عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة ونصف الساعة	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the table below, only one among the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, the corresponding answer.

№	Questions	Answers a	Answers b	Answers c
1	Let f be the function given by $f(x)=\dfrac{\ln(x-2)}{x}$ The domain of definition of f is	$[0\,;+\infty[$	$]2\,;+\infty[$	$]0\,;2[\cup]2\,;+\infty[$
2	The solution of the equation $\ln(x-2)=\ln(-x+4)$ is	1	2	3
3	Let f be the function defined over $]0\,;+\infty[$ as $f(x)=\dfrac{\ln(x)}{x}$. An antiderivative of f is	$(\ln(x))^2$	$\dfrac{(\ln(x))^2}{2}$	$2(\ln(x))^2$
4	$\lim\limits_{x\to+\infty}\dfrac{1+\ln(x)}{\ln(x)}$ is equal to	0	1	$+\infty$

II- (6 points)

U and V are two urns such that:

- U contains 3 red balls and 5 blue balls.
- V contains 4 red balls and 3 blue balls.

Part A

One ball is randomly selected from U and one ball is randomly selected from V.

1) Show that the probability of selecting two red balls is $\frac{3}{14}$.
2) Calculate the probability of selecting two balls having the same color.
3) Calculate the probability of selecting two balls with different colors.

Part B

In this part, one ball is randomly selected from U:

- if the selected ball from U is red, then two balls are randomly and simultaneously selected from V.
- if the selected ball from U is blue, then three balls are randomly and simultaneously selected from V.

Consider the following events:

R: “The selected ball from U is red”

S: “The selected balls from V have the same color”.

1) Determine the probability P(R).
2) Show that $P(S / R)=\frac{3}{7}$ and deduce $P(S\cap R)$.
3) The probability $P\left(S\cap\overline{R}\right)=\frac{5}{56}$, calculate P(S).

III- (10 points)

Consider the function f defined on $\mathbb{R}$ as $f(x) = e^x - x - 2$.

Denote by (C) its representative curve in an orthonormal system $\left(O; \vec{i}, \vec{j}\right)$.

Let (d) be the line with equation $y = -x - 2$.

1) Show that $\lim_{x \to +\infty} f(x) = +\infty$. Calculate $f(2)$.

2) **a-** Determine $\lim_{x \to -\infty} f(x)$.

b- Show that (d) is an asymptote to (C) at $-\infty$.

c- Show that (C) is above (d) for all $x \in \mathbb{R}$.

3) Determine $f'(x)$, then set up the table of variations of f.

4) The equation $f(x) = 0$ has two solutions $\alpha > 0$ and $\beta < 0$.

Verify that $1.1 < \alpha < 1.2$.

5) Knowing that $-1.9 < \beta < -1.8$, draw (d) and (C).

6) Let $A(\alpha)$ be the area of the region limited by the curve (C), the line $(d)$and the two lines with equations $x = 0$ and $x = \alpha$.

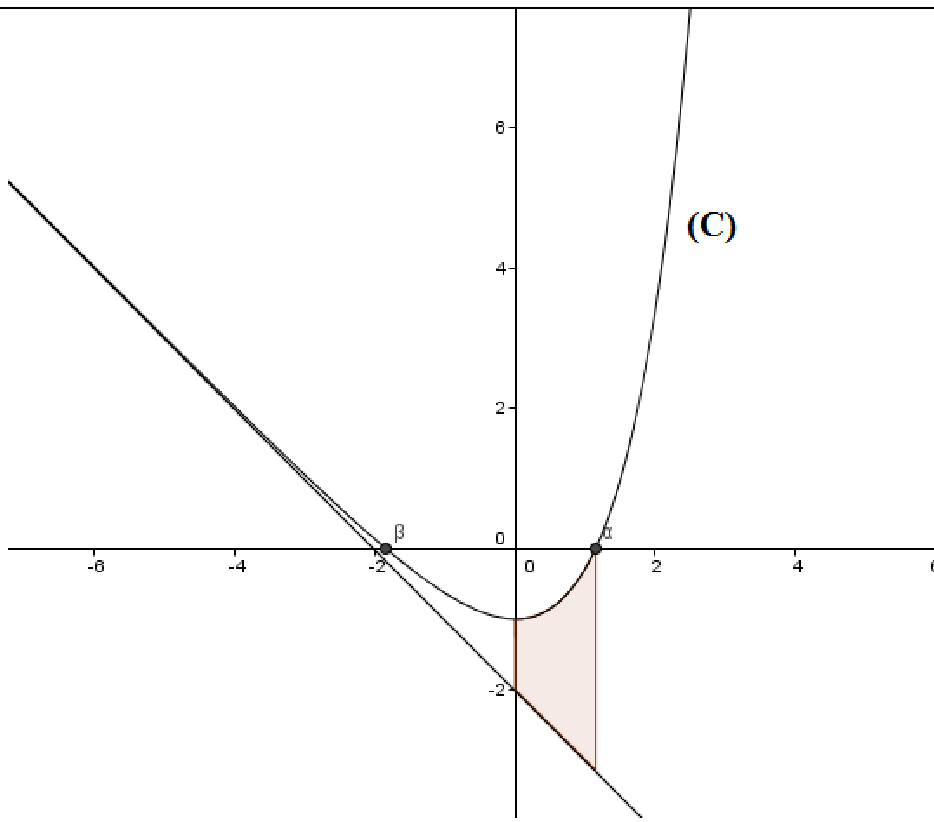
a- Verify that $e^\alpha = \alpha + 2$.

b- Prove that $A(\alpha) = (\alpha + 1)$ units of area.

