

الاسم: الرقم:	مسابقة في مادة الرياضيات المدة: ساعة ونصف
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ملاحظة: - يتكوّن هذا الإمتحان من ست مسائل، يجب اختيار أربع مسائل منها فقط.

- في حال الإجابة عن أكثر من أربع مسائل، عليك شطب الإجابات المتعلقة بالمسألة التي لم تعد من ضمن اختيارك، لأنّ التصحيح سيقترص على إجابات المسائل الأربعة الأولى غير المشطوبة.
- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- Distribution in two variables (5 points)

The table below represents the average number of Artificial Intelligence (AI) users, worldwide, y_i in millions, in terms of the rank of the year x_i from 2019 to 2024.

Year	2019	2020	2021	2022	2023	2024
Rank of the year: x_i	1	2	3	4	5	6
Number of users in millions: y_i	80	100	120	150	200	240

All the values are rounded to the nearest 10^{-2} .

- 1) Calculate the percentage increase of AI users from 2023 to 2024.
- 2) Calculate the coordinates \bar{x} and \bar{y} of G, the center of gravity of the statistical data (x_i, y_i) .
- 3) Determine the coefficient of correlation r and interpret the obtained result.
- 4) Write an equation of the regression line $(D_{y/x})$ of y in terms of x .
- 5) The model remains valid until the year 2030.

Suppose that the average number of AI users in Lebanon is 1% of the world's AI users.

Starting from which year will the number of AI users in Lebanon exceed 3 million? Justify.

II- Exponential Functions (5 points)

Consider the function f defined over $]-\infty, +\infty[$ as $f(x) = x + e^{-x} + 1$, and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$. Let (d) be the line with equation $y = x$.

- 1) a) Determine $\lim_{x \rightarrow +\infty} f(x)$.
b) Show that (d) is an oblique asymptote to (C) at $+\infty$.
c) Show that (C) is above (d) for all values of x .
- 2) a) Calculate $f'(x)$.
b) Solve $f'(x) = 0$.
c) Write an equation of (T) the tangent to (C) at the point with abscissa 1.
d) Copy and complete the table of variations of the function f .

x	$-\infty$		$+\infty$
$f'(x)$		- 0 +	
$f(x)$	$+\infty$		

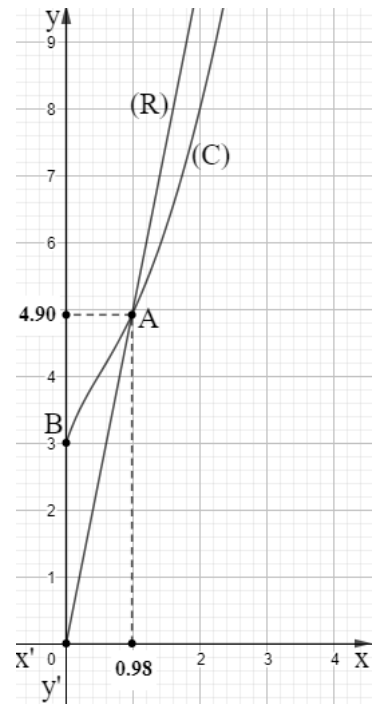
- 3) a) Calculate $f(-1)$ and $f(0)$.
b) Draw (d) and (C).

III- Cost, Revenue and Profit (5 points)

A company produces a certain type of objects.

In the adjacent figure:

- The curve (C) represents the total cost function C.
- The line (R) represents the revenue function R.
- (C) and (R) intersect at the point A(0.98 , 4.90).
- (C) intersects the y-axis at the point B(0 , 3).
- x is the quantity produced, in thousands of objects, where $0 \leq x \leq 4$.
- C(x) and R(x) are expressed in 10 million LL.



