امتحانات الشهادة الثانوية العامة
فرع: العلوم العامة

دورة العام ٢٠٢١ العاديّة
الاثنين ٢٦ تموز ٢٠٢١

عدد المسائل: خمس

مسابقة في مادة الرياضيات
المدة: ثلاث ساعات

الاسم:
الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (2 points)

In the table below, only one among the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, the corresponding answer.

№	Questions	Answers a	Answers b	Answers c
1	Let f be the function given by $f(x) = \ln(x^2 - 3x)$ The domain of definition of f is	$[0 ; +\infty[$	$[1 ; 3[$	$]-\infty ; 0[\cup]3 ; +\infty[$
2	For all real numbers x, $\ln(e^x + 2) - x$ is equal to	$\ln\left(\frac{e^x+2}{x}\right)$	$\ln(2)$	$\ln\left(\frac{e^x+2}{e^x}\right)$
3	Let $I = \int_0^1 \frac{e^x}{3+e^x}dx$. The value of I is	$\ln\left(\frac{e+3}{4}\right)$	$\ln\left(\frac{e+3}{3}\right)$	$\ln(e+3)$
4	Given below the table of variations of a continuous function f: x: 2 ; 4 ; 5 f′(x): − ; 0 ; + f(x): 3 ↘ −1 ↗ 6 The equation f(x) = 4 has	one root only	two roots	no roots

II- (3 points)

The complex plane is referred to the direct orthonormal system $(O;\ \vec{u}, \vec{v})$.

Consider the points A, B and C with affixes $z_A = -2 + 2i$, $z_B = -2i$ and $z_C = 4$.

For every point M with affix z, assign the point M′ with affix z′ such that $z' = \frac{2z + 4i}{iz + 2 + 2i}$ with $z \neq -2 + 2i$.

1) In the case where z = 0, give the exponential form of z′.
2) Write $\frac{z_A - z_B}{z_C - z_B}$ in algebraic form. Deduce the nature of triangle ABC.
3) **a-** Verify that $z' = \frac{2(z - z_B)}{i(z - z_A)}$

   b- Deduce that $OM' = \frac{2BM}{AM}$

   c- Show that when M moves on the perpendicular bisector of [AB], the point M′ moves on a circle whose center and radius are to be determined.

III- (3 points)

An urn U contains 10 balls: 6 blue balls and 4 red balls.

Part A

Two balls are selected randomly and simultaneously from U.

Consider the following events:

A: “The two selected balls have the same color”

B: “The two selected balls have different colors”.

1) Verify that the number of possible outcomes is 45.
2) Show that the probability $P(A) = \frac{7}{15}$ and deduce P(B).

Part B

In this part, a fair die numbered from 1 to 6 is rolled.

- If the die shows an even number, then two balls are selected randomly and simultaneously from U.
- If the die shows an odd number, then two balls are selected randomly and successively with replacement from U.

Consider the following events:

E: “The die shows an even number”

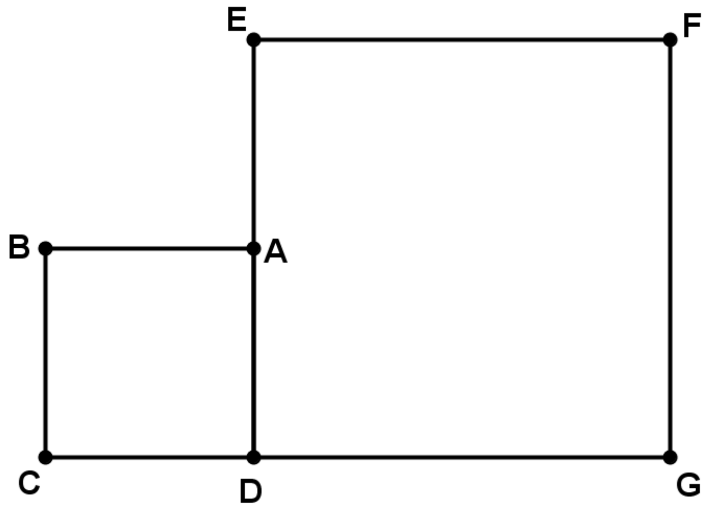
F: “The selected balls have the same color”.

