عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة ونصف	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (5 points)

A survey was implemented on the active labor force in Lebanon.
The table below shows an estimation of the number of unemployed people y_i (among the labor force in Lebanon) each year (rank x_i) in June from 2016 till 2021.

Year	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**
Rank x_i	1	2	3	4	5	6
Yearly number y_i **in 100 000 of unemployed**	2	3	4	6	11	13

1) Determine the percentage increase of unemployed from June 2019 till June 2021.
2) Calculate the coordinates of the center of gravity G.
3) Calculate the coefficient of linear correlation r and interpret the value thus obtained.
4) Write an equation of the regression line $(D_{y/x})$ of y in x.
5) Represent, in an orthogonal system,
- the scatter plot of points (x_i , y_i),
- the center of gravity G,
- the regression line $(D_{y/x})$.

6) The above model remains valid only till the year 2025
Suppose that the labor force in Lebanon is estimated to be 3 100 000 in the year 2025.
Samer claims: "The percentage of unemployed will reach 75 % of the labor force in June 2025".
Is he right? Justify.

II- (5 points)

Part A

Consider two urns U and V such that:
- U contains five balls: three red and two black.
- V contains three balls: one red and two black.

One ball is selected randomly from the urn U and two balls are selected randomly and simultaneously from the urn V.

1) Verify that the total number of all possible selections is 15.
2) Calculate the number of selections consisting of one red ball from U and two balls with different colors from V.
3) Calculate the number of selections consisting of three balls with the same color.

Part B

Answer, **with justification**, by **true** or **false**.

1) The equation $\ln(x) = -2$ has a solution in $\mathbb{R}$.
2) For $x > 0$ if $g(x) = \ln(x^2 + x)$ then $g'(x) = \frac{1}{x^2+x}$.
3) The domain of definition of the function h given by $h(x) = \ln(1 + x) + \ln(x - 2)$ is $]-\infty; -1[\cup]2; +\infty[$.

III- (10 points)

Part A

Consider the function f defined over $[0 ; +\infty[$ as $f(x) = e^{0.5x} - 1$.

Denote by (C) the representative curve of f in an orthonormal system $(O ; \vec{i}, \vec{j})$.

1) Determine $\lim\limits_{x \to +\infty} f(x)$. Calculate f(2) and f(4).

2) **a-** Calculate f '(x) and set up the table of variations of f.

b- Deduce the sign of f(x).

3) **a-** Show that the line (T) with equation y = 0.5x is tangent to (C) at O.

b- Draw (T) and (C). (Note that (C) is above (T))

Part B

A gas station sells fuel.

The supply function f and the demand function g are modeled, for all $x \in [1 ; 8]$, as

$f(x) = e^{0.5x} - 1$ and $g(x) = \frac{5}{e^{0.5x}+1}$ where:

- x is the sale unit price in hundred millions of LL,
- f(x) and g(x) are expressed in ten thousand liters (One unit is worth ten thousands liters).

1) **a-** Solve the equation f(x) = g(x).

b- Deduce the equilibrium price (in tens of millions) then find the corresponding quantity.

2) **a-** Show that the elasticity of the demand with respect to the unit price is $E(x) = \frac{0.5xe^{0.5x}}{e^{0.5x}+1}$.

b- Calculate E(2) and interpret the result found.

3) In this part, the sale price of one liter of fuel is 15 000 LL.

a- Estimate, in liters, the demanded quantity of fuel as well as the supply quantity.

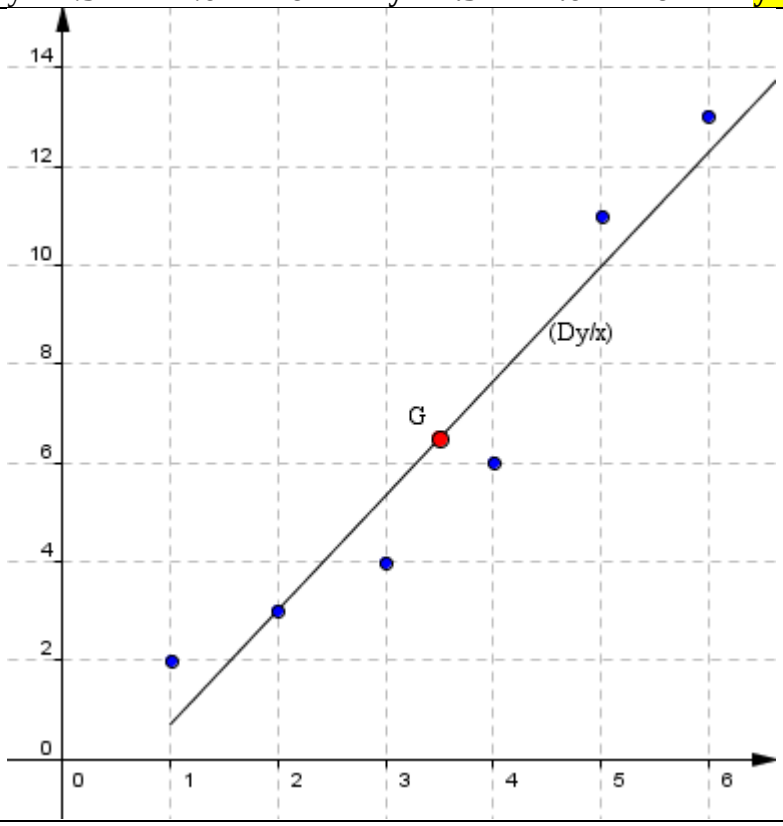
b- If all the quantity supplied is sold and if the profit achieved by this sale is 11 170 000 LL, calculate, then, the average cost of 20 liters in LL.

