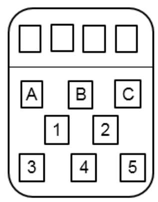
وزارة التربية والتعليم العالي المديريّة العامّة للتربية دائرة الامتحانات الرسميّة	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	دورة العام ٢٠٢١ الاستثنائية السبت في ٤ أيلول ٢٠٢١
عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة ونصف	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the table below, only one among the proposed answers to each question is correct.

Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Proposed Answers a	Proposed Answers b	Proposed Answers c
1	The inequality $\ln x < 1$ is verified for	$x < 0$	$0 < x < e$	$x > e$
2	The equation $\ln^2 x + \ln x - 6 = 0$ has two roots x_1 and x_2. The product $x_1 . x_2$ is equal to	-6	e^{-1}	e^{30}
3	$\lim\limits_{x \to +\infty} \left(\frac{x + \ln x}{x} + 1\right) =$	$+\infty$	1	2
4	A security entrance keyboard of a building is formed of three letters A, B and C and five digits 1, 2, 3, 4 and 5. The entrance code is formed of one letter followed by a number consisting of three distinct digits. The number of all possible codes is 	15	180	375

II- (6 points)

An urn U contains red balls and black balls holding distinct natural numbers.

- 60 % of the balls are red of which 80 % hold odd numbers.
- 70 % of the black balls hold odd numbers.

Part A

One ball is selected from the urn. Consider the following events:

R: "the selected ball is red" and O: "the selected ball holds an odd number".

1) Show that the probability $P(O \cap R)$ is equal to 0.48 and calculate $P(O \cap \overline{R})$.
2) Deduce that $P(O) = 0.76$.
3) Are the events R and O independent? Justify your answer.

Part B

Suppose in this part that the number of balls in the urn U is 50.

1) Show that the number of red balls holding odd numbers is equal to 24.
2) Copy and complete the following table :

	Red	Black	Total
Odd			38
Even			
Total	30		50

3) Three balls are selected randomly and simultaneously from the urn U.
   a- Calculate the probability of selecting at least one red ball holding an odd number.
   b- The even numbered balls hold the numbers 2, 4, 6, …, 24.
   Knowing that the three selected balls hold even numbers, calculate the probability that each of these balls holds a number greater than 15.

III- (10 points)

The plane is referred to an orthonormal system $(O;\vec{i},\vec{j})$.

Consider the function f defined, on $\mathbb{R}$, as $f(x) = 2xe^{-x+1} + 1$ and denote by (C) its representative curve.

1) Determine $\lim_{x\to-\infty} f(x)$.

2) Show that $\lim_{x\to+\infty} f(x) = 1$. Deduce an asymptote (d) to (C).

3) Show that $f'(x) = 2(1 - x)e^{-x+1}$.

4) Copy and complete the following table of variations of f.

x	$-\infty$	1	$+\infty$
$f'(x)$		0	
$f(x)$			

5) **a-** Show that the equation $f(x) = 0$ has, on $\mathbb{R}$, a unique solution α.

   b- Verify that $-0.16 < \alpha < -0.15$.

6) Calculate $f(-0.5)$ and $f(0)$ then draw (C) and (d).

7) **a-** Show that $\int xe^{-x+1}dx = (-x-1)e^{-x+1} + K$ where K is a real number.

   b- Deduce the area limited by (C), the straight line with equation $y = 3$ and the two straight lines with equations $x = 0$ and $x = 4$.

عدد المسائل: ثلاث

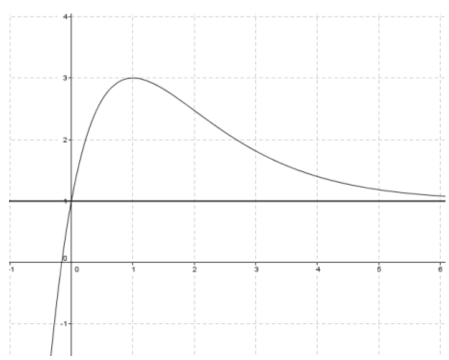
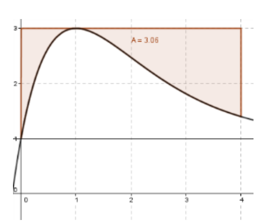
اسس تصحيح مادة الرياضيات