1) Calculate P(F / E) and verify that $P(F \cap E) = \frac{7}{30}$.
2) Verify that $P(F \cap \overline{E}) = \frac{13}{50}$ and deduce P(F).
3) Knowing that the two selected balls have the same color, calculate the probability that the die shows an even number.

IV- (4 points)

In the following figure,

- ABCD and EDGF are two direct squares.
- CD = 1 and DG = 2.



Denote by S the direct plane similitude with angle $\frac{\pi}{2}$ that maps B onto D and maps A onto E.

1) Calculate the ratio k of S and show that S(C) = G.

2) Denote by (T) and (T′) the circles with diameters [BD] and [AE] respectively.

(T) and (T′) intersect in two points W and A.

Show that W is the center of S.

3) **a-** Show that the image of line (BD) by S is the line (DF).

b- Determine the image of line (AD) by S.

c- Show that S(D) = F.

4) Let h be the transformation defined as $h = S \circ S$.

a- Determine the nature and the characteristic elements of h.

b- Determine h(B) and deduce that $\overrightarrow{WF} = -4\overrightarrow{WB}$.

5) The complex plane is referred to a direct orthonormal system $\left(C; \overrightarrow{CD}, \overrightarrow{CB}\right)$.

a- Determine the complex form of h.

b- Calculate the affix of point W.

V- (8 points)

Part A

Let g be the function defined on $\mathbb{R}$ as $g(x) = (x+1)e^x - 1$.

1) Verify that $\lim_{x \to -\infty} g(x) = -1$ and determine $\lim_{x \to +\infty} g(x)$.

2) Copy and complete the following table of variations of g:

x	$-\infty$		-2		$+\infty$
$g'(x)$		$-$	0	$+$	
$g(x)$					

3) Calculate g(0). Verify that g(x) < 0 for all x < 0 and that g(x) > 0 for all x > 0.

Part B

Let f be the function defined on $\mathbb{R}$ as $f(x) = x(e^x - 1)$.

Denote by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

Let (d) be the line with equation $y = -x$.

1) **a-** Determine $\lim_{x \to -\infty} f(x)$ and show that (d) is an asymptote to (C).

   b- Study the relative positions of (C) and (d).

2) Determine $\lim_{x \to +\infty} f(x)$ and calculate f(2).

3) Verify that $f'(x) = g(x)$ and set up the table of variations of the function f.

4) Show that the curve (C) has a point of inflection I with abscissa –2.

5) Draw (d) and (C).

6) The equation f(x) = 1 has two real roots α and β such that $\alpha < 0 < \beta$.

   a- Prove that $\int xe^x dx = (x-1)e^x + k$, with $k \in \mathbb{R}$.

   b- Let $A(\alpha)$ be the area of the region bounded by the curve (C), (d), the line $x = \alpha$ and the y-axis.

   Show that $A(\alpha) = \left(1 + \alpha - \frac{1}{\alpha}\right)$ units of area.

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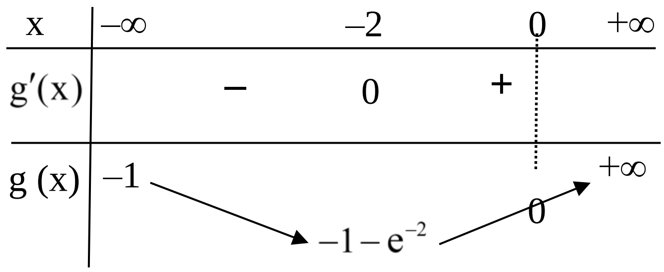
عدد المسائل: خمس	أسس تصحيح مسابقة الرياضيات	

