دورة العام ٢٠٢١ الإستثنائية
الإثنين ٦ أيلول ٢٠٢١

امتحانات الشهادة الثانوية العامة
فرع العلوم العامّة

وزارة التربية والتعليم العالي
المديرية العامّة للتربية
دائرة الامتحانات الرسميّة

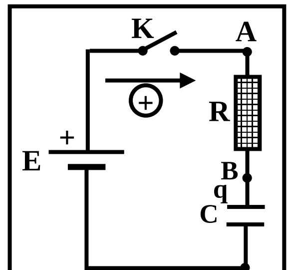
الاسم:	مسابقة في مادة الفيزياء
الرقم:	المدة: ساعتان ونصف

This exam is formed of four obligatory exercises in four pages.
The use of non-programmable calculator is recommended.

Exercise 1 (5 pts)

Charging a capacitor

The aim of this exercise is to determine the capacitance C of a capacitor and the average electric power consumed by this capacitor during a certain time interval.
For this purpose, consider the series circuit of document 1 that includes a resistor of resistance R = 1 kΩ, an initially uncharged capacitor of capacitance C, an ideal battery of electromotive force E, and a switch K.
The switch K is closed at $t_0 = 0$.



Doc. 1

1) Name the physical phenomenon that takes place in the circuit.

2) Determine the differential equation that governs the variation of the voltage $u_{BD} = u_C$ across the capacitor.

3) The solution of this differential equation is:

$u_C = A + B\,e^{\frac{-t}{\tau}}$.

Determine the expressions of the constants A, B and τ in terms of E, R and C.

4) The curve of document 2 represents u_C as a function of time.

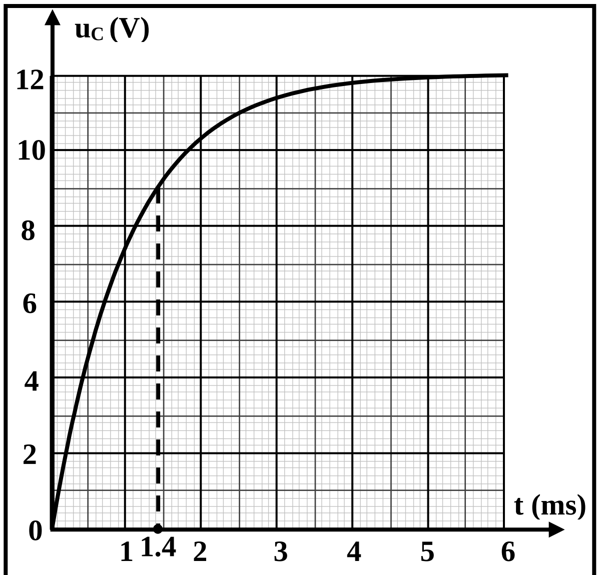
4.1) Referring to document 2 indicate the value of E.

4.2) Use document 2 to determine the time constant τ of the circuit.

4.3) Deduce the value of C.

4.4) Use document 2 to determine the electric energy stored in the capacitor at t = 1.4 ms.

4.5) Deduce the average electric power consumed by the capacitor between $t_0 = 0$ and t = 1.4 ms.

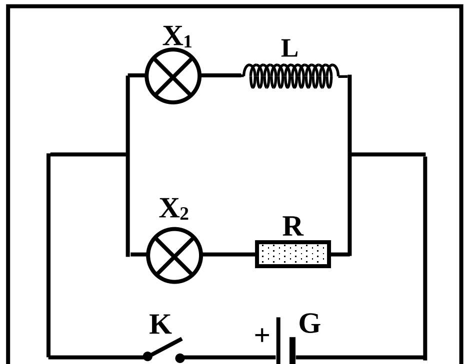


Doc. 2

Exercise 2 (5 pts)

Self-induction

Consider an ideal battery G of electromotive force (emf) E, a coil of inductance L and negligible resistance, a resistor of resistance R, two lamps X_1 and X_2, an oscilloscope, and a switch K.
The aim of this exercise is to study the effect of the inductance of a coil on the growth of the current and to determine the value of this inductance.
Two experiments are set up for this purpose:



Doc. 3

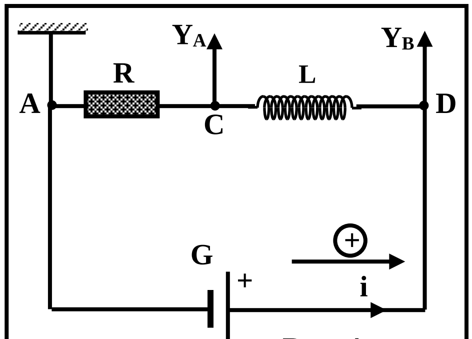
1) First experiment

Consider the circuit of document 3. When we close the switch K, we observe that X_2 glows instantly whereas X_1 glows with some time delay.

