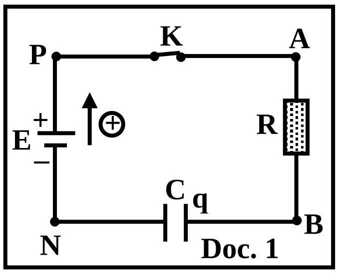
مسابقة في مادة الفيزياء	الاسم:
المدة: ساعتان ونصف	الرقم:

This exam is formed of four obligatory exercises in four pages.
The use of non-programmable calculator is recommended.

Exercise 1 (4.5 pts)

Charging a capacitor

The aim of this exercise is to determine the minimum duration needed for a capacitor to store the electric energy needed to feed an electronic flash later.
For this purpose, we set up the circuit represented in document 1.
This series circuit is composed of: a resistor of resistance R = 100 Ω, a capacitor, initially uncharged, of capacitance C = 10 mF, a switch K and an ideal battery of emf $E = u_{PN} = 12$ V.
Switch K is closed at the instant $t_0 = 0$, thus a current i flows in the circuit.



Doc. 1

1) Redraw the circuit of document 1 and show on it the direction of i.

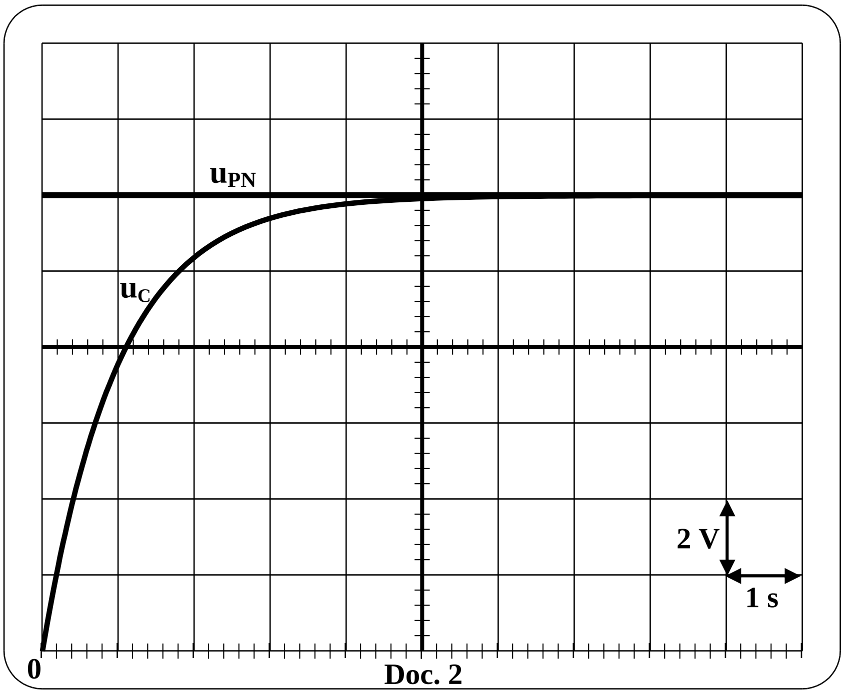
2) Determine the differential equation that governs the variation of the voltage $u_C = u_{BN}$ across the capacitor.

3) The solution of this differential equation is of the form: $u_C = E\,(1 - e^{\frac{-t}{\tau}})$, where τ is constant.

3.1) Determine the expression of τ in terms of R and C.

3.2) Calculate the value of τ.

4) An oscilloscope is used to display on channel Y_1 the voltage $u_C = u_{BN}$ across the capacitor, and on channel Y_2 the voltage u_{PN} across the battery.
The obtained curves are displayed on the screen of the oscilloscope (Doc. 2).



Doc. 2

4.1) Show on the redrawn circuit, of document 1, the connections of the oscilloscope.

4.2) Referring to document 2, determine again the value of τ.

5) During the charging process, the minimum electric energy needed to be stored in the capacitor in order to feed the flash later is W = 0.18 J.

5.1) Calculate the value U_1 of u_C when the energy stored in the capacitor becomes 0.18 J.