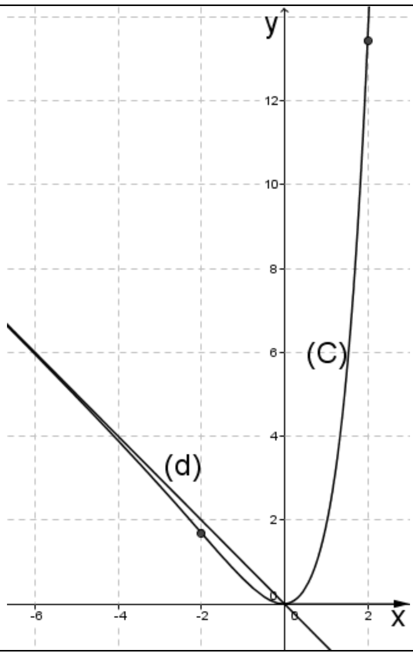
I	Answers	Mark
1	$f(x) = \ln(x^2 - 3x)$. $x^2 - 3x > 0$ then $x(x-3) > 0$ so $x \in]-\infty ; 0[\cup]3 ; +\infty[$. **Then c**	0.5
2	$\ln(e^x + 2) - x = \ln(e^x + 2) - \ln(e^x) = \ln\left(\frac{e^x+2}{e^x}\right)$. **Then c**	0.5
3	$\int_0^1 \frac{e^x}{3+e^x}dx = \ln(3+e^x)\|_0^1 = \ln(3+e) - \ln 4 = \ln\left(\frac{e+3}{4}\right)$. **Then a**	1
4	Since f is continuous and strictly decreasing over [2 ; 4] from 3 < 4 to –1 < 4 then the equation f(x) = 4 has no solution over this interval. Since f is continuous and strictly increasing over [4 ; 5] from –1 < 4 to 6 > 4 then the equation f(x) = 4 has one root over this interval. **Then a**	1

II	Answers	Mark
1	For z = 0, $z' = \frac{4i}{2+2i} = \sqrt{2}e^{i\frac{\pi}{4}}$	1
2	$\frac{z_A - z_B}{z_C - z_B} = \frac{-2+2i+2i}{4+2i} = i$. Thus $\left\|\frac{z_A-z_B}{z_C-z_B}\right\| = 1$ and $\arg\left(\frac{z_A-z_B}{z_C-z_B}\right) = \frac{\pi}{2}\ (2\pi)$. Triangle ABC is right isosceles with vertex B.	1
3a	$z' = \frac{2(z+2i)}{i(z+2-2i)} = \frac{2(Z - Z_B)}{i(Z - Z_A)}$	0.5
3b	$\|z'\| = \frac{\|2\|\|Z - Z_B\|}{\|i\|\|Z - Z_A\|} = \frac{2BM}{AM}$	1
3c	AM = BM then OM' = $\|Z'\| = 2$, then M' varies on the circle with center O and radius 2.	1

III	Answers	Mark
A1	The number of possible outcomes is $C_{10}^2 = 45$	1
A2	$P(A) = \frac{C_6^2 + C_4^2}{C_{10}^2} = \frac{7}{15}$ $P(B) = 1 - P(A) = \frac{8}{15}$ or $P(B) = \frac{C_6^1 \cdot C_4^1}{C_{10}^2} = \frac{8}{15}$	1
B1	$P(F / E) = P(A) = \frac{C_6^2 + C_4^2}{C_{10}^2} = \frac{7}{15}$ $P(F \cap E) = P(F / E) \times P(E) = \frac{7}{15} \times \frac{1}{2} = \frac{7}{30}$	1
B2	$P(F \cap \overline{E}) = P(F / \overline{E}) \times P(\overline{E}) = \left(\frac{6\times6}{10\times10} + \frac{4\times4}{10\times10}\right) \times \frac{1}{2} = \frac{52}{100} \times \frac{1}{2} = \frac{13}{50}$ $P(F) = P(F \cap E) + P(F \cap \overline{E}) = \frac{7}{30} + \frac{13}{50} = \frac{37}{75}$	1
B3	$P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{7}{30}}{\frac{37}{75}} = \frac{35}{74}$	0.5