Explain the cause of this time delay.

2) Second experiment

Consider the circuit of document 4. Given that R = 100 Ω.



Doc. 4

2.1) An oscilloscope is connected as indicated in document 4 in order to display two voltages as functions of time. Indicate the voltage displayed by channel Y_A and that displayed by channel Y_B.

2.2) Determine the differential equation that governs the variation of the voltage $u_{CA} = u_R$ across the resistor.

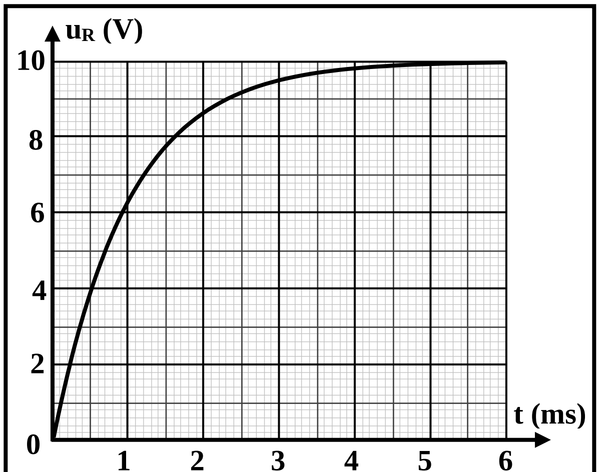
2.3) The solution of this differential equation is:

$$u_R = A\,(1 - e^{\frac{-t}{\tau}}).$$

Determine the expressions of the constants A and τ in terms of E, L and R.

2.4) Show that the voltage across the resistor in the steady state is $u_R = E$.

2.5) The curve of document 5 represents u_R as a function of time.



Doc. 5

2.5.1) Referring to document 5 indicate the value of E.

2.5.2) Define the time constant τ of the series (RL) circuit.

2.5.3) Use document 5 to determine the value of τ.

2.5.4) Deduce the value of L.

Exercise 3 (5 pts)

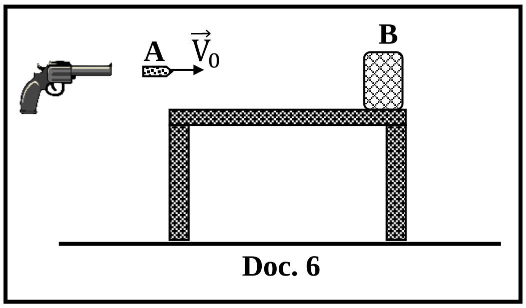
Motion of a block in a vertical plane

A gun shoots a bullet (A) of mass $m_1 = 10$ g towards a block (B), considered as a particle of mass $m_2 = 240$ g, initially at rest on the edge of a horizontal table (Doc. 6).

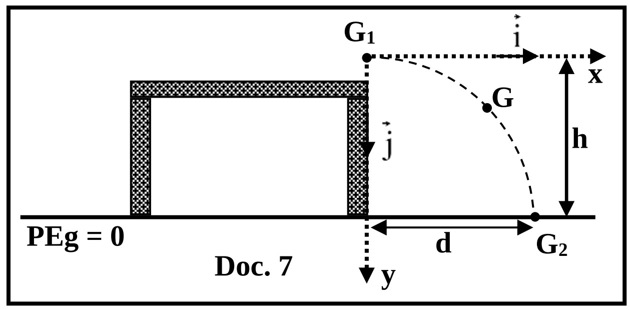
The bullet (A) hits the block (B) with a horizontal velocity $\vec{V}_0$ of magnitude $V_0 = 125$ m/s and becomes embedded in it.

The system [(A) - (B)] is taken as a particle G of mass $M = m_1 + m_2$. Just after the collision, G leaves the table at position G_1 of height h = 80 m with a horizontal velocity $\vec{V}_1$. G moves in the vertical plane G_1xy containing $\vec{V}_1$ and then it reaches the ground at position G_2 (Doc. 7).

Neglect air resistance.



Doc. 6



Doc. 7

Take:

- the horizontal plane containing G_2 as a reference level for gravitational potential energy;
- $g = 10$ m/s^2.

1) During the collision between (A) and (B), the linear momentum of the system [(A) - (B)] is conserved. Why?

2) Deduce that the magnitude of $\vec{V}_1$ is $V_1 = 5$ m/s.