I	Answers		Grade 4 pts
1	Lnx < 1 . So, x < e but x > 0 . Thus , 0 < x < e	(b)	1
2	The roots of the equation : $\ln^2x+\ln x-6=0$ are $x_1=e^{-3}$ et $x_2=e^2$. then $x_1.x_2=e^{-1}$	(b)	1
3	$\lim\limits_{x\to+\infty}\left(\frac{x+\ln x}{x}+1\right)=\lim\limits_{x\to+\infty}\left(\frac{x}{x}+\frac{\ln x}{x}+1\right)=1+0+1=2$ since $\lim\limits_{x\to+\infty}\left(\frac{\ln x}{x}\right)=0$	(c)	1
4	The number of all codes is : $3\times A_5^3=180$	(b)	1

II	Answers	Grade 6 pts
A1	$P(O\cap R)=P(O/R)\times P(R)=0.8\times0.6=0.48$ $P(O\cap\overline{R})=P(O/\overline{R})\times P(\overline{R})=0.7\times0.4=0.28$	1
A2	$P(O)=P(O\cap R)+P(O\cap\overline{R})=0.48+0.28=0.76$	0.5
A3	Since $P(O\cap R)=0.48\neq P(O)\times P(R)=0.76\times0.6=0.456$ Then, the events R and O are not independent.	0.5
B1	The number of red balls holding odd numbers is $50\times0.48=24$	1
B2	(see table below)	1
B3.a	P (selecting at least one red ball holding an odd number) = $1-\frac{C_{26}^3}{C_{50}^3}=\frac{85}{98}$	1
B3.b	2 ; 4 ; 6 ; 8 ; 10 ; 12 ; 14 ; <u>16</u> ; <u>18</u> ; <u>20</u> ; <u>22</u> ; <u>24</u> P (each of the balls holds a number greater than 15/ knowing that the numbers on the balls are even) = $\frac{C_5^3}{C_{12}^3}=\frac{1}{22}$	1

B2:

	Red	Black	Total
Odd	24	14	38
Even	6	6	12
Total	30	20	50

III	Answers	Grade 10 pts
1	$\lim\limits_{x\to-\infty} f(x) = (-\infty)(+\infty)+1 = -\infty$	1
2	$\lim\limits_{x\to+\infty} (2x)e^{-x+1} = +\infty \cdot 0$ Indeterminate form ; $\lim\limits_{x\to+\infty} \frac{2x}{e^{x-1}} = \frac{0}{0}$ I.F. then $\lim\limits_{x\to+\infty} \frac{2x}{e^{x-1}}$ Using HR $\lim\limits_{x\to+\infty} \frac{2}{e^{x-1}} = 0$. Thus, $\lim\limits_{x\to+\infty} f(x) = 0+1 = 1$ (d) : y = 1 is an asymptote to (C).	1
3	$f'(x) = (2x)'.e^{-x+1} + (-e^{-x+1}).2x + 0 = 2(1-x)e^{-x+1}$	0.5
4	x: $-\infty$; 1 ; $+\infty$ $f'(x)$: + ; 0 ; – $f(x)$: $-\infty$ ↗ 3 ↘ 1	1.5
5.a	• Over $]-\infty ; 1[$: f is continuous , strictly increasing from $-\infty$ to 3 then the equation f(x) = 0 has one solution α • Over $[1 ; +\infty[$: f is continuousstrictly decreasing from 3 to 1 then , the equation f(x) = 0 has no solution. Therefore, the equation f(x) = 0 has, on ℝ, a uniques solution α	1
5.b	$f(-0.16) \approx -0.02 < 0$ $f(-0.15) \approx +0.05 > 0$	0.5
6	$f(-0.5) = -e^{1.5}+1 \approx -3.481$ $f(0) = 1$ 	2
7.a	$(-x-1)'e^{-x+1} + (-x-1)(e^{-x+1})' = (-1+x+1)\, e^{-x+1} = xe^{-x+1}$	1
7.b	$A = \int_0^4 [3 - f(x)]dx = \int_0^4 [2 - 2xe^{-x+1}]dx$ $= [\,2x + 2(x+1)e^{-x+1}]_0^4 = 8 + 10\,e^{-3} - 2\,e \approx 3.06$ (units)^2. 	1.5