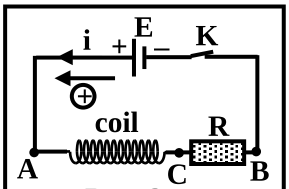
5.2) Deduce, graphically, the minimum duration taken by the capacitor to store the electric energy needed to feed the flash.

Exercise 2 (5 pts)

Sparks due to switching off a circuit of large inductance

Consider a circuit consisting of a coil of inductance L and resistance r connected in series with a resistor of resistance $R = 10^4\ \Omega$ and a switch K, across an ideal battery of constant voltage $u_{AB} = E = 20$ V.

Switch K is closed at $t_0 = 0$, thus a current i flows in the circuit (Doc. 3).



Doc. 3

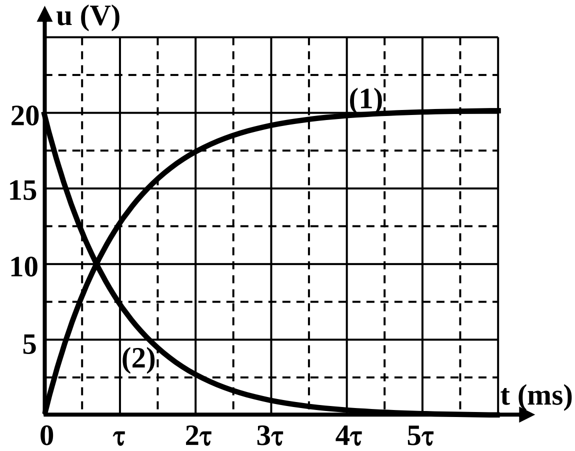
1) Theoretical study

1.1) Determine the differential equation that governs the variation of i.

1.2) The solution of this differential equation is of the form $i = I_m\left(1 - e^{\frac{-t}{\tau}}\right)$, where I_m and τ are constants. Determine the expressions of I_m and τ in terms of E, R, r and L.

2) Experimental study

Curves (1) and (2) in document 4 represent the voltages u_{AC} across the coil and u_{CB} across the resistor as functions of time t.



Doc. 4

2.1) Curve (1) represents $u_{CB} = u_R$. Why?

2.2) Use curve (2) to prove that the resistance r of the coil can be neglected.

2.3) The steady state is practically attained at t = 0.25 ms. Calculate the value of the time constant τ of the circuit.

2.4) Deduce that L = 0.5 H.

2.5) Determine the maximum magnetic energy stored in the coil in steady state.

3) Switching off the circuit

The circuit is switched off abruptly. Assume that the current decreases linearly with time and has the following expression $i = -2000\,t + 0.002$ (SI units).

3.1) Calculate the self-induced emf "e" in the coil during the decay of the current.

3.2) Deduce that sparks appear at the switch contacts during the decay of the current.

3.3) Propose a method used to protect the switch from sparks.

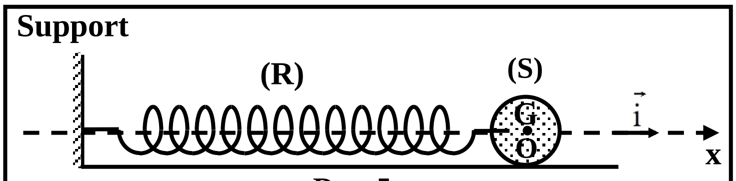
Exercise 3 (5 pts)

Determination of the force exerted by a wall on a ball

The aim of this exercise is to determine the magnitude of the force exerted by a wall on a ball during the collision between them.

For this purpose, we use a massless spring (R) of force constant k = 51 N/m.

The spring is placed horizontally connected from one of its ends to a fixed support.



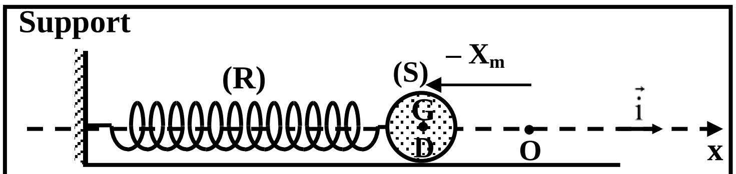
Doc. 5

A ball (S) of mass m = 1 kg is attached to the other end of the spring. (S) may slide without friction on a horizontal surface and its center of mass G can move along a horizontal x-axis of unit vector $\vec{i}$.