- Using the figure, answer each of the questions below.
 - Determine, in LL, the fixed cost of this company.
 - Estimate, in LL, the average cost of one object when 980 objects are produced.
 - Give an economical interpretation of the abscissa of A.
- Suppose in this part that $R(x) = 5x$ and $C(x) = x^2 - e^{-3x} + 4$.
 - Write, in terms of x, the profit function P.
 - Calculate $P(0.5)$. Deduce that the company does not realize a gain for this level of production.

IV- Logarithm, Exponential and Economic functions (5 points)

In the following table, only one of the proposed answers to each question is correct.

Write the number of each question and give, with justification, the answer that corresponds to it

N°	Questions	Proposed answers		
		a	b	c
1	For $a > 0$, $\ln(3a) - \ln\left(\frac{3}{e}\right) =$	$1 + \ln(a)$	$-1 - \ln(a)$	$\ln\left(3a - \frac{3}{e}\right)$
2	Let f be the function given by $f(x) = \ln(x - 1) + \ln(-x + 3)$. The domain of definition of f is	$] -\infty \ 3[$	$]1 \ 3[$	$]1 \ +\infty[$
3	The total cost, in millions LL, of the production of x object is modeled as $C(x) = x^2 - e^{-5x} + 2$, where $0 \leq x \leq 10$. The marginal cost, in millions LL, when 6 objects are produced is	$\frac{38 - e^{-30}}{6}$	$12 + 5e^{-30}$	$38 - e^{-30}$
4	The elasticity of the demand of a product with respect to the unit price is modeled as $E(x) = \frac{1}{2 - \ln(x)}$ where $0 < x < 6$. If the demand is unit elastic, then the unit price is	$\frac{1}{e}$	1	e

V- Supply, Demand and Elasticity (5 points)

A company produces units of a certain product.

The demand function f and the supply function g , defined over $[0.1, 6]$ are respectively modeled as $f(x) = (x + 12)e^{-x+1}$ and $g(x) = x + 12$, where $f(x)$ and $g(x)$ are expressed in thousands of units and x is the price of one unit expressed in hundred million of LL.

- 1) Calculate the number of demanded units for a unit price of 300 000 000 LL.
- 2) Find the unit price for a supply of 14 000 units.
- 3) a) Prove that the market equilibrium price is 100 000 000 LL.
b) Deduce the corresponding number of units.
- 4) Denote by $E(x)$ the elasticity of the demand with respect to the unit price x .
 - a) Knowing that $f'(x) = (-x - 11)e^{-x+1}$, show that $E(x) = \frac{x^2 + 11x}{x + 12}$.
 - b) Calculate $E(3)$. Is the demand elastic? Justify.
 - c) The unit price of 300 million LL increases by 1%. Calculate the number of demanded units.

VI- Counting and Probability (5 points)

The students of the third secondary of a school are distributed in three sections: GS, LS and ES.

All the students of this class sit for the official exams where History is an optional subject.

The administration of the school collected the following information:

- 20% of the students are in the GS section, among whom 30% chose History.
- 30% of the students are in the LS section, among whom 40% chose History.
- The remaining students are in the ES section, among whom 80% chose History.

A student is randomly selected from the third secondary class of this school and interviewed.

Consider the following events:

G: "The selected student is in the GS section"

L: "The selected student is in the LS section"

E: "The selected student is in the ES section"

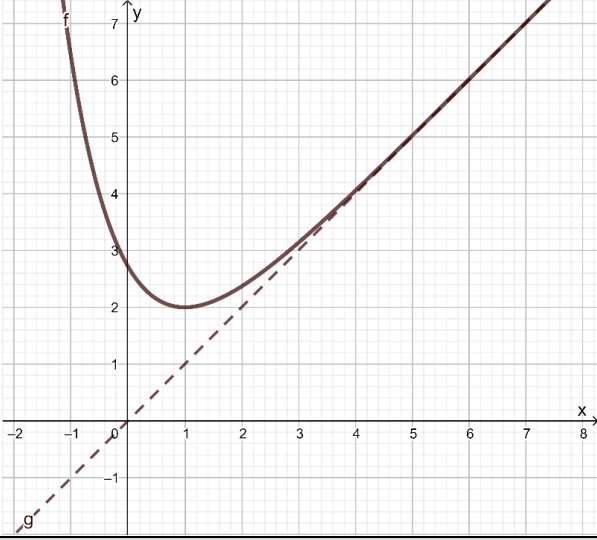
H: "The selected student chose History".

