

عدد المسائل: ثلاث	مسابقة في مادة الرياضيات	الاسم:
	المدّة: ساعة واحدة	الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.  
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه ( دون الالتزام بترتيب المسائل الواردة في المسابقة).

### I- (5 points)

The following table shows the distribution of a group of students of the third secondary class (LH section) based on gender and the elective subject chosen (mathematics or another subject).

	Girls	Boys	Total
Mathematics	20	40	60
Another subject	35	5	40
Total	55	45	100

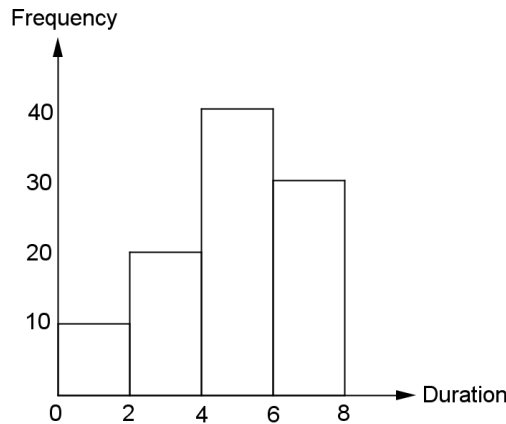
- One student is randomly selected and interviewed.  
Consider the following events:  
A: "The interviewed student is a boy"  
M: "The interviewed student chose mathematics".  
a- Calculate the probabilities  $P(M)$  and  $P(A \cap M)$ .  
b- Knowing that the interviewed student chose mathematics, calculate the probability that this student is a boy.  
c- Show that  $P(A \cup M) = \frac{13}{20}$ .
- The name of each of the 100 students is written on a separate card, then we place the 100 cards in a box.  
Two cards are selected randomly and successively without replacement from the box.  
Calculate the probability that the two selected cards hold the names of two students who chose mathematics.

### II- (5 points)

The following table shows the daily duration (in hours) spent by 100 students on social media.

Duration (in hours)	[0 ; 2[	[2 ; 4[	[4 ; 6[	[6 ; 8]	Total
Frequency	10	20	40	30	100
Increasing cumulative frequency		30			

- a- Copy and complete the given table with the increasing cumulative frequencies.  
b- Determine the number of students who spent, daily, at least 4 hours on social media.
- The frequency histogram of the given statistical data is represented below.



