

اسم:
الرقم:
مسابقة في مادة الرياضيات
المدة: ساعة واحدة

ملاحظة: - يتكوّن هذا الإمتحان من أربع مسائل، يجب اختيار مسألتين منها فقط.

- في حال الإجابة عن أكثر من مسألتين، عليك شطب الإجابات المتعلقة بالمسألة التي لم تعد من ضمن اختيارك، لأنّ التصحيح سيقترصر على إجابات أوّل مسألتين غير مشطوبتين.
- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- Functions (10 points)

Consider the function f defined over $] -\infty, 0[\cup] 0, +\infty[$ as $f(x) = \frac{x^2 + 1}{x}$ and denote by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$. Let (d) be the line with equation $y = x$.

- 1) a) Determine $\lim_{\substack{x \rightarrow 0 \\ x < 0}} f(x)$ and $\lim_{\substack{x \rightarrow 0 \\ x > 0}} f(x)$.
b) Deduce an equation of an asymptote to (C).
- 2) Given that $f(x) = x + \frac{1}{x}$.
a) Determine $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow +\infty} f(x)$.
b) Show that (d) is an oblique asymptote to (C) at $+\infty$ and at $-\infty$.
- 3) a) Verify that $f'(x) = \frac{x^2 - 1}{x^2}$.
b) Copy and complete the following table of variations of f :

x	$-\infty$	-1	0	1	$+\infty$
$f'(x)$		0		0	
$f(x)$					

- 4) Draw (d) and (C).

II- Statistics (10 points)

The following table shows the distribution of grades (out of 20) of the students of a given class.

Grades	[0 , 5[[5 , 10[[10 , 15[[15 , 20]
Number of students	4	5	8	3

- 1) What is the number of students whose grades are greater than or equal to 10?
- 2) Calculate the mean grade of these students.
- 3) a) Construct a frequency histogram.
b) Find the modal class.
c) Find, graphically the mode. Interpret the obtained result.
- 4) a) Set up the increasing cumulative frequency table.
b) Find the median class.

III- Probability (10 points)

The following table shows the types of movies preferred by 100 students at a school.

	Science fiction	Drama	Horror	Total
Girls	15	33	9	57
Boys	16	17	10	43
Total	31	50	19	100

- 1) One student is randomly selected and interviewed. Consider the following events:
 - B: "The interviewed student is a boy"
 - S: "The interviewed student prefers Science fiction movies".
 - a) Calculate the probabilities $P(S)$ and $P(B)$.
 - b) Calculate $P(S \cap B)$ and $P(S \cup B)$.
 - c) Calculate $P(S / B)$.
- 2) Two students are randomly and successively selected and interviewed one after the other.
 - a) Calculate the probability that the first student prefers the Science fiction movies and the second student prefers the horror movies.
 - b) Calculate the probability that the two students prefer horror movies.

IV- Functions (10 points)

The plane is referred to an orthonormal system $(O ; \vec{i}, \vec{j})$.

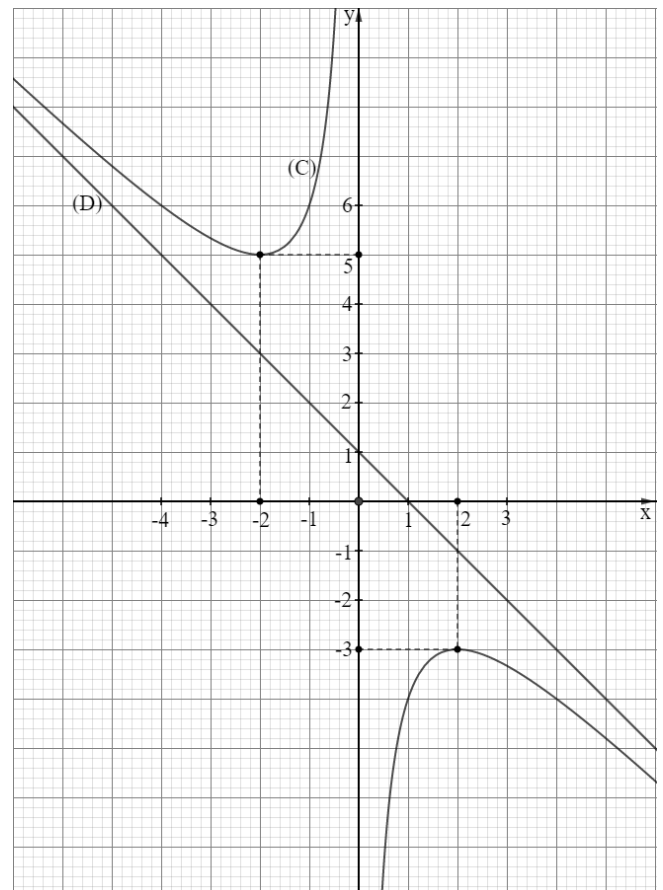
The curve (C) in the adjacent figure represents a function f defined over $]-\infty, 0[\cup]0, +\infty[$.