- 1) a) Calculate the probabilities $P(L \cap H)$ and $P(G \cap H)$.
b) Prove that $P(E \cap H) = 0.4$.
c) Deduce that $P(H) = 0.58$.
- 2) The selected student chose History. Calculate the probability that this student is from the ES section.
- 3) The number of students in the third secondary class is 200.
 - a) Copy and complete the table below:

	GS	LS	ES	Total
The number of students who chose History				
The number of students who did not choose History	28	36		
Total			100	200

- b) A group of three students are randomly selected from the third secondary class of this school and interviewed. Calculate the probability that only one student is in ES and that the two other students did not choose History.

أسس تصحيح مسابقة الرياضيات

Q.I	Answers	6.25 pts												
1	$\frac{240-200}{200} \times 100 = 20$ then The percentage increase is 20%	1.5												
2	$\bar{x} = 3.5$ and $\bar{y} = 148.33$ then $G(3.5, 148.33)$	1												
3	$r = 0.98 \approx 1$ then the relation between x and y is strong linear positive.	1.5												
4	$(D_{y/x}): y = 32.28x + 35.33$	1												
5	$0.01y > 3$ then $y > 300$, then $32.28x + 35.33 > 300$, then $x > 8.1$ Thus, $x = 9$. Thus, in 2027.	1.25												
Q.II	Answers	6.25 pts												
1a	$\lim_{x \rightarrow +\infty} f(x) = +\infty + 0 = +\infty$	0.5												
1b	$\lim_{x \rightarrow +\infty} (f(x) - y_d) = \lim_{x \rightarrow +\infty} e^{-x+1} = 0$ So, (d): $y = x$ is an oblique asymptote at $+\infty$.	0.75												
1c	$f(x) - y_d = e^{-x+1} > 0$ for all values of x So, (C) is above (d) for all values of x.	0.5												
2a	$f'(x) = 1 - e^{-x+1}$	0.5												
2b	$f'(x) = 0$, when $e^{-x+1} = 1$, then $-x + 1 = 0$, then $x = 1$	0.75												
2c	$f'(1) = 0$, so (T) is parallel to the x-axis. $f(1) = 2$. Therefore, (T): $y = 2$.	0.75												
2d	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>$-\infty$</td> <td>1</td> <td>$+\infty$</td> </tr> <tr> <td>f'(x)</td> <td>-</td> <td>0</td> <td>+</td> </tr> <tr> <td>f(x)</td> <td>$+\infty$</td> <td>2</td> <td>$+\infty$</td> </tr> </table>	x	$-\infty$	1	$+\infty$	f'(x)	-	0	+	f(x)	$+\infty$	2	$+\infty$	0.5
x	$-\infty$	1	$+\infty$											
f'(x)	-	0	+											
f(x)	$+\infty$	2	$+\infty$											
3a	$f(0) = e \approx 2.7$ and $f(-1) = -1 + e^2 \approx 6.38$	0.5												
3b		1.5												
Q.III	Answers	6.25 pts												
1a	$C_F = C(0) = 3$ in millions of LL = 3 000 000 LL	1.5												
1b	$\bar{C}(0.98) = \frac{C(0.98)}{0.98} = \frac{4.90}{0.98} = 5$ in ten millions of LL Average cost of 1 object is $\frac{50000000}{1000} = 50\ 000$ LL	1.5												

