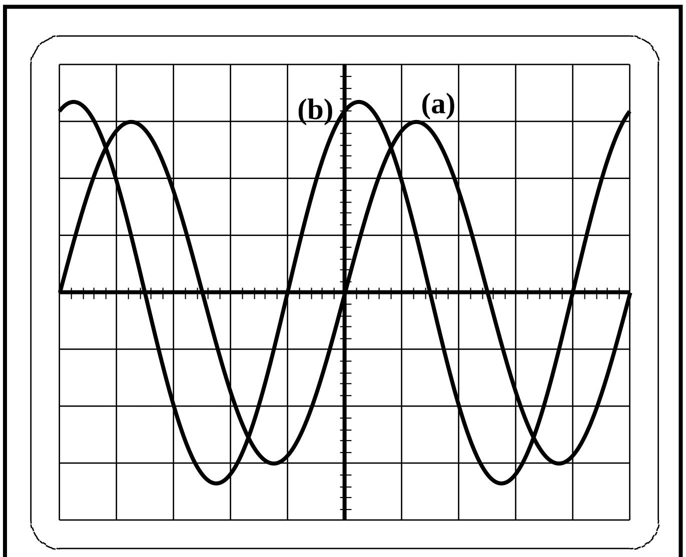
In order to determine the inductance L and the resistance r of a coil, we connect it in series with a resistor of resistance R = 30 Ω across a function generator (G) providing an alternating sinusoidal voltage of angular frequency ω.
The circuit thus carries an alternating sinusoidal current of expression $i = I_m \sin(\omega t)$ (Doc. 6).
An oscilloscope allows us to display the voltage $u_{AB} = u_R$ across the resistor and the voltage $u_{BC} = u_L$ across the coil.
The obtained waveforms are shown in document 7.
The adjustments of the oscilloscope are:

- vertical sensitivity for both channels: S_v = 2 V/div;
- horizontal sensitivity: S_h = 0.4 ms/div.



Doc. 6

1) The voltage u_R represents the image of i. Why?
2) Referring to document 7, specify which of the curves, (a) or (b), leads the other.
3) Deduce that curve (a) corresponds to u_{AB}.
4) Using document 7, determine:
  4.1) the angular frequency ω;
  4.2) the maximum value I_m of i ;
  4.3) the phase difference φ between u_L and i.
5) Prove that $u_L = 6.8 \sin(\omega t + 0.4\pi)$ (SI).
6) Knowing that the voltage across the coil is given by $u_L = r\,i + L\frac{di}{dt}$, write the expression of u_L in terms of r, L, ω and t.
7) Using the two expressions of u_L found in parts 5 and 6 and by giving «ωt» two particular values, determine the values of L and r.



Doc.7

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Exercise 1 (7 pts) **Mechanical oscillator**

Part		Answer	Note
1		$ME = KE + EPE = \frac{1}{2} M v^2 + \frac{1}{2} kx^2$	0.5
2		The sum of the works done by the non-conservative forces is zero, then the mechanical energy is conserved. (Or: Friction is neglected, then the mechanical energy is conserved). ME = constant , then $\frac{dME}{dt} = 0$, so $M v v' + k x x' = 0$, but $v = x'$ and $v' = x''$, hence $v (M x'' + k x) = 0$ $v = 0$ is rejected , then $x'' + \frac{k}{M} x = 0$	1
3		The differential equation is of the form: $x'' + \omega_0^2 x = 0$, with $\omega_o = \sqrt{\frac{k}{M}}$ $T_1 = \frac{2\pi}{\omega_o}$; therefore, $T_1 = 2\pi\sqrt{\frac{M}{k}}$	1
4	4.1	Free undamped mechanical oscillations	0.5
	4.2	$X_m = 8$ cm	0.5
	4.3	From the curve: $T_1 = 0.1\,\pi$ s = 0.314 s	0.5
5	5.1	$T_2 = 2\pi\sqrt{\frac{M+m}{k}}$	0.5
	5.2	$T_1^2 = 4\pi^2 \frac{M}{k}$ and $T_2^2 = 4\pi^2 (\frac{M+m}{k})$ $T_2^2 - T_1^2 = 4\pi^2 (\frac{M+m}{k} - \frac{M}{k}) = \frac{4\pi^2 m}{k}$, so $k = \frac{4\pi^2 m}{(T_2^2 - T_1^2)}$	1
	5.3	$T_2 = \frac{3.67}{10} = 0.367$ s $k = \frac{4\pi^2 \times 0.05}{0.367^2 - 0.314^2}$, then $k = 54.7$ N/m $T_1^2 = 4\pi^2 \frac{M}{k}$, substituting the value of k into this expression gives: $0.314^2 = 4\pi^2 \frac{M}{54.7}$; therefore, $M = 0.1366$ kg = 136.6 g	0.5 0.5 0.5

Exercise 2 (7 pts) **Charging and discharging of a capacitor**

Part		Answer	Note
1	1.1	Curve (a): $u_{AB} = u_{R_1} = R_1 i$; u_{R_1} is directly proportional to the current in the circuit. During the charging process the current decreases so u_{R_1} decreases. Curve (b) : $u_{BD} = u_C = \frac{q}{C}$; During charging process q increases so u_C increases	0.5 0.5
	1.2.1	At $t = \tau_1$: $u_C = 0.63\,E = 6.3$ V From document 4: $u_C = 6.3$ V at $t = 0.4$ s , then $\tau_1 = 0.4$ s	1
	1.2.2	$\tau_1 = R_1 C$, so $C = \frac{\tau_1}{R_1} = \frac{0.4}{4000}$, hence $C = 1 \times 10^{-4}$ F = 100 μF	0.5
	1.3	$t_1 = 5\tau_1 = 5 \times 0.4$, then $t_1 = 2$ s	0.5
2	2.1	$u_{BD} = u_{BA} + u_{AD}$ $u_C = R_1 i + R_2 i$, then $u_C = (R_2 + R_1)\, i = R\, i$ But, $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$, hence $u_C = -RC\frac{du_C}{dt}$ Therefore , $RC\frac{du_C}{dt} + u_C = 0$	1.5
	2.2	$u_C = E\, e^{\frac{-t}{\tau_2}}$, then $\frac{du_C}{dt} = -\frac{E}{\tau_2} e^{\frac{-t}{\tau_2}}$ Substituting u_C and $\frac{du_C}{dt}$ into the differential equation gives: $RC(-\frac{E}{\tau_2} e^{\frac{-t}{\tau_2}}) + E\, e^{\frac{-t}{\tau_2}} = 0$, so $E\, e^{\frac{-t}{\tau_2}}(1 - \frac{RC}{\tau_2}) = 0$ $E\, e^{\frac{-t}{\tau_2}} = 0$ is rejected , then $1 - \frac{RC}{\tau_2} = 0$, so $\tau_2 = RC$	1.5
	2.3	At $t = 5\,\tau_2$: $u_C = E\, e^{\frac{-5\tau_2}{\tau_2}} = E\, e^{-5} \cong 0$, so the capacitor is practically completely. discharged.	0.5
3		$t_1 = 5\, R_1 C$ and $t_2 = 5\, RC = 5\,(R_1 + R_2)\, C$ $(R_1 + R_2) > R_1$, then $t_2 > t_1$	0.5

Exercise 3 (6 pts) **Characteristics of a coil**

Part		Answer	Note
1		$u_R = Ri$, but R is a positive constant , then u_R and i are directly proportional ; therefore, u_R is the image of current.	0.5
2		Curve (b) leads curve (a), since curve (b) becomes maximum before curve (a).	0.5
3		The voltage across the coil u_L leads u_R (or i). Curve (b) leads curve (a), then curve (a) corresponds to $u_R = u_{AB}$.	0.5
4	4.1	$T = 5 \times 0.4 = 2$ ms $= 2\times10^{-3}$ s $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\times10^{-3}}$, hence $\omega = 1000\,\pi$ rad/s	0.25 0.5
	4.2	Curve (a): $U_{R(max)} = 3 \times 2 = 6$ V $U_{R(max)} = R \times I_m$, then $I_m = \frac{6}{30} = 0.2$ A	0.25 0.5
	4.3	$\varphi = \frac{2\pi d}{D} = \frac{2\pi\times1}{5}$, then $\varphi = 0.4\,\pi$ rad	0.5
5		From curve (b): $U_{L(max)} = 3.4 \times 2 = 6.8$ V , and u_L leads i by $\varphi = 0.4\pi$ rad $u_L = U_{L(max)} \sin(\omega t + \varphi)$; therefore, $u_L = 6.8 \sin(\omega t + 0.4\,\pi)$	0.25 0.25
6		$u_L = r\,i + L\frac{di}{dt} = r I_m \sin(\omega t) + L I_m \omega \cos(\omega t)$ $u_L = 0.2\, r \sin(\omega t) + L\,(0.2)\,(1000\pi) \cos(\omega t) = 0.2\, r \sin(\omega t) + 200\pi L \cos(\omega t)$ (SI) **<u>Or</u>** $u_L = 0.2\, r \sin(\omega t) + \omega L\,(0.2) \cos(\omega t)$ (SI)	0.5
7		$6.8 \sin(\omega t + 0.4\,\pi) = 0.2\, r \sin(\omega t) + 200\,\pi L \cos(\omega t)$ For $\omega t = 0$: $6.8 \sin(0.4\,\pi) = 0 + 200\pi L$, then $L = 0.01$ H For $\omega t = \frac{\pi}{2}$ rad : $6.8 \sin(\frac{\pi}{2} + 0.4\,\pi) = 0.2\, r + 0$, then $r = 10.5\ \Omega$	0.75 0.75