IV	Answers	Mark
1	S: B→ D, A→ E then the ratio $k = \frac{DE}{AB} = \frac{2}{1} = 2$. $(\overrightarrow{BC} ; \overrightarrow{DG}) = \frac{\pi}{2} [2\pi]$, $\frac{DG}{BC} = 2$ et S(B) = D then S(C) = G.	1
2	The center of S belongs (T) and (T') because $\alpha = \frac{\pi}{2}$ and S(B) = D and S(A) = E Then the center is W or A. But S(A) = E then A is not invariant by S then W is the center.	1
3a	S(BD) = line passing through D and perpendicular to (BD), then S(BD) = (DF)	0.5
3b	S(AD) = line passing through E and perpendicular to (AD), then S(AD) = (EF)	0.5
3c	$\{D\} = (BD) \cap (AD)$ then $\{S(D)\} = S(BD) \cap S(AD) = (DF) \cap (EF) = \{F\}$. Then S(D) = F.	0.5
4a	$h = S \text{ o } S = \text{Sim}(W , 4 , \pi) = \text{hom}(W , -4)$	0.5
4b	h(B) = S(S(B)) = S(D) = F h(B) = F, then $\overrightarrow{WF} = -4\overrightarrow{WB}$	1
5a	C(0 ; 0), D(1 ; 0) , B(0 ; 1) et F(3 ; 2). h: z' = az + b then z' = –4z + b. h(B) = F then 3 + 2i = –4(i) + b then b = 3 + 6i then z' = –4z + 3 + 6i	0.5
5b	W is invariant, then z = –4z + 3 + 6i then W $(\frac{3}{5} ; \frac{6}{5})$	0.5

V	Answers	Mark
A1	$\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty} [(x+1)e^x - 1] = \lim_{x\to-\infty} (xe^x + e^x - 1) = 0 + 0 - 1 = -1$ since $\lim_{x\to-\infty} e^x = 0$ and $\lim_{x\to-\infty} xe^x = 0$. $\lim_{x\to+\infty} g(x) = \lim_{x\to+\infty} [(x+1)e^x - 1] = +\infty$.	1
A2	$g'(x) = e^x + (x+1)e^x = (x+2)e^x$ 	1
A3	g(0) = 0. Over $]-\infty ; 0[$, $g(x) < 0$ because the maximum of g is less than zero. Over $]0; +\infty[$, $g(x) > 0$ because the minimum of g is greater than zero.	1.5
B1a	$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} [x(e^x - 1)] = -\infty(0 - 1) = +\infty$ $\lim_{x\to-\infty} [f(x) + x] = \lim_{x\to-\infty} xe^x = 0$ then (d) is an asymptote to (C) at $-\infty$.	1
B1b	$f(x) + x = xe^x$ If $x \in]-\infty; 0[$, $f(x) + x < 0$, then (C) is below (d) If $x \in]0; +\infty[$, $f(x) + x > 0$, then (C) is above (d) If x = 0, f(x) + x = 0, then (d) and (C) intersect at point O.	1
B2	$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} [x(e^x - 1)] = +\infty(+\infty + 1) = +\infty$ $f(2) = 2(e^2 - 1) = 12.77$	1

B3	$f'(x) = e^x - 1 + xe^x = (x+1)e^x - 1 = g(x)$. x: $-\infty$; 0 ; $+\infty$ $f'(x)$: $-$; 0 ; $+$ $f(x)$: $+\infty$ ↘ 0 ↗ $+\infty$	1.5
B4	$f''(x) = g'(x)$; then $f''(x)$ vanishes at -2 and changes its sign, then (C) has a point of inflection $I(-2\,;\ 2-2e^{-2})$.	0.5
B5		1
B6a	$[(x-1)e^x]' = e^x + (x-1)e^x = xe^x$, then $\int xe^x\,dx = (x-1)e^x + k$, with $k \in \mathbb{R}$.	1
B6b	Over $[-2; 0]$, (C) is below (d), then $A = \int_\alpha^0 (-x - f(x))\,dx = \int_\alpha^0 -xe^x\,dx = [(1-x)e^x]_\alpha^0 = 1 - (1-\alpha)e^\alpha$ but $f(\alpha) = 1$ then $\alpha(e^\alpha - 1) = 1$ then $e^\alpha = \frac{1}{\alpha} + 1$ Then $A(\alpha) = 1 - (1-\alpha)(\frac{1}{\alpha} + 1) = 1 - \frac{1}{\alpha} - 1 + 1 + \alpha = 1 + \alpha - \frac{1}{\alpha}$ units of area.	1.5