3) Show that the collision between (A) and (B) is inelastic.

4) G leaves the table at G_1 at an instant $t_0 = 0$ taken as an initial time.

4.1) During the motion of G between G_1 and G_2, the mechanical energy of the system [(A) - (B) - Earth] is conserved. Why?

4.2) Deduce the value of the speed V_2 with which G reaches the ground at G_2.

4.3) Apply Newton's second law to show that the expression of the linear momentum of the system [(A) - (B)] is: $\vec{P} = 1.25\,\vec{i} + 2.5\,t\,\vec{j}$ (SI).

4.4) Deduce the parametric equations x(t) and y(t) of G in the plane G_1xy.

4.5) Given that the coordinates of G_2 are ($x_{G2} = d$, $y_{G2} = 80$ m). Deduce:

4.5.1) the time taken by G to pass from G_1 to G_2;

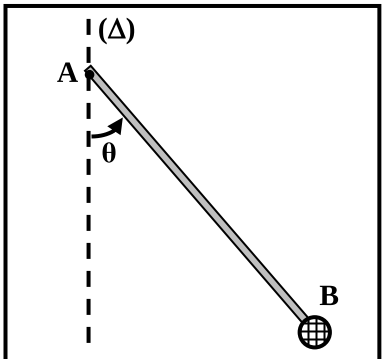
4.5.2) the value of the distance $d = x_{G2}$.

Exercise 4 (5 pts)

Compound pendulum

Consider a uniform rigid thin rod AB of mass $M = 0.5$ kg and length $L = AB = 2$ m.

This rod may rotate about a horizontal axis (Δ) passing through its upper end A (Doc. 8).



Doc. 8

The aim of this exercise is to determine the moment of inertia I_1 of the rod about (Δ).

For this purpose, we fix a particle of mass $m = 0.1$ kg at the lower end B of the rod.

The system (S), formed by the rod and the particle, constitutes a compound pendulum whose center of mass is G.

(S) is shifted from its stable equilibrium position ($\theta_0 = 0$) by a small angle and then it is released without initial velocity at the instant $t_0 = 0$.

(S) oscillates without friction about (Δ).

At an instant t, the angular abscissa of the pendulum is θ and its angular velocity is $\theta' = \frac{d\theta}{dt}$.

Take:

- the horizontal plane containing A as a reference level for gravitational potential energy;
- $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2}$ for $\theta \leq 10^0$ (θ in rad);
- $g = 10$ m/s^2 ; $\pi^2 = 10$.

1) Write the expression of the moment of inertia I of (S) about (Δ) in terms of I_1, m and L.

2) Show that the position of G relative to A is: $AG = a = \frac{L(M+2m)}{2(M+m)}$.

3) (S) undergoes free un-damped oscillations. Why?

4) Write the expression of the mechanical energy of the system [(S) - Earth] in terms of m, M, g, θ, θ' and I.

5) Determine the differential equation in θ that governs the motion of (S).

6) Deduce that the expression of the proper period (natural) of the pendulum is $T_0 = 2\pi\sqrt{\frac{2I}{(M+2m)gL}}$.

7) The pendulum performs 20 oscillations during 49 seconds.

7.1) Determine the value of I.

7.2) Deduce the value of I_1.

مسابقة في مادة الفيزياء
أسس التصحيح - إنكليزي

Exercise 1 (5 pts) Charging a capacitor

Part		Answer	Mark
1		Charging the capacitor	0.25
2		$u_{AD} = u_{AB} + u_{BD}$ $E = Ri + u_C$ $i = \frac{dq}{dt}$ and $q = C\,u_C$, then $i = C\frac{du_C}{dt}$ Substituting in the above equation gives : $E = RC\frac{du_C}{dt} + u_C$	1
3		$u_C = A + Be^{-t/\tau}$, then $\frac{du_C}{dt} = -\frac{B}{\tau}e^{-t/\tau}$ Substituting u_C and $\frac{du_C}{dt}$ into the differential equation gives: $E = -RC\frac{B}{\tau}e^{-t/\tau} + A + Be^{-t/\tau}$ $E = Be^{-t/\tau}(-RC\frac{1}{\tau} + 1) + A$ This equation is valid at any instant, then : $E = A$ and $Be^{-t/\tau}(-RC\frac{1}{\tau} + 1) = 0$ $Be^{-t/\tau} = 0$ is rejected , so $(-RC\frac{1}{\tau} + 1) = 0$; therefore, $\tau = RC$	1
4	4.1	$E = 12$ V	0.25
	4.2	At $t = \tau$: $u_C = 0.63\,E = 0.63 \times 12 = 7.56$ V Graphically, for $u_C = 7.56$ V , $t = \tau = 1$ ms	1
	4.3	$\tau = RC$, then $1 \times 10^{-3} = 1000 \times C$, hence $C = 1 \times 10^{-6}$ F	0.5
	4.4	Graphically: At t = 1.4 ms , $u_C = 9$ V $W_C = \frac{1}{2}C\,u_C^2 = \frac{1}{2} \times 1 \times 10^{-6} \times 9^2$, then $W_C = 4.05 \times 10^{-5}$ J	0.5
	4.5	$P = \frac{W_C}{\Delta t} = \frac{4.05 \times 10^{-5}}{1.4 \times 10^{-3}} = 0.029$ W	0.5