At equilibrium, G coincides with the origin O of the x-axis (Doc. 5).

1) Oscillatory motion of (S)

(S) is shifted from the equilibrium position to the left, in the negative direction, by a displacement $\overline{OD} = -X_m$ and then it is released from rest at $t_0 = 0$ as shown in document 6.



Doc. 6

Thus, the elastic pendulum formed by the spring and the ball oscillates without friction with a proper (natural) period T_0.

At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity is $v = \dfrac{dx}{dt}$.

G reaches point O for the first time at an instant t_1 with a velocity $\vec{V_1} = 1\,\vec{i}$ (m/s).

Take the horizontal plane containing G as a reference level for gravitational potential energy.

1.1) By applying the principle of conservation of mechanical energy of the system [(S)-Spring-Earth], determine the value of X_m.

1.2) Write the expression of the mechanical energy ME of this system at an instant t in terms of m, k, x and v.

1.3) Determine the second order differential equation that governs the variation of x.

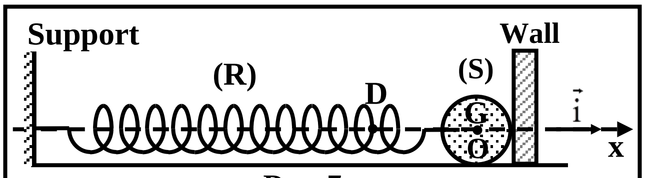
1.4) Deduce the value of T_0.

1.5) Choose from the table below the correct relation between t_1 and T_0.

Relation 1	Relation 2	Relation 3	Relation 4
$t_1 = \dfrac{T_0}{4}$	$t_1 = \dfrac{T_0}{2}$	$t_1 = \dfrac{3T_0}{4}$	$t_1 = T_0$

2) Collision of (S) with a wall

As G reaches point O at the instant t_1, ball (S) makes a head-on elastic collision with the wall (Doc.7).



Doc. 7

At an instant t_2, just after the collision, (S) rebounds with a velocity $\vec{V_2} = -1\,\vec{i}$ (m/s).

The duration of this collision is Δt. G continues its oscillation with the same proper (natural) period T_0 and returns back to point D at an instant t_3.

2.1) Show that $t_3 = \dfrac{T_0}{2} + \Delta t$.

2.2) Calculate Δt knowing that $t_3 = 0.5$ s.

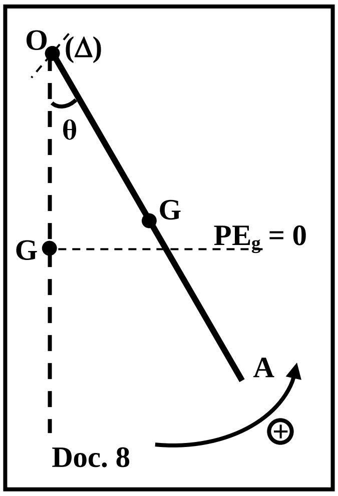
2.3) Determine the variation $\Delta\vec{P}$ in the linear momentum of (S), during its collision with the wall, between t_1 and t_2.

2.4) Apply Newton's second law $\Sigma\vec{F}_{ext} = \dfrac{d\vec{P}}{dt} \cong \dfrac{\Delta\vec{P}}{\Delta t}$ on (S) to determine the magnitude of the force $\vec{F}_{wall/(S)}$ (supposed constant) exerted by the wall on (S) during the collision. Neglect the magnitude of the spring force exerted by the spring on (S) during the collision.

Exercise 4 (5.5 pts)

Oscillation of a rigid rod

A compound pendulum consists of a thin uniform metallic rod of length ℓ = OA and mass m = 0.5 kg. The rod of center of mass G can rotate without friction in the vertical plane about a horizontal axis (Δ) passing through its upper end O.