The line (D) is an asymptote to (C) at $+\infty$ and at $-\infty$.

- 1) Find $f(2)$ and $f(-2)$.
- 2) Determine $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow +\infty} f(x)$.
- 3) Show that the y-axis is an asymptote to (C).
- 4) Copy and complete the table of variations of f :

x	$-\infty$	-2	0	2	$+\infty$
$f'(x)$		0		0	
$f(x)$					

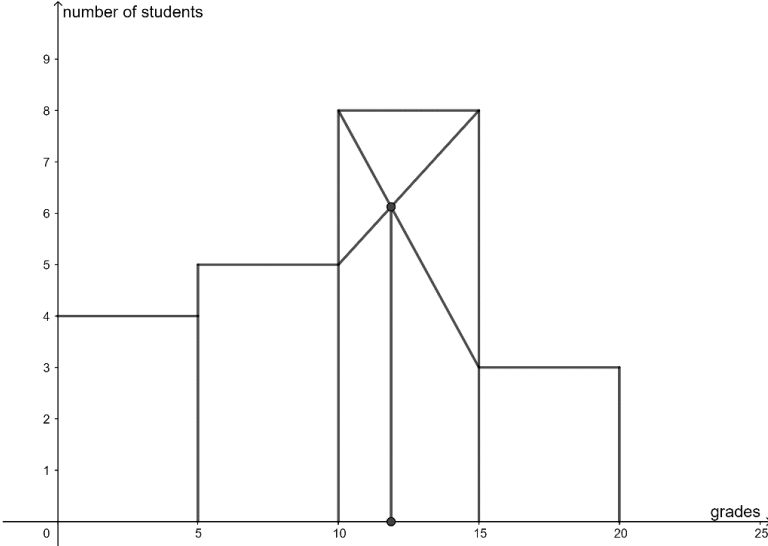
- 5) Find the solutions of the equation $f(x) = 6$.
- 6) Compare $f(2.5)$ and $f(3)$. Justify your answer.
- 7) Compare $f'(0.5)$ and $f'(3.5)$. Justify your answer.



أسس تصحيح مسابقة الرياضيات

Q.I	Answers	10 pts																			
1a	$\lim_{x \rightarrow -0^+} f(x) = +\infty$; $\lim_{x \rightarrow -0^-} f(x) = -\infty$	1																			
1b	$x = 0$ is a vertical asymptote	1																			
2a	$\lim_{x \rightarrow +\infty} f(x) = +\infty$; $\lim_{x \rightarrow -\infty} f(x) = -\infty$	1																			
2b	$\lim_{x \rightarrow \pm\infty} (f(x) - y_d) = \lim_{x \rightarrow \pm\infty} \frac{1}{x} = 0$ Thus, (d) is an oblique asymptote to (C) at $\pm\infty$	1.5																			
3a	$f'(x) = \frac{(2x)(x) - (1)(x^2+1)}{x^2} = \frac{x^2-1}{x^2}$	1.5																			
3b	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>$-\infty$</td> <td>-1</td> <td>0</td> <td>1</td> <td>$+\infty$</td> </tr> <tr> <td>$f'(x)$</td> <td>+</td> <td>0</td> <td>-</td> <td>-</td> <td>0</td> <td>+</td> </tr> <tr> <td>$f(x)$</td> <td>$-\infty$</td> <td>-2</td> <td>$+\infty$</td> <td>2</td> <td>$+\infty$</td> </tr> </table>	x	$-\infty$	-1	0	1	$+\infty$	$f'(x)$	+	0	-	-	0	+	$f(x)$	$-\infty$	-2	$+\infty$	2	$+\infty$	2
x	$-\infty$	-1	0	1	$+\infty$																
$f'(x)$	+	0	-	-	0	+															
$f(x)$	$-\infty$	-2	$+\infty$	2	$+\infty$																
4		2																			

