مسابقة في مادة الرياضيات — الاسم:
المدة: ساعة ونصف — الرقم:

ملاحظة: - يتكوّن هذا الامتحان من ست مسائل، يجب اختيار **أربع مسائل** منها فقط.
- في حال الإجابة عن أكثر من أربع مسائل، عليك شطب الإجابات المتعلّقة بالمسألة التي لم تعد من ضمن اختيارك، لأنّ التصحيح سيقتصر على إجابات المسائل الأربع الأولى غير المشطوبة.
- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- MCQ (5 points)

In the table below, only one among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Proposed answers		
		a	b	c
1	$\lim\limits_{x\to-\infty} \dfrac{e^x+2}{e^x+1} =$	$+\infty$	1	2
2	The domain of definition of the function f given by $f(x) = \dfrac{\ln(x-4)}{x-5}$ is	$]0;+\infty[$	$]4;+\infty[$	$]4;5[\cup]5;+\infty[$
3	The derivative of the function f defined over $\mathbb{R}$ as $f(x) = \ln(2+e^{-x})$ is	$\dfrac{-1}{1+2e^x}$	$\dfrac{1}{2+e^{-x}}$	$-e^{-x}$
4	An urn contains 7 balls: 4 red balls and 3 black balls. The number of ways to select, at random successively and with replacement 3 red balls and 1 black ball is	72	768	192
5	The number of solutions of the equation $(\ln x)^2 = 4\ln x$ is	0	1	2

II- Probability (5 points)

In a university, a study on the usage of Artificial Intelligence Apps COPILOT and GEMINI showed that:

- 60% of the students use the COPILOT among whom 30% use the GEMINI.
- 40% of the students do not use COPILOT among whom 50% use GEMINI.

A student is randomly selected from the university.
Consider the following events:
C: "The selected student uses COPILOT"
G: "The selected student uses GEMINI"

1) a) Show that the probability $P(C\cap G) = 0.18$ and calculate $P(\overline{C}\cap G)$.
b) Calculate $P(G)$.

2) Knowing that the selected student did not use GEMINI, calculate the probability that he/she uses COPILOT.

3) Calculate $P(C\cup G)$.

4) The number of students in this university is 400.
a) Show that the number of students that use both COPILOT and GEMINI is 72.
b) Four students are randomly and simultaneously selected. Calculate the probability that exactly one student among the four uses both COPILOT and GEMINI.

III- Probability (5 points)

U and V are two urns:

- U contains 3 red and 2 black balls.
- V contains 3 red and 3 black balls.

One urn is randomly chosen, then 3 balls are simultaneously and randomly selected from the chosen urn.

The following events are considered:

U: "The selected urn is U"

R: "The three selected balls are red"

1) **a)** Calculate the probability P(R / U) and deduce that $P(R \cap U) = \frac{1}{20}$.

b) Calculate $P(R \cap \overline{U})$ and show that P(R) = 0.075.

2) Knowing that the three selected balls are not red, calculate the probability that they are selected from U.

3) The balls from the two urns U and V are placed in the same urn W and then three balls are selected randomly and successively without replacement from W.

a) Verify that the number of possible cases is 990.

b) Calculate the probability of having at least one red ball among the three selected balls.

IV- Functions (5 points)

Consider the function f defined over $\mathbb{R}$ as $f(x) = (-x - 2)e^{-x} + 2$ and denote by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

1) **a)** Determine $\lim_{x \to -\infty} f(x)$ and calculate f(–2.5).

b) Show that $\lim_{x \to +\infty} f(x) = 2$. Deduce an asymptote (d) to (C).

2) **a)** Show that $f'(x) = (x + 1)e^{-x}$ then set up the table of variations of f.

b) Calculate f(0) and show that f(x) = 0 has, on $]-\infty; -1[$, a unique root α.

c) Verify that $-1.6 < \alpha < -1.5$.

3) Calculate f(–2) then draw (d) and (C).

4) Let g be the function given by $g(x) = \ln[f(x) - 2]$.

a) Determine the domain of definition of g.

b) Prove that g is strictly decreasing.

V- Functions (5 points)

Consider the function f defined on]0 ; +∞[as $f(x) = \frac{\ln x}{x} + x + 1$ and denote by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

Let (d) be the line with equation y = x + 1.