Exercise 2 (5 pts) **Self-induction**

Part		Answer	Mark
1		When we close the switch the electric current in the circuit increases, then the coil is crossed by a variable self-flux; therefore, the coil becomes the seat of self-induced electromotive force which according to Lenz's law tends to oppose the increase in the current (delays its growth). X_1 is connected in series with the coil, thus it glows with some time delay.	**0.75**
2	**2.1**	Channel Y_A displays the voltage u_{CA} Channel Y_B displays the voltage u_{DA}	**0.25** **0.25**
	2.2	$u_{DA} = u_{DC} + u_{CA}$ $E = L\frac{di}{dt} + u_R$; $i = \frac{u_R}{R}$, so $\frac{di}{dt} = \frac{1}{R}\frac{du_R}{dt}$ Substituting $\frac{di}{dt}$ into the above equation gives: $E = \frac{L}{R}(\frac{du_R}{dt}) + u_R$	**1**
	2.3	$u_R = A(1 - e^{-t/\tau}) = A - A e^{-t/\tau}$, then $\frac{du_R}{dt} = \frac{A}{\tau} e^{-t/\tau}$ Substituting u_R and $\frac{du_R}{dt}$ into the differential equation gives: $E = \frac{L\,A}{R\,\tau} e^{-t/\tau} + A - A e^{-t/\tau}$ This equation is valid at any instant, then $E = A$ $Ae^{-t/\tau}(\frac{L}{R\,\tau} - 1) = 0$, but $A e^{-t/\tau} = 0$ is rejected Hence, $(\frac{L}{R\,\tau} - 1) = 0$, then $\tau = \frac{L}{R}$	**1**
	2.4	In the steady state $t = 5\tau$, then $u_R = E(1 - e^{-5}) \cong E$	**0.25**
	2.5.1	$E = 10V$	**0.25**
	2.5.2	τ is the time needed by the current flowing in the coil to grow or to decay 63% of its maximum value.	**0.5**
	2.5.3	At $t = \tau$: $u_R = 0.63E = 0.63\times 10 = 6.3$ V Graphically : for $u_R = 6.3$ V , $t = \tau = 1$ ms	**0.25**
	2.5.4	$\tau = \frac{L}{R}$, then $1\times10^{-3} = \frac{L}{100}$, hence $L = 0.1$ H	**0.5**