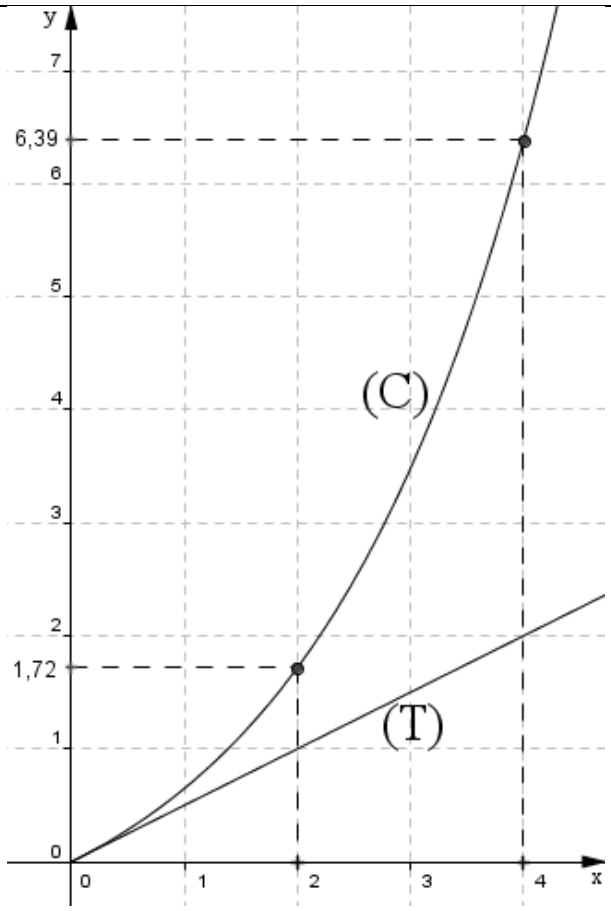
وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة الثانوية العامة
فرع: الاجتماع والاقتصاد

دورة العام ٢٠١٨ العاديّة
الإثنين ١١ حزيران ٢٠١٧

أسس تصحيح مسابقة الرياضيات

Q.I	Answers	6.25 pts
1	The percentage increase = $\frac{13-6}{6} \times 100 = 117\ \%$	1
2	G(3.5 ; 6.5)	1
3	r = 0.961 then there is a strong positive correlation between x and y.	1
4	y = 2.314x – 1.6 or **y = 2.31x – 1.6** or **y = 2.3x – 1.6**	1
5		1.25
6	In June 2025, x = 10 then y = 21,54 or **y = 21,8** or **y = 21,4** Then the percentage is $\frac{100\,000 \times 21.54}{3\,100\,000} = 69.4\ \%$ **where the percentage is $\frac{100\,000 \times 21.8}{3\,100\,000} = 70.3\ \%$** Or **the percentage is $\frac{100\,000 \times 21.4}{3\,100\,000} = 69.03\ \%$** Thus, Samer is not right since the percentage does not reach 75 %.	1
Q.II	**Answers**	**6.25 pts**
A1	$C_5^1 \times C_3^2 = 5 \times 3 = 15$	1
A2	$C_3^1 \times (C_1^1 \times C_2^1) = 6$	1
A3	$C_2^1 \times C_2^2 = 2$	1.25
B1	True since lnx = –2.lne = $\ln e^{-2}$ then $x = e^{-2}$.	1
B2	False, since $g'(x) = \frac{2x+1}{x^2+x}$	1
B3	False, since h is defined if x > –1 and x > 2 then x > 2 .Thus, $D_f =]2 ; +\infty[$.	1

Q.III	Answers	12.5 pts
A1	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} (e^{0.5x} - 1) = +\infty$; f(2) = 1.72 ; f(4) = 6.39	**2**
A2a	$f'(x) = 0.5e^{0.5x}$ x | 0 | $+\infty$ f '(x) | + | f(x) | 0 → $+\infty$	**2**
A2b	$f(x) \geq 0$.	**0,5**
A3	f '(0) = 0.5 then (T) : y = f '(0)(x – 0) + f(0) then y = 0.5x	**1**
A4		**1**
B1a	$e^{0.5x} - 1 = \frac{5}{e^{0.5x}+1}$ then $e^x - 1 = 5$ thus x = ln6	**1**
B1b	ln6 = 1.791 then the price of equilibrium is 179 100 000 LL. f(ln6) = 1.449 then the equilibrium quantity is 14 490 liters.	**1**
B2a	$g'(x) = \frac{5(-0.5e^{0.5x})}{(e^{0.5x}+1)^2}$ then $E(x) = -\frac{xg'(x)}{g(x)} = \frac{0.5xe^{0.5x}}{e^{0.5x}+1}$	**1**
B2b	E(2) = 0.73 As the unit price x=2 increases by 1%, then the demand g(2) decreases by 0.73 %.	**1**
B3a	Price of one liter = 15 000 LL then the unit price in 10 millions LL becomes $\frac{10\,000 \times 15\,000}{100\,000\,000}$ = 1.5 then f(1.5) = 1.117 thus the supply quantity is 11 170 liters. g(1.5) = 1.604 thus the demanded quantity is 16 040 liters. Remark that : f(1.5) < g(1.5).	**1**
B3b	C_T = R – P with R = quantity × unit price = **f(1.5)** × 1.5 in ten millions LL = 1.117 × 1.5 × 100 000 000 LL = 167 550 000 LL. then C_T = 167 550 000 – 11 170 000 = 156 380 000 LL. Thus the average cost of 1 liter is $\frac{156\,380\,000}{11\,170}$ = 14 000 LL. So, the average cost of 20 liters is 280 000 LL.	**1**