Doc. 8

The moment of inertia of the rod about (Δ) is $I = \frac{m\ell^2}{3}$.

The pendulum is shifted in the vertical plane by a small angle θ_m from its stable equilibrium position, and then it is released from rest.

At the instant $t_0 = 0$, the pendulum passes through the equilibrium position ($\theta_0 = 0$) in the positive sense (Counterclockwise).

At an instant t, the angular abscissa of the pendulum is θ and its angular velocity is $\theta' = \frac{d\theta}{dt}$ (Doc. 8).

Take:

- the horizontal plane passing through the lowest position of G as a reference level for gravitational potential energy;
- $\sin\theta \cong \theta$ and $\cos\theta \cong 1 - \frac{\theta^2}{2}$ for small angles measured in radians (rad);
- $g = 10\ m/s^2$;
- $\pi^2 = 10$.

1) Time equation of motion of the pendulum

1.1) Determine the expression of the mechanical energy of the system (Pendulum-Earth) in terms of m, g , I, θ , θ' and a = OG.

1.2) Prove that the differential equation that governs the variation of θ is: $\theta'' + \frac{3g}{2\ell}\theta = 0$.

1.3) The solution of the obtained differential equation is $\theta = \theta_m \sin(\omega_0 t + \varphi)$, where ω_0 and φ are constants. Determine:

1.3.1) the expression of the proper (natural) angular frequency ω_0 in terms of g and ℓ;

1.3.2) the value of φ.

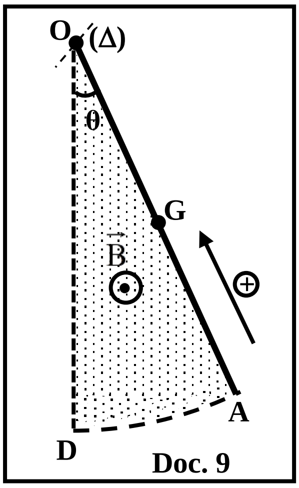
1.4) The rod completes 10 oscillations during 16 s.

1.4.1) Deduce the value of ω_0.

1.4.2) Calculate the value of ℓ.

2) Electromagnetic induction

The above experiment is repeated in a horizontal uniform magnetic field $\vec{B}$, of magnitude B = 0.19 T, parallel to (Δ) as shown in document 9.



Doc. 9

An emf "e" is induced in the rod and no current passes through the rod since the circuit is open.

During the motion of the rod from its equilibrium position ($\theta_0 = 0$) to the extreme position ($\theta = \theta_m$), the magnetic flux crossing the shaded area ODA at an instant t is given by: $\phi = \frac{B\ell^2}{2}\theta$, where $\theta = \theta_m \sin(\omega_0 t)$ and $t \in [0, \frac{T_0}{4}]$. T_0 is the proper period of the pendulum.

During the time interval $[0, \frac{T_0}{4}]$:

2.1) Determine the expression of "e" in terms of B, θ_m, ℓ, ω_0 and t.

2.2) The voltage across the rod is $u_{AO} = ri - e$. Prove that $u_{AO} = \frac{B\ell^2\omega_0\theta_m}{2}\cos(\omega_0 t)$.

2.3) Given that $u_{AO} = 0.06\cos(\omega_0 t)$ in SI units. Deduce the value of θ_m.

وزارة التربية والتعليم العالي
المديرية العامّة للتربية
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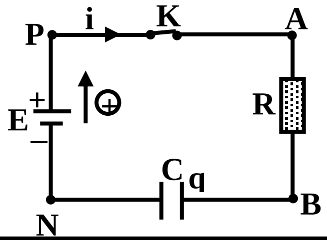
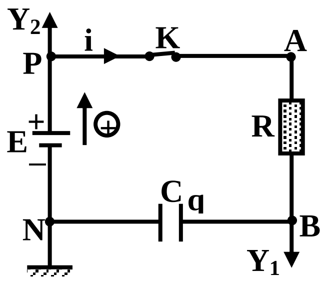
امتحانات الشهادة الثانوية العامة
فرع العلوم العامّة

