وزارة التربية والتعليم العالي
المديريّة العامّة للتربية
دائرة الامتحانات الرسميّة

امتحانات الشهادة الثانوية العامة
فرع: علوم الحياة

دورة العام ٢٠٢٣ العاديّة
الاثنين ١٠ تموز ٢٠٢٣

عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة ونصف	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the table below, only one among the proposed answers to each question is correct.

Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Proposed answers		
		a	**b**	**c**
1	The solution of the equation $2\ln(x) = \ln(25)$ is	5	$\frac{25}{2}$	–5
2	Consider the function f defined over]e , +∞[as $f(x) = x - 3 - \frac{3\ln x}{1 - \ln x}$ and denote by (C) its curve in an orthonormal system $(O ; \vec{i}, \vec{j})$. (C) admits two asymptotes with equations	x = 1 and y = x – 3	x = e and y = x	x = – e and y = x – 3
3	The table below is the table of variations of a continuous function f over [0 , +∞[. x: 0 … 1 … e … +∞ f '(x): – 0 + 0 – f(x): 5 ↘ –2 ↗ 3 ↘ –1 The image of the interval I = [1 , +∞[by f is	[–2 , 3]	[–2 , –1[	]–1 , 3]
4	A code is a number formed of three digits using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The number of possible even codes greater than or equal to 300 is	280	350	500

II- (6 points)

During the financial crisis in Lebanon, a study on a group of teachers showed that:

- 30% currently work abroad out of which:
  - 40% teach at private schools only.
  - 20% teach at public schools only.
  - The remaining teachers started working in another domain.
- Out of those who stayed in Lebanon:
  - 50% teach at private schools only.
  - 40% teach at public schools only.
  - The remaining teachers retired and stopped working.

A member from the group is randomly interviewed. Consider the following events:

A: “The interviewed member currently work abroad”

R: “The interviewed member teaches at private schools only”

U: “The interviewed member teaches at public schools only”

D: “The interviewed member works in another domain”

N: “The interviewed member is retired and stopped working”.

1) **a)** Calculate the probabilities $P(A \cap R)$ and $P(\overline{A} \cap R)$.

b) Verify that $P(R) = 0.47$.

2) Calculate P(U).

3) Show that the probability that the interviewed member is still teaching or is working in another domain is 0.93.

4) The interviewed member does not teach at private schools.
Calculate the probability that the member stayed in Lebanon.

5) The group consists of 500 teachers.

a) Show that the number of teachers that teach at private schools is 235.

b) Three members are interviewed from this group.
Calculate the probability of interviewing at least two members who teach at private schools.

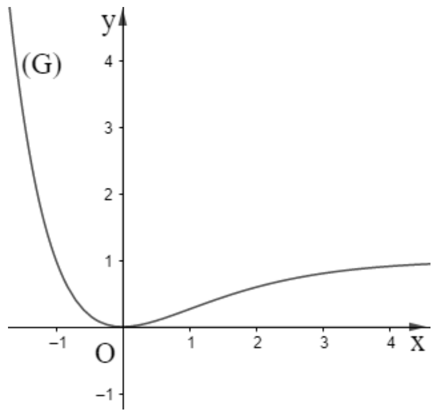
III- (10 points)

The plane is referred to an orthonormal system $(O\,;\vec{i},\vec{j})$.

Part A

The adjacent curve (G) is the representative curve of a differentiable function g over $]-\infty;+\infty[$.

(G) is tangent to the x-axis at O.



1) Using the curve (G):

a) Verify that $g(x) \geq 0$ for all real numbers x.

b) The function g' is the derivative of g.
Study, according to the values of x, the sign of g'.

2) Knowing that $g(x) = (ax + b)e^{-x} + 1$ where a and b are two real numbers, show that $a = b = -1$.

Part B

Consider the function f defined, on $\mathbb{R}$, as $f(x) = (x + 2)e^{-x} + x$.

Denote by (C) the representative curve of f.

Let (D) be the line with equation $y = x$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$ and calculate $f(-2.5)$.

2) **a)** Determine $\lim\limits_{x\to+\infty} f(x)$.

b) Show that the line (D) is an asymptote to (C).

c) Study, according to the values of x, the position of (C) with respect to (D).

3) Show that $f'(x) = g(x)$ then set up the table of variations of f.

4) **a)** Show that the equation $f(x) = 0$ has, on $\mathbb{R}$, a unique root α.

b) Verify that $-1.7 < \alpha < -1.6$.