1) Determine $\lim\limits_{\substack{x \to 0 \\ x > 0}} f(x)$. Deduce an asymptote to (C).

2) **a)** Determine $\lim\limits_{x \to +\infty} f(x)$.

b) Show that (d) is an asymptote to (C) at +∞.

c) Study, according to the values of x, the relative positions of (C) and (d).

3) Complete the following table of variations of f:

x	0		+∞
f '(x)		+	
f(x)			

4) **a)** Show that the equation f(x) = 0 has a unique solution α.

b) Verify that 0.4 < α < 0.5.

5) Draw (d) and (C).

VI- Exponential Functions and Integrals (5 points)

Consider the function f defined over ℝ as $f(x) = x(1 - e^{-x}) - 1$ and denote by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

Let (d) be the line with equation y = x – 1.

1) Calculate f(–1.5).

2) **a)** Show that (d) is an asymptote to (C) at +∞.

b) Study, according to the values of x, the relative positions of (C) and (d).

3) Copy and complete the following table of variations of f:

x	–∞		0		+∞
f '(x)		–	0	+	
f(x)	+∞				+∞

4) Show that the equation f(x) = 0 has exactly two solutions α and β such that –0.9 < α < –0.8 and 1.3 < β < 1.4.

5) Draw (d) and (C).

6) **a)** Calculate $f'(x) + f(x)$.

b) Calculate, in terms of α, the area of the region limited by (C), the x-axis, the y-axis and the line with equation x = α.

عدد المسائل: ست	مسابقة في مادة الرياضيات المدة: ساعة ونصف	الاسم: الرقم:

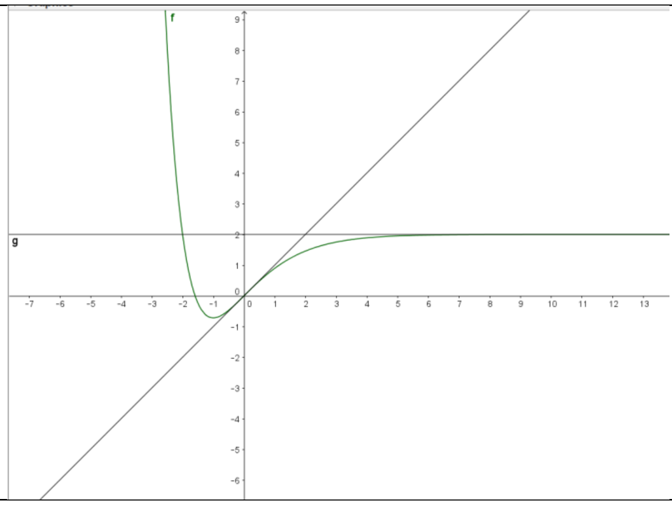
مشروع أسس التصحيح

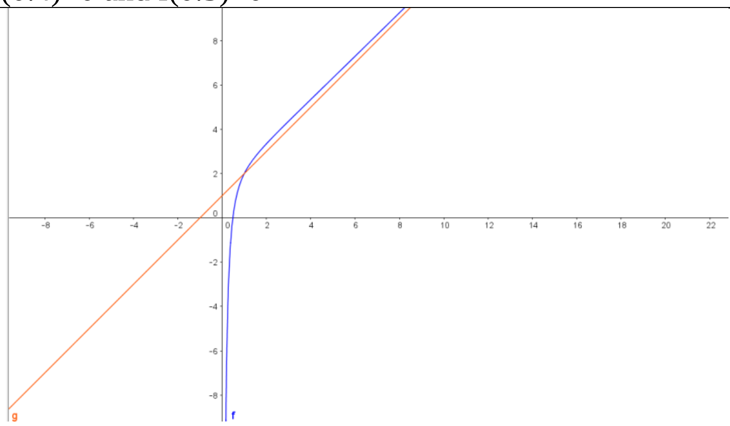
	Q1: Réponses	7.5 pts
1	$\lim_{x\to-\infty}\frac{e^x+2}{e^x+1}=\frac{0+2}{0+1}=2$ Answer : c	1.5
2	$f(x)=\frac{\ln(x-4)}{x-5}$ is defined for $\begin{cases}x-4>0\\x-5\neq0\end{cases}$ so $\begin{cases}x>4\\x\neq5\end{cases}$ Thus, $x\in\ ]4;5[\ \cup\ ]5;+\infty[$ Solution: c	1.5
3	$f'(x)=\frac{-e^{-x}}{2+e^{-x}}=\frac{-1}{2e^x+1}$ Answer: a	1.5
4	RRRB, then $4^3\times3^1\times\frac{4!}{3!}=768$ Answer: b	1.5
5	$\ln^2(x)=4\ln(x)$, then $\ln(x)\,(\ln(x)-4))=0$, then $\ln(x)=0$ or $\ln(x)=4$ Then $x=1$ or $x=e^4$ Answer: c	1.5