Exercise 3 (5 pts) **Motion of a block in a vertical plane**

Part		Answer	Mark
1		During the collision the internal forces are much stronger than the external forces acting on the system, then the external forces can be considered neglected relative to the internal forces. Therefore, the system of the two colliding objects is considered isolated during the collision : $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0}$, then $\vec{P}$ is constant.	0.25
2		$\vec{P}_{befor} = \vec{P}_{after}$ $m\ \vec{V}_0 = (m_1 + m_2)\ \vec{V}_1$ Then, $0.01 \times 125\ \vec{i} = (0.01 + 0.24)\ \vec{V}_1$, so $V_1 = 5$ m/s	0.75
3		$KE_{before} = \frac{1}{2} m_1 V_0^2 = \frac{1}{2} \times 0.01 \times 125^2$, then $KE_{before} = 78.125$ J $KE_{after} = \frac{1}{2}(m_1 + m_2) V_1^2 = \frac{1}{2}(0.01 + 0.24) \times 5^2$, then $KE_{after} = 3.125$ J $KE_{after} < KE_{before}$, then this collision is non-elastic	0.5
4	4.1	Since air resistance is neglected Or : The sum of the works done by the non-conservative forces is zero; therefore, the mechanical energy of the system is conserved.	0.25
	4.2	$ME_{G1} = ME_{G2}$ $KE_{G1} + GPE_{G1} = KE_{G2} + GPE_{G2}$ ($GPE_{G2} = 0$ since G is at the reference level of GPE) $KE_{G1} + (m_1 + m_2)\ g\ h = \frac{1}{2}(m_1 + m_2) V_2^2 + 0$ $3.125 + (0.25 \times 10 \times 80) = \frac{1}{2} \times 0.25 \times V_2^2$ Then $V_2 = 40.3$ m/s	0.75
	4.3	$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$, then $M\ \vec{g} = Mg\ \vec{j} = \frac{d\vec{P}}{dt}$, hence $d\vec{p} = Mg\ \vec{j}\ dt$ $\vec{p} = Mg\ t\ \vec{j} + \vec{P}_1$, but $\vec{P}_1 = M\ \vec{V}_1 = 0.25 \times 5\ \vec{i}$, so $\vec{P}_1 = 1.25\ \vec{i}$ (kg.m/s) $\vec{p} = (0.25 \times 10)\ t\ \vec{j} + 1.25\ \vec{i}$; therefore, $\vec{P} = 1.25\ \vec{i} + 2.5t\ \vec{j}$	0.75
	4.4	$\vec{V} = \frac{\vec{P}}{M} = \frac{1{,}25\ \vec{i} + 2{,}5t\ \vec{j}}{0{,}25} = 5\ \vec{i} + 10\ t\ \vec{j}$ (SI) $\vec{r} = \int \vec{V}\ dt = 5t\ \vec{i} + 5\ t^2\ \vec{j} + \vec{r}_1$, but $\vec{r}_1 = \vec{0}$, so $\vec{r} = 5t\ \vec{i} + 5\ t^2\ \vec{j}$ $x = 5\ t$ (SI) and $y = 5\ t^2$ (SI)	0.5 0.5
	4.5.1	G reaches G_2 when y = 80 m $80 = 5t^2$, so t = 4 s	0.5
	4.5.2	Substituting t = 4 into x(t) gives : $x_{G2} = d = 5 \times 4$, then d = 20 m	0.25

Exercise 4 (5 pts) **Compound pendulum**

Part		Answer	Mark
1		$I = I_{rod/(\Delta)} + I_{p/(\Delta)}$, then $I = I_1 + mL^2$	0.5
2		$\overrightarrow{AG} = \frac{M\,\overrightarrow{AO} + m\,\overrightarrow{AB}}{M+m}$, then $a = AG = \frac{M\frac{L}{2} + m\,L}{M+m}$, hence $a = AG = \frac{L(M+2m)}{2(M+m)}$	0.75
3		Since (S) oscillates about (Δ) without friction.	0.25
4		$ME = KE + GPE = \frac{1}{2} I (\theta')^2 + (m+M)g\,Z$ with $Z = -\,a\cos\theta$ $ME = \frac{1}{2} I (\theta')^2 - (m+M)g\,a\cos\theta$ $ME = \frac{1}{2} I (\theta')^2 - (m+M)g\,(\frac{L\,(M+2m)}{2\,(M+m)})\cos\theta$ $ME = \frac{1}{2} I (\theta')^2 - \frac{g\,L\,(M+2m)}{2}\cos\theta$	1
5		Friction is neglected, then the mechanical energy is conserved, hence $\frac{dME}{dt} = 0$ $I\theta'\,\theta'' + \frac{g\,L\,(M+2m)}{2}\,\theta'\sin\theta = 0$, then $\theta'\,(I\theta'' + \frac{g\,L\,(M+2m)}{2}\sin\theta) = 0$ $\theta' = 0$ is rejected , then $I\theta'' + \frac{g\,L\,(M+2m)}{2}\sin\theta = 0$ θ is small, so $\sin\theta \approx \theta$, hence $\theta'' + \frac{(M+2\,m)\,g\,L}{2I}\,\theta = 0$	0.5
6		The differential equation is of the form $\theta'' + \omega_o^2\,\theta = 0$, then $\omega_o^2 = \frac{(M+2\,m)\,g\,L}{2I}$ Then, $\omega_o = \sqrt{\frac{(M+2m)\,g\,L}{2I}}$ But, $\omega_o = \frac{2\pi}{T_0}$, hence $T_o = 2\pi\sqrt{\frac{2I}{(M+2m)gL}}$	1
7	7.1	$T_0 = \frac{49}{20} = 2.45$ s $T_o^2 = \frac{8\,\pi^2\,I}{(M+2m)gL}$ $2.45^2 = \frac{8\,(10)\,I}{(0.5+0.2)\times 10\times 2}$, then $I = 1.05$ kgm^2	0.5
	7.2	$I = I_1 + m\,L^2$ $I_1 = 1.05 - (0.1\times 2^2)$, then $I_1 = 0.65$ kg.m^2	0.5