دورة العام 2022 العاديّة

الاسم:	مسابقة في مادة الفيزياء
الرقم:	المدة: ساعتان ونصف

Exercise 1 (4.5pts) **Charging a capacitor**

Part		Answer	Note
1			0.25
2		$u_{PN} = u_{PA} + u_{AB} + u_{BN}$, then $E = R\,i + u_C$ But, $i = \frac{dq}{dt}$ and $q = C\,u_C$, so $i = C\frac{du_C}{dt}$; therefore, $E = RC\frac{du_C}{dt} + u_C$	1
3	3.1	$u_C = E(1 - e^{\frac{-t}{\tau}}) = E - E\,e^{\frac{-t}{\tau}}$, so $\frac{du_C}{dt} = \frac{E}{\tau}e^{\frac{-t}{\tau}}$ Substituting the expressions of u_C and $\frac{du_C}{dt}$ into the differential equation, gives: $E = RC\frac{E}{\tau}e^{\frac{-t}{\tau}} + E - E\,e^{\frac{-t}{\tau}}$, then $E\,e^{\frac{-t}{\tau}}[\frac{RC}{\tau} - 1] = 0$ The equation is valid at any instant: $E\,e^{\frac{-t}{\tau}} = 0$ is rejected. Then, $\frac{RC}{\tau} - 1 = 0$, hence $\tau = RC$	1
	3.2	$\tau = RC = 100 \times 10^{-2} = 1$ s	0.25
4	4.1		0.25
	4.2	At $t = \tau$, $u_C = 0.63 \times 12 = 7.56$ V ; Graphically: $u_C = 7.56$ V at t = 1 s , so $\tau = 1$ s	0.75
5	5.1	$W = \frac{1}{2}C\,u_C^2$, then $0.18 = \frac{1}{2}10^{-2}\,U_1^2$, so $U_1 = 6$ V	0.75
	5.2	Graphically: $U_1 = 6$ V at t = 0.7 s	0.25

Exercise 2 (5pts)

Sparks due to switching off a circuit of large inductance

Part		Answer	Note
1	1.1	$u_{AB} = u_{AC} + u_{CB}$, then $E = r\,i + L\frac{di}{dt} + R\,i$, then $E = (R+r)\,i + L\frac{di}{dt}$	0.75
	1.2	$i = I_m\left(1 - e^{\frac{-t}{\tau}}\right) = I_m - I_m e^{-\frac{t}{\tau}}$, then $\frac{di}{dt} = \frac{I_m}{\tau} e^{\frac{-t}{\tau}}$ Substituting the expressions of i and $\frac{di}{dt}$ into the differential equation, gives: $E = (R+r)(I_m - I_m e^{-\frac{t}{\tau}}) + L\frac{I_m}{\tau} e^{\frac{-t}{\tau}} = I_m(R+r) - I_m(R+r)e^{-\frac{t}{\tau}} + L\frac{I_m}{\tau}e^{\frac{-t}{\tau}}$ Then, $I_m e^{\frac{-t}{\tau}}\left[\frac{L}{\tau} - (R+r)\right] + (R+r)I_m = E$ This equation is true for any instant, by comparison: $\frac{L}{\tau} - (R+r) = 0$, then $\tau = \frac{L}{R+r}$, and $(R+r)I_m = E$, then $I_m = \frac{E}{R+r}$	1
2	2.1	$u_{CB} = u_R = R\,i$. The current increases during its growth and R is a positive constant, therefore, u_R increases with time.	0.25
	2.2	<u>In steady state</u>: $i = I_m$ = constant , so $\frac{di}{dt} = 0$; Graphically $u_{AC} = 0$ But, $u_{AC} = r\,i + L\frac{di}{dt}$, then $0 = r\,I_m + 0$. But, $I_m \neq 0$; therefore, $r = 0$	0.5
	2.3	Steady state is attained at t = 0.25 ms = 5 τ , so $\tau = \frac{0.25}{5} = 0.05$ ms	0.25
	2.4	$\tau = \frac{L}{R+r}$, then $L = \tau(R+r) = 0.05\times10^{-3}\left(10^4 + 0\right)$ so L = 0.5 H	0.5
	2.5	$W_{magnetic} = \frac{1}{2} L\, I_m^2$, but $I_m = \frac{E}{R+r} = \frac{20}{10^4} = 2\times10^{-3}$ A $W_{magnetic} = \frac{1}{2}\times 0.5\times(2\times10^{-3})^2 = 1\times10^{-6}$ J	0.75
3	3.1	$e = -L\frac{di}{dt} = -0.5(-2000)$, so e = 1000 V	0.5
	3.2	e = 1000 V which is a very large value. Then, the voltage across the switch would be very large; therefore, sparks appear at the switch contacts.	0.25
	3.3	We connect a capacitor across the switch. Or: We connect a diode and a resistor in parallel across the coil.	0.25