5) **a)** Prove that (C) has an inflection point W whose coordinates are to be determined.

b) Show that the line (T) with equation $y = 2$ is tangent to (C) at W.

6) Draw (T), (D) and (C).

7) Consider the function h defined over $]-2\,,0[$ as $h(x) = \dfrac{\ln(x+2) - \ln(-x)}{x}$.

Prove that $h(\alpha)$ is a natural number to be determined.

عدد المسائل: ثلاث

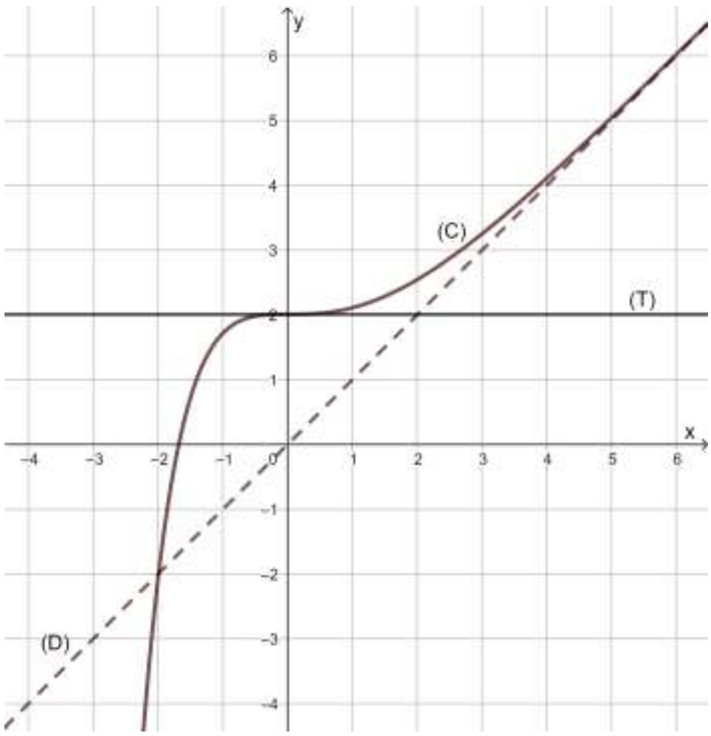
اسس تصحيح مادة الرياضيات

I	Answers	Grade 4 pts
1	$\ln x^2 = \ln 25$; $x > 0$ $x^2 = 25$ then $x = 5$ Answer : a	**1**
2	$\lim\limits_{x \to e^+} f(x) = e - 3 - \frac{3}{0^+} = -\infty$, so $x = e$ is a vertical asymptote. <u>OR:</u> $\lim\limits_{x \to +\infty} [f(x) - x] = -3 + 3 = 0$, so $y = x$ is an oblique asymptote at $+\infty$. Answer: b	**1**
3	f changes variation over $I = [1; +\infty[$, so $f(I) = [\min(f) ; \max(f)] = [-2; 3]$. Answer: a	**1**
4	Number of possible even codes greater or equal to 300 is: $7 \times 10 \times 5 = 350$. Answer: b	**1**

II	Answers	Grade 6 pts
1a	$P(A \cap R) = P(A) \times P(R/A) = 0.3 \times 0.4 = 0.12$ $P(\overline{A} \cap R) = P(\overline{A}) \times P(R/\overline{A}) = 0.7 \times 0.5 = 0.35$	**0.5** **0.5**
1b	$P(R) = P(A \cap R) + P(\overline{A} \cap R) = 0.12 + 0.35 = 0.47$ verified	**0.5**
2	$P(U) = P(A \cap U) + P(\overline{A} \cap U) = 0.3 \times 0.2 + 0.7 \times 0.4 = 0.34$	**1**
3	P(still working in teaching or other domains) $= 0.3 + 0.7 \times 0.5 + 0.7 \times 0.4 = 0.93$	**1**
4	$P(\overline{A}/\overline{R}) = \frac{P(\overline{A} \cap \overline{R})}{P(\overline{R})} = \frac{P(\overline{A}) - P(\overline{A} \cap R)}{1 - P(R)} = \frac{0.7 - 0.35}{1 - 0.47} = \frac{35}{53}$	**1**
5a	Number of teachers that teach in private schools $= P(R) \times 500 = 0.47 \times 500 = 235$	**0.5**
5b	P(at least 2 teachers who teach at private schools) $= \frac{235C2 \times 265C1 + 235C3}{500C3} \approx 0.455$	**1**