	Q2: Réponses	7.5 pts
1a	$P(G/C)=0.3$ $P(C\cap G)=P(G/C)\times P(C)=(0.3)(0.6)=0.18$ $P(\overline{C}\cap G)=P(G/\overline{C})\times P(\overline{C})=(0.5)(0.4)=0.2$	1.5
1b	$P(G)=P(C\cap G)+P(\overline{C}\cap G)=0.18+0.2=0.38$	0.75
2	$P(C/\overline{G})=\frac{P(C\cap\overline{G})}{P(\overline{G})}=\frac{0.6\times0.7}{1-0.38}=\frac{21}{31}$	1.5
3	$P(C\cup G)=P(C)+P(G)-P(C\cap G)=0.6+0.38-0.18=0.8$	1.5
4a	$P(C\cap G)=0.18$ Then $N=0.18\times400=72$	0.75
4b	P(1 out of 4 uses both AI’s) = $\frac{C_{72}^1\times C_{328}^3}{C_{400}^4}=0.399$	1.5

	Q3: Réponses	7.5 pts
1a	$P(R/U) = \frac{C_3^3}{C_5^3} = \frac{1}{10}$ $P(R \cap U) = P(R/U) \times P(U) = \frac{1}{10} \times \frac{1}{2} = \frac{1}{20}$	1.5
1b	$P(R/V) = \frac{C_3^3}{C_6^3} = \frac{1}{20}$ $P(R \cap \overline{U}) = P(R/V) \times P(V) = \frac{1}{20} \times \frac{1}{2} = \frac{1}{40}$ $P(R) = P(R \cap U) + P(R \cap V) = P(R/U) \times P(U) + P(R/V) \times P(V) = 0.075$	2.25
2	$P(U/\overline{R}) = \frac{P(U \cap \overline{R})}{P(\overline{R})} = \frac{P(U) - P(U \cap R)}{1 - P(R)} = \frac{18}{37}$	1.5
3a	$A_{11}^3 = 990$	0.75
3b	$P(\text{at least one red}) = 1 - P(\text{none is red}) = 1 - \frac{A_6^3}{A_{11}^3} = \frac{29}{33}$	1.5

<table>
<tr><th></th><th>Q4: Réponses</th><th>7.55 pts</th></tr>
<tr><td>1a</td><td>$\lim_{x \to -\infty} [(-x-2)e^{-x}+2] = (+\infty)(+\infty) + 2 = +\infty$
$f(-2,5) = 8,09$</td><td>1.5</td></tr>
<tr><td>1b</td><td>$\lim_{x \to +\infty} [(-x-2)e^{-x}+2] = \lim_{x \to +\infty} [(-x)e^{-x} - 2e^{-x}+2] = 2$
Then y=2 is the equation of a horizontal asymptote to (C) at $+\infty$</td><td>0.75</td></tr>
<tr><td>2a</td><td>$f'(x) = -e^{-x} - (-x-2)e^{-x} = (-1+x+2)e^{-x} = (x+1)e^{-x}$
<table>
<tr><td>x</td><td colspan="2">-1</td></tr>
<tr><td>$f'(x)$</td><td>-</td><td>+</td></tr>
<tr><td>$f(x)$</td><td>↘</td><td>→</td></tr>
</table>
f(-1)= -e+2</td><td>1.5</td></tr>
<tr><td>2b</td><td>f(0)= 0</td><td>0.75</td></tr>
<tr><td>2c</td><td>Since f is continuous and strictly decreasing over $]-\infty, -1[$ from $+\infty$ to -e+2<0 then f(x)=0 has a unique solution over this interval x=0. Since f is continuous and strictly increasing over $]-1, +\infty[$ $from -e+2 < 0$ to $2 > 0$ then f(x)=0 has a unique solution α .</td><td>0.5</td></tr>
<tr><td>3</td><td>$f''(x) = e^{-x} - (x+1)e^{-x} = e^{-x}(-x)$
Since f''(x) vanishes and changes sign at x=0 from positive to negative the (C) has an inflection point O(0,0)</td><td>0.75</td></tr>
<tr><td>4a</td><td>The equation of the tangent to (C) at x=0 is: y=f'(0)(x-0)+f(0)=x</td><td>0.75</td></tr>
<tr><td>4b</td><td></td><td>0.75</td></tr>
</table>