Exercise 3 (5pts)

Determination of the force exerted by a wall on a ball

Part		Answer	Note
1	**1.1**	$ME_D = ME_O$, then $GPE_D + KE_D + EPE_D = GPE_O + KE_O + EPE_O$ $GPE_D = GPE_O = 0$ since G is at the reference level; $KE_D = 0$ since $V_D = 0$ $EPE_O = 0$ since the spring is not deformed at O. $\frac{1}{2} k\ X_m^2 = \frac{1}{2} m\ V_1^2$, then $\frac{1}{2} \times 51 \times X_m^2 = \frac{1}{2} \times 1 \times 1$, so $X_m = 0.14$ m	0.5
	1.2	$ME = KE + EPE + GPE$, then $ME = \frac{1}{2} k\ x^2 + \frac{1}{2} m\ v^2$	0.5
	1.3	Friction is neglected, so ME is conserved. Or: The sum of the works done by the nonconservative forces is zero, then ME is constant. Therefore, $\frac{dME}{dt} = 0$ $k\,x\,x' + m\,v\,v' = 0$, but $x' = v$ and $v' = x''$, then $v\,(k\,x + m\,x'') = 0$ $v = 0$ is rejected , then $k\,x + m\,x'' = 0$ Therefore, $x'' + \frac{k}{m} x = 0$	0.75
	1.4	This differential equation is of the form: $x'' + \omega_0^2 x = 0$, with $\omega_0^2 = \frac{k}{m}$. $T_0 = \frac{2\pi}{\omega_0}$, then $T_0 = 2\pi\sqrt{\frac{m}{k}}$; $T_0 = 2\pi\sqrt{\frac{1}{51}}$, so $T_0 = 0.88$ s	0.75
	1.5	$t_1 = \frac{T_0}{4}$	0.25
2	**2.1**	G moves from D to O during a time $\frac{T_0}{4}$. The duration of the collision is Δt. G moves from O to D during a time $\frac{T_0}{4}$. Then, $t_3 = \frac{T_0}{4} + \Delta t + \frac{T_0}{4}$; therefore, $t_3 = \frac{T_0}{2} + \Delta t$	0.5
	2.2	$t_3 = \frac{T_0}{2} + \Delta t$, then $0.5 = \frac{0.88}{2} + \Delta t$, so $\Delta t = 0.06$ s	0.25
	2.3	$\Delta\vec{P} = \vec{P}_{t_2} - \vec{P}_{t_1} = m\vec{V}_2 - m\vec{V}_1$, then $\Delta\vec{P} = -1\,\vec{i} - 1\,\vec{i} = -2\,\vec{i}$ (kg.m/s)	0.5
	2.4	Newton's 2nd law on (S): $\Sigma\,\vec{F}_{ext} = \frac{d\vec{P}}{dt}$, then $m\,\vec{g} + \vec{N} + \vec{F}_{wall/S} = \frac{\Delta\vec{P}}{\Delta t}$, but $m\,\vec{g} + \vec{N} = \vec{0}$ $\vec{F}_{wall/S} = \frac{-2\,\vec{i}}{0.06} = -33.3\,\vec{i}$ (N)	1

Exercise 4 (5.5pts)

