

الاسم:
الرقم:

مسابقة في مادة الرياضيات
المدّة: ساعة ونصف السّاعة

عدد المسائل: أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (6 points)

The price of 1 kg of rice was affected by an economic crisis. The following table shows the change in price of 1 kg of rice, in thousands LL, from January 2021 till July 2021.

Month	January	February	March	April	May	June	July
Rank: x_i	1	2	3	4	5	6	7
Price of 1 kg in thousands LL: y_i	6	10	14	15	18	20	22

Numbers are rounded to the nearest 10^{-3} .

- 1) Determine the coordinates of the center of gravity $G (\bar{X}; \bar{Y})$.
- 2) Determine the coefficient of correlation r and interpret the obtained value.
- 3) An equation of the regression line ($D_{y/x}$), of y in terms of x , is: $y = 2.571x + m$ where m is a real number.
Find an approximate value of m to the nearest 10^{-3} .
- 4) In an orthogonal system, represent the scatter plot of points $(x_i; y_i)$, plot the center of gravity G and draw the regression line ($D_{y/x}$).
- 5) Suppose that the preceding model remains valid till the end of the year 2021.
Show that the price of 1 kg of rice would exceed 30 000 LL in 2021.

II- (2 points)

Consider the two functions f and g expressed as:

$$f(x) = \ln(x + 2) \text{ and } g(x) = \ln x + \ln 8 - \ln 4.$$

- 1) Determine the domain of definition of f .
- 2) For all $x > 0$:
 - a- Show that $g(x) = \ln(2x)$.
 - b- Solve the equation: $f(x) = g(x)$.

III- (4 points)

An urn U contains fifteen balls: 8 red balls, 5 white balls and 2 black balls.

Three balls are randomly and simultaneously selected from U .

- 1) Verify that the total number of possible selections is 455.
- 2) Determine the number of different selections containing:
 - a- three red balls.
 - b- three balls of the same color.
 - c- at least one red ball.

IV- (8 points)

Consider the function f defined, on $[0; +\infty[$, as $f(x) = xe^{-x+2}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

Part A

- 1) Calculate $f(0)$ and $f(1.5)$.
- 2) Show that $\lim_{x \rightarrow +\infty} f(x) = 0$. Deduce an asymptote to (C) .
- 3) **a-** Show that $f'(x) = (1 - x)e^{-x+2}$.
b- Copy and complete the following table of variations of f :

x	0	1	$+\infty$
$f'(x)$	+	0	-
$f(x)$			

- 4) Draw (C) .

Part B

A factory produces pens.

The demand d and the supply g , in thousands of pens, are modeled as $d(x) = e^{-x+2}$ and $g(x) = e^{x-2}$ where x is the unit price in ten millions LL with $0 < x \leq 8$.

- 1) Calculate the number of demanded pens for a unit price of 30 000 000 LL.
- 2) **a-** Solve the equation $d(x) = g(x)$.
b- Deduce the equilibrium price in LL and the corresponding number of pens.
- 3) The revenue, in ten millions LL, is modeled as $f(x) = xe^{-x+2}$ with $0 < x \leq 8$.
a- For what unit price is the revenue maximal?
 Show, in this case, that the price p of one pen is 10 000 LL.
b- If p increases by 5 000 LL, Sami claims that the revenue will be 35 000 000 LL.
 Is Sami's claim true? Justify.

دورة العام ٢٠٢١ العادية السبت في ٣١ تموز ٢٠٢١	امتحانات الشهادة الثانوية العامة فرع: الاجتماع والاقتصاد	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسميّة عدد المسائل: أربع مسائل
	أسس تصحيح مسابقة الرياضيات	

QI	Answer key	Mark
1	Using the calculator, $G(4 ; 15)$	1
2	Using the calculator, $r = 0.987$ Interpretation : strong positive correlation between x and y	1 ½
3	Using the calculator, $m = A = 4.714$ Second method: using point G, $y_G = 2.571x_G + m \Rightarrow m = 4.716$	1
4		3
5	$y > \frac{30\,000}{1\,000}$ then $y > 30$ then $x > 9,83$ In October 2021 the price of 1 kg of rice will exceed 30 000	1

QII	Answer key	Mark
1	$x + 2 > 0; x > -2; D_f =] - 2; +\infty[$	1
2a	$g(x) = \ln x + \ln 8 - \ln 4 = \ln \left(\frac{8x}{4} \right) = \ln (2x)$	½
2b	$\ln(x + 2) = \ln(2x)$ $x + 2 = 2x$ $x = 2 > 0$	1

QIII	Answer key	Mark
1	$C_{15}^3 = 455$	1
2a	$C_8^3 = 56$	1 ½
2b	$C_8^3 + C_5^3 = 56 + 10 = 66$	1 ½
2c	$C_{15}^3 - C_7^3 = 455 - 35 = 420$ Ou, $C_8^1 \times C_7^2 + C_8^2 \times C_7^1 + C_8^3 = 420$	1

QIV	Answer key	Mark												
A1	$f(0) = 0; f(1,5) = 2,473$	1												
A2	$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \left(\frac{x}{e^{x-2}} \right) = \text{R.H} = \lim_{x \rightarrow +\infty} \left(\frac{1}{e^{x-2}} \right) = \frac{1}{+\infty} = 0$ Or $\lim_{x \rightarrow +\infty} x e^{-x} e^2 = 0$ because $\lim_{x \rightarrow +\infty} x e^{-x} = 0$ $y = 0$ is an asymptote to (C).	1												
A3a	$f'(x) = e^{-x+2} - x e^{-x+2} = (1-x)e^{-x+2}$	1												
A3b	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">$+\infty$</td> </tr> <tr> <td style="padding: 5px;">$f'(x)$</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">+</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">$f(x)$</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">-</td> </tr> </table> 	x	0	1	$+\infty$	$f'(x)$		+	0	$f(x)$			-	1
x	0	1	$+\infty$											
$f'(x)$		+	0											
$f(x)$			-											
A4		1												
B1	$x = \frac{30\,000\,000}{10\,000\,000} = 3; d(3) = e^{-1} = 0,3678$ The number of demanded pens for a unit price of 30 000 000 is estimated for 368	1												
B2a	$d(x) = g(x)$ $e^{-x+2} = e^{x-2}$ $-x + 2 = x - 2$ $2x = 4; x = 2$	1												
B2b	The equilibrium price in ten millions LL is 2 or it is 20 000 000 LL $d(2) = 1$ The corresponding number of pens is: 1 000 pens	1												
B3a	Using the table of variations of f, f is maximal when $x = 1$ The revenue is maximal for a unit price of 1 in ten millions LL or 10 000 000 LL So the price of a pen $p = \frac{10\,000\,000}{1\,000} = 10\,000$ LL	1												
B3b	$f(1) = 2.718$, the maximal revenue is 27 180 000 LL < 35 000 000 LL Second method: $10\,000 + 5\,000 = 15\,000 \rightarrow \frac{1\,000 \times 15\,000}{10\,000\,000} = 1.5$ $f(1.5) = 2.473 < \frac{35\,000\,000}{10\,000\,000}$ Sami is not right.	1												