III	Answers	Grade 10 pts
A1a	Over $]-\infty; +\infty[$, (G) is above the x-axis and intersects it at O, so $g(x) \geq 0$ for all values of x.	**0.5**
A1b	If $x \in]-\infty; 0[$, (G) is strictly decreasing, so $g'(x) < 0$. If $x \in]0; +\infty[$, (G) is strictly increasing, so $g'(x) > 0$. If $x = 0$, $g'(x) = 0$.	**0.5**
A2	$g(x) = (ax + b)e^{-x} + 1$ $g(0) = 0$, gives $b + 1 = 0$, so $b = -1$ $g'(x) = (a - ax + 1)e^{-x}$ $g'(0) = 0$, gives $a + 1 = 0$, so $a = -1$	**0.5**
B1	$\lim\limits_{x \to -\infty} f(x) = -\infty \times +\infty - \infty = -\infty - \infty = -\infty$ $f(-2.5) \approx -8.59$	**0.5** **0.25**
B2a	$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \left(\frac{x+2}{e^x} + x\right) = \lim\limits_{x \to +\infty} \left(\frac{1}{e^x} + x\right) = 0 + \infty = +\infty$ (using Hopital's rule)	**0.5**

<table>
<tr><td>B2b</td><td>$\lim_{x \to +\infty}[f(x) - x] = \lim_{x \to +\infty} \frac{x+2}{e^x} = 0$ (proved).
Thus (d): $y = x$ is an asymptote to (C).</td><td>0.5</td></tr>
<tr><td>B2c</td><td>$f(x) - y_D = (x+2)e^{-x}$
If $x \in]-\infty; -2[$, (C) is below (D).
If $x \in]-2; +\infty[$, (C) is above (D).
(C) and (D) intersect at the point $(-2; -2)$.</td><td>1</td></tr>
<tr><td>B3</td><td>$f'(x) = (1 - x - 2)e^{-x} + 1 = (-x - 1)e^{-x} + 1 = g(x)$
So f' and g have the same sign and roots, so $f'(x) \geq 0$.
<table>
<tr><td>x</td><td>$-\infty$</td><td>0</td><td>$+\infty$</td></tr>
<tr><td>$f'(x)$</td><td>+</td><td>0</td><td>+</td></tr>
<tr><td>f (x)</td><td>$-\infty$</td><td>↗</td><td>$+\infty$</td></tr>
</table></td><td>0.5

0.5</td></tr>
<tr><td>B4a</td><td>Over $]-\infty; +\infty[$: f is continuous and strictly increasing from $-\infty$ to $+\infty$, so (C) cuts the x-axis at one point only, then the equation $f(x) = 0$ has a unique root α.</td><td>0.5</td></tr>
<tr><td>B4b</td><td>$f(-1.7) \approx -0.05 < 0$ and $f(-1.6) \approx 0.38 > 0$.
Thus, $-1.7 < \alpha < -1.6$</td><td>0.5</td></tr>
<tr><td>B5a</td><td>$f''(x) = g'(x) = xe^{-x}$.
Thus, $f''(x)$ vanishes at $x = 0$ and changes sign from negative to positive, so (C) admits at $x = 0$ a point of inflection W of coordinates $(0; 2)$.</td><td>1</td></tr>
<tr><td>B5b</td><td>$f'(0) = g(0) = 0$, so the tangent to (C) at $x = 0$ is parallel to the x-axis.
Also, $f(0) = 2$ so the equation of this tangent is $y = 2$.
Thus (T): $y = 2$ is tangent to (C) at $W(0 , 2)$.</td><td>0.5</td></tr>
<tr><td>B6</td><td>
</td><td>1.25</td></tr>
<tr><td>B7b</td><td>$f(\alpha) = 0$ gives $(\alpha + 2)e^{-\alpha} + \alpha = 0$, then $e^{-\alpha} = \frac{-\alpha}{\alpha+2}$, then $-\alpha = \ln(\frac{-\alpha}{\alpha+2})$,
then $\ln(\alpha + 2) - \ln(-\alpha) = \alpha$.
Therefore, $h(\alpha) = \frac{\ln(\alpha+2) - \ln(-\alpha)}{\alpha} = \frac{\alpha}{\alpha} = 1$.</td><td>1</td></tr>
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