Oscillations of a rigid rod

Part			Answer	Note
1	1.1		$ME = KE + GPE = \frac{1}{2}\ I\ \theta'^2 + m\ g\ Z_G$, where $Z_G = a\ (1 - \cos\theta)$ So, $ME = \frac{1}{2}\ I\ \theta'^2 + mg\ a\ (1 - \cos\theta)$	0.75
	1.2		Friction is neglected, or the sum of the works done by the non-conservative forces is zero, therefore the mechanical energy is conserved. Therefore, $\frac{dME}{dt} = 0$ $I\ \theta'\ \theta'' + mg\ a\ \theta'\ \sin\theta = 0$, then $\theta'\ (I\ \theta'' + mg\ a\ \sin\theta) = 0$; $\sin\theta \cong \theta$ So, $\theta'\ (I\ \theta'' + mg\ a\ \theta) = 0$; $\theta' = 0$ is rejected , then $I\ \theta'' + mg\ a\ \theta = 0$ $\frac{m\ell^2}{3}\ \theta'' + mg\frac{\ell}{2}\theta = 0$, so $\frac{\ell}{3}\ \theta'' + \frac{1}{2}g\ \theta = 0$, hence $\theta'' + \frac{3g}{2\ell}\ \theta = 0$	0.75
	1.3	1.3.1	$\theta = \theta_m \sin(\omega_0\ t + \varphi)$; $\theta' = \omega_0\theta_m \cos(\omega_0\ t + \varphi)$; $\theta'' = -\omega_0^2\theta_m \sin(\omega_0\ t + \varphi)$ Then, $\theta'' = -\omega_0^2\theta$. Replace θ'' by its expression in the differential equation: $-\omega_0^2\theta + \frac{3\ g}{2\ \ell}\ \theta = 0$, then $\theta\ (-\omega_0^2 + \frac{3\ g}{2\ \ell}) = 0$ $\theta = 0$ is rejected , then $-\omega_0^2 + \frac{3\ g}{2\ \ell} = 0$, so $\omega_0 = \sqrt{\frac{3\ g}{2\ \ell}}$	1
		1.3.2	$\theta = \theta_m \sin(\omega_0\ t + \varphi)$: At $t_o = 0$: $\theta_o = 0 = \theta_m \sin\varphi$ $\theta_m \neq 0$, then $\sin\varphi = 0$, hence $\varphi = 0$ or $\varphi = \pi$ rad $\theta' = \omega_0\theta_m \cos(\omega_0\ t + \varphi)$: At $t_o = 0$: $\theta'_0 = \omega_0\theta_m \cos(\varphi)$ $\theta'_0 > 0$, so $\cos(\varphi) > 0$; therefore, $\varphi = 0$	0.5
	1.4	1.4.1	$10\ T_0 = 16$ s , then $T_0 = 1.6$ s ; $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{1.6} = 1.25\ \pi$ rad/s = 3.9 rad/s	0.5
		1.4.2	$\omega_0 = \sqrt{\frac{3\ g}{2\ \ell}}$, then $\ell = \frac{3\ g}{2\ \omega_0^2} = \frac{3\ (10)}{2 \times 1.25^2 \times 10}$, so $\ell = 0.96$ m	0.5
2	2.1		$e = -\frac{d\phi}{dt} = -\ (\frac{B\ell^2}{2}\theta')$, then $e = -\frac{B\ell^2}{2}\theta_m\ \omega_0 \cos(\omega_0\ t)$	0.5
	2.2		The circuit is open, then i = 0 ; $u_{AO} = ir - e = -e$ Therefore, $u_{AO} = \frac{B\ell^2\omega_0\theta_m}{2} \cos(\omega_0\ t)$	0.5
	2.3		$u_{AO} = 0.06 \cos(\omega_0\ t) = \frac{B\ell^2\omega_0\theta_m}{2} \cos(\omega_0\ t)$, then $0.06 = \frac{B\ell^2\omega_0\theta_m}{2}$ $0.06 = \frac{(0.19)(0.96)^2\ (1.25\ \pi)\ \theta_m}{2}$, then $\theta_m = 0.17$ rad	0.5