Q.II	Answers	10 pts															
1	Number of students whose grades are greater or equal to 10 is $8 + 3 = 11$	1.5															
2	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Grades</td> <td>$[0, 5[$</td> <td>$[5, 10[$</td> <td>$[10, 15[$</td> <td>$[15, 20[$</td> </tr> <tr> <td>Center</td> <td>2.5</td> <td>7.5</td> <td>12.5</td> <td>17.5</td> </tr> <tr> <td>Number of students</td> <td>4</td> <td>5</td> <td>8</td> <td>3</td> </tr> </table> <p>Mean grade = 1.5 (by calculator)</p>	Grades	$[0, 5[$	$[5, 10[$	$[10, 15[$	$[15, 20[$	Center	2.5	7.5	12.5	17.5	Number of students	4	5	8	3	1.5
Grades	$[0, 5[$	$[5, 10[$	$[10, 15[$	$[15, 20[$													
Center	2.5	7.5	12.5	17.5													
Number of students	4	5	8	3													

3a		2																								
3b	Modal class is : $[10, 15[$ (highest frequency class)	1																								
3c	Mode ≈ 11.8 The grade 11.8 is the most frequent grade	1.5																								
4a	<table border="1" data-bbox="240 833 1219 922"> <thead> <tr> <th>Grades</th> <th>$[0, 5[$</th> <th>$[5, 10[$</th> <th>$[10, 15[$</th> <th>$[15, 20[$</th> </tr> </thead> <tbody> <tr> <td>ICF</td> <td>4</td> <td>9</td> <td>17</td> <td>20</td> </tr> </tbody> </table>	Grades	$[0, 5[$	$[5, 10[$	$[10, 15[$	$[15, 20[$	ICF	4	9	17	20	1.5														
Grades	$[0, 5[$	$[5, 10[$	$[10, 15[$	$[15, 20[$																						
ICF	4	9	17	20																						
4b	$\frac{20}{2} = 10$ 10 is between the two ICF's 9 and 17, so the median class is $[10, 15[$	1																								
Q.III	Answers	10 pts																								
1a	$P(S) = 0.31$; $P(B) = 0.43$	2																								
1b	$P(S \cap B) = 0.16$ $P(S \cup B) = 0.31 + 0.43 - 0.16 = 0.58$	1.5 2																								
1c	$P(S / B) = \frac{16}{43}$	1.5																								
2a	$P(S, H) = \frac{31}{100} \times \frac{19}{99} = \frac{589}{9900}$	1.5																								
2b	$P(H, H) = \frac{19}{100} \times \frac{18}{99} = \frac{19}{550}$	1.5																								
Q.IV	Answers	10 pts																								
1	$f(2) = -3$; $f(-2) = 5$	1																								
2	$\lim_{x \rightarrow -\infty} f(x) = +\infty$; $\lim_{x \rightarrow +\infty} f(x) = -\infty$	1																								
3	$\lim_{x \rightarrow 0} f(x) = \pm\infty$, so (y'y) is a vertical asymptote to (C).	1.5																								
4	<table border="1" data-bbox="252 1662 1058 1839"> <tbody> <tr> <td>x</td> <td>$-\infty$</td> <td>-2</td> <td>0</td> <td>2</td> <td>$+\infty$</td> </tr> <tr> <td>$f'(x)$</td> <td>-</td> <td>0</td> <td>+</td> <td>+</td> <td>0</td> <td>-</td> </tr> <tr> <td>$f(x)$</td> <td>$+\infty$</td> <td>\searrow</td> <td>5</td> <td>\nearrow</td> <td>$+\infty$</td> <td>$-\infty$</td> <td>\nearrow</td> <td>-3</td> <td>\searrow</td> <td>$-\infty$</td> </tr> </tbody> </table>	x	$-\infty$	-2	0	2	$+\infty$	$f'(x)$	-	0	+	+	0	-	$f(x)$	$+\infty$	\searrow	5	\nearrow	$+\infty$	$-\infty$	\nearrow	-3	\searrow	$-\infty$	2
x	$-\infty$	-2	0	2	$+\infty$																					
$f'(x)$	-	0	+	+	0	-																				
$f(x)$	$+\infty$	\searrow	5	\nearrow	$+\infty$	$-\infty$	\nearrow	-3	\searrow	$-\infty$																
5	The line of equation $y = 6$ cuts (C) at two points. The equation $f(x) = 6$ has solutions $x = -4$ or $x = -1$	1.5																								
6	2.5 and 3 belong to the interval $]2; +\infty[$ where f is strictly decreasing, so $f(2.5) > f(3)$.	1.5																								
7	$f'(0.5) > 0$ and $f'(3.5) < 0$ So $f'(3.5) < f'(0.5)$	1.5																								