اسس تصحيح

I	Answer key	4 pts
1	**b,** $x-2>0$ *and* $x\neq 0$; so domain = $]2,+\infty[$	1
2	**c,** $2<x<4; x-2=-x+4; x=3$ second way: by verification for x=3, ln (1) = ln (1)	1
3	**b,** let u = ln x, $u'=\frac{1}{x}$, $\int\frac{ln(x)}{x}dx=\int u'\times u dx=\frac{(ln(x))^2}{2}+c$ second way: by verification $\left(\frac{(ln(x))^2}{2}\right)'=\frac{ln(x)}{x}$	1
4	**b,** $\lim_{x\to+\infty}\frac{1+\ln(x)}{\ln(x)}=\lim_{x\to+\infty}\frac{1}{\ln(x)}+1=1$ second way: $\lim_{x\to+\infty}\frac{1+\ln(x)}{\ln(x)}=H.R\lim_{x\to+\infty}\frac{\frac{1}{x}}{\frac{1}{x}}=1$	1

II	Answer key	6 pts
A1	$P(2\ red\ balls)=\frac{C_3^1}{C_8^1}\times\frac{C_4^1}{C_7^1}=\frac{3}{8}\times\frac{4}{7}=\frac{3}{14}$	1
A2	$P(\text{same color})=\frac{3}{14}+\frac{C_5^1}{C_8^1}\times\frac{C_3^1}{C_7^1}=\frac{3}{14}+\frac{5}{8}\times\frac{3}{7}=\frac{27}{56}$	1
A3	$P(different\ colors)=1-\frac{27}{56}=\frac{29}{56}$	1
B1	$P(R)=\frac{C_3^1}{C_8^1}=\frac{3}{8}$	1
B2	P(S/R)= $\frac{C_4^2}{C_7^2}+\frac{C_3^2}{C_7^2}=\frac{3}{7}$ P(R)×P(S/R)= $\frac{3}{8}\times\frac{3}{7}=\frac{9}{56}$	1
B3	P(S)= $P(S\cap R)+P(S\cap\overline{R})=\frac{9}{56}+\frac{5}{56}=\frac{1}{4}$	1

<table>
<tr><th>III</th><th>Answer key</th><th>10 pts</th></tr>
<tr><td>1</td><td>$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x\left(\frac{e^x}{x} - 1\right) - 2 = +\infty(+\infty - 1) - 2 = +\infty$
$f(2) = e^2 - 4 \cong 3{,}39$</td><td>1</td></tr>
<tr><td>2a</td><td>$\lim_{x\to-\infty} f(x) = +\infty$</td><td>½</td></tr>
<tr><td>2b</td><td>$\lim_{x\to-\infty} [f(x) - (-x-2)] = \lim_{x\to-\infty} e^x = 0$
Then $y = -x - 2$ is an oblique asymptote to (C).</td><td>1</td></tr>
<tr><td>2c</td><td>$f(x) - y_d = e^x > 0$ for all x
Then (C) is above (d) for all $x \in \mathbb{R}$.</td><td>1</td></tr>
<tr><td>3</td><td>$f'(x) = e^x - 1; f'(x) = 0\ for\ x = 0$
<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>0</td><td></td><td>$+\infty$</td></tr>
<tr><td>f'</td><td></td><td>$-$</td><td>0</td><td>$+$</td><td></td></tr>
<tr><td>f</td><td>$+\infty$</td><td>↘</td><td>-1</td><td>↗</td><td>$+\infty$</td></tr>
</table></td><td>2</td></tr>
<tr><td>4</td><td>$f(1.1) = -0.096$ (negative)
$f(1.2) = 0.12$ (positive) and f is continuous so $1.1 < \alpha < 1.2$
Or second way $f(1.1) \times f(1.2) < 0$ and f is continuous so $1.1 < \alpha < 1.2$</td><td>1</td></tr>
<tr><td>5</td><td>
</td><td>2</td></tr>
<tr><td>6a)</td><td>$f(\alpha) = 0, e^\alpha - \alpha - 2 = 0 \Rightarrow e^\alpha = \alpha + 2$</td><td>½</td></tr>
<tr><td>6b)</td><td>$Area = \int_0^\alpha [f(x) - y_{(d)}]\,dx = \int_0^\alpha e^x\,dx = e^x]_0^\alpha = e^\alpha - 1 = (\alpha + 1)$ units of area</td><td>1</td></tr>
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