1c	$x_A = 0.98$ At A, $C = R$, so the abscissa of A represent the break-even level of this production. The break-even level is 980 objects.	1.25																				
2a	$P(x) = R(x) - C(x) = 5x - x^2 + e^{-3x} - 4$	1																				
2b	$P(0.5) = -1.52$ Since $P(0.5) = -1.52 < 0$, then the company does not realize a gain for a production of 500 objects	0.5 0.5																				
Q.IV	Answers	6.25 pts																				
1	$\ln(3a) - \ln\left(\frac{3}{e}\right) = \ln 3 + \ln a - \ln 3 + 1 = \ln a + 1$. Answer: a	1.5																				
2	$x - 1 > 0$ and $-x + 3 > 0$, then $1 < x < 3$. Answer: b	1.75																				
3	$C(x) = x^2 - e^{-5x} + 2$ $C_m(x) = 2x + 5e^{-5x}$ then $C_m(6) = 12 + 5e^{-30}$. Answer: b	1.5																				
4	$E(x) = 1$, then $\frac{1}{2 - \ln(x)} = 1$, then $2 - \ln x = 1$, then $\ln x = 1$, $x = e$. Answer: b	1.5																				
Q.V	Answers	6.25 pts																				
1	The number of demanded units for a unit price of 300 000 000 is $f(3) \times 10^3 = 2.03 \times 1000 = 2030$ units	1																				
2	$x + 12 = 14$ then $x = 2$ so the unit price for a supply of 14 000 units is 200 000 000 LL	1																				
3a	$f(1) = g(1) = 13$, so the demand and supply are equal for $x = 1$. The market equilibrium price is 100 000 000 LL	1																				
3b	The corresponding number of units is 13 000 units	0.5																				
4a	$E(x) = \frac{-x(-x-11)e^{-x+1}}{(x+12)e^{-x+1}} = \frac{x^2+11x}{x+12}$	0.75																				
4b	$E(3) = \frac{14}{5} > 1$ then the demand is elastic	1																				
4c	The number of demanded units will be $f(3) \times 10^3 \times \frac{100-14}{100} \cong 1973$ units	1																				
Q.VI	Answers	6.25 pts																				
1a	$P(L \cap H) = P(L) \times P(H/L) = 0.3 \times 0.4 = 0.12$ $P(G \cap H) = P(G) \times P(H/G) = 0.2 \times 0.3 = 0.06$	0.75 0.75																				
1b	$P(E \cap H) = P(E) \times P(H/E) = 0.5 \times 0.8 = 0.4$	0.75																				
1c	$P(H) = P(E \cap H) + P(L \cap H) + P(G \cap H) = 0.06 + 0.12 + 0.4 = 0.58$	0.75																				
2	$P(E/H) = \frac{P(E \cap H)}{P(H)} = \frac{0.4}{0.58} = \frac{20}{29}$	1.25																				
3a	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th></th> <th>GS</th> <th>LS</th> <th>ES</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>The number of students who chose History</td> <td style="text-align: center;">12</td> <td style="text-align: center;">24</td> <td style="text-align: center;">80</td> <td style="text-align: center;">116</td> </tr> <tr> <td>The number of students who did not choose History</td> <td style="text-align: center;">28</td> <td style="text-align: center;">36</td> <td style="text-align: center;">20</td> <td style="text-align: center;">84</td> </tr> <tr> <td>Total</td> <td style="text-align: center;">40</td> <td style="text-align: center;">60</td> <td style="text-align: center;">100</td> <td style="text-align: center;">200</td> </tr> </tbody> </table>		GS	LS	ES	Total	The number of students who chose History	12	24	80	116	The number of students who did not choose History	28	36	20	84	Total	40	60	100	200	1
	GS	LS	ES	Total																		
The number of students who chose History	12	24	80	116																		
The number of students who did not choose History	28	36	20	84																		
Total	40	60	100	200																		
3b	$P(T) = \frac{C_{100}^1 \times C_{64}^2}{C_{200}^3} = \frac{336}{2189}$	1																				