5		1.5
6a	g is defined for f(x)>2 that is for x<-2	0.5
6b	$g'(x) = \frac{f'(x)}{f(x)-2}$ For x<-2, f'(x) is negative anf f(x)-2 >0 so g'(x)<0 . Thus g is always decreasing	0.5

	Q5: Réponses	5 pts
1	$\lim_{x\to 0^+}[(\frac{lnx}{x}+x+1]=\frac{-\infty}{0}+0+1=-\infty$ thus the y-axis is a vertical asymptote for (C)	1
2a	$\lim [\frac{lnx}{x}+x+1]=$ 0+ +∞ +1= +∞	1
2b	$\lim_{x\to+\infty}[\frac{lnx}{x}+x+1-x-1]=\lim_{x\to 0^+}[\frac{lnx}{x}]=0$ Thus, (d) is an asymptote to (C) at +∞	1
2c	Since $\frac{lnx}{x}>0$ *for* $x>1$ then (C) is above (d) for x>1	1
3	x: 0 … +∞ $f'(x)$: + $f(x)$: ↗	0.5
4a	Since f is continuous and strictly increasing over $]0,+\infty[$ from $-\infty$ to $+\infty$ then f(x)=0 has a unique solution ∝.	1
4b	f(0.4)<0 and f(0.5)>0	0.5
5	(C) and (d) intersect at (1,2)	1.5
6a	f(∝) = 0 then $\frac{ln\propto}{\propto}+\propto+1=0$ so $ln\propto+\propto^2+\propto=0$ thus $ln\propto=-\propto-\propto^2$	0.5

6b	$f(\propto^2) = \frac{\ln(\propto^2)}{\propto^2}+\propto^2+1 = \frac{2ln\propto}{\propto^2}+\propto^2+1 = \frac{2(-\propto^2-\propto)}{\propto^2}+\propto^2+1$= -2-$\frac{2}{\propto}+\propto^2+1$ $f(\propto^2) -\propto^2= -1-\frac{2}{\propto} < 0$	0.5

	Q6: Réponses	5 pts
1	$\lim_{x\to-\infty} [x(1-e^{-x})-1] = +\infty - 1 = +\infty$ $f(-1{,}5) = 4{,}22$	1
2a	$\lim_{x\to+\infty} [x(1-e^{-x})-1] = \lim_{x\to+\infty} [x-(x)e^{-x}-1 = +\infty$	1
2b	$\lim_{x\to+\infty} [x(1-e^{-x})-1-x+1] = \lim_{x\to+\infty} [-x(e^{-x})] = 0$ Thus, (d) is an asymptote to (C) at $+\infty$	1
2c	$f(x)-x+1= -xe^{-x}$ (C) is above (d) for x>0 (C) is below (d) for x<0 (C) and (d) intersect at (0,1)	1
3a	$f'(x) = 1-e^{-x}+xe^{-x}$ f(0)=0	1
3b	x: 0 … $+\infty$ $f'(x)$: + $f(x)$: ↗	0.5
4)		
4	The tangent to (C) is parallel to (d) then f '(x)=1 So $e^{-x}(x-1)=0$ so x=-1 and y=-e+2 (T): y=1(x+1)-1/e+2	1
5		1
6a	$f'(x)+f(x) = 1-e^{-x}+xe^{-x}+x-1-xe^{-x} = -e^{-x}+x$	0.5
6b	A = $\int_0^1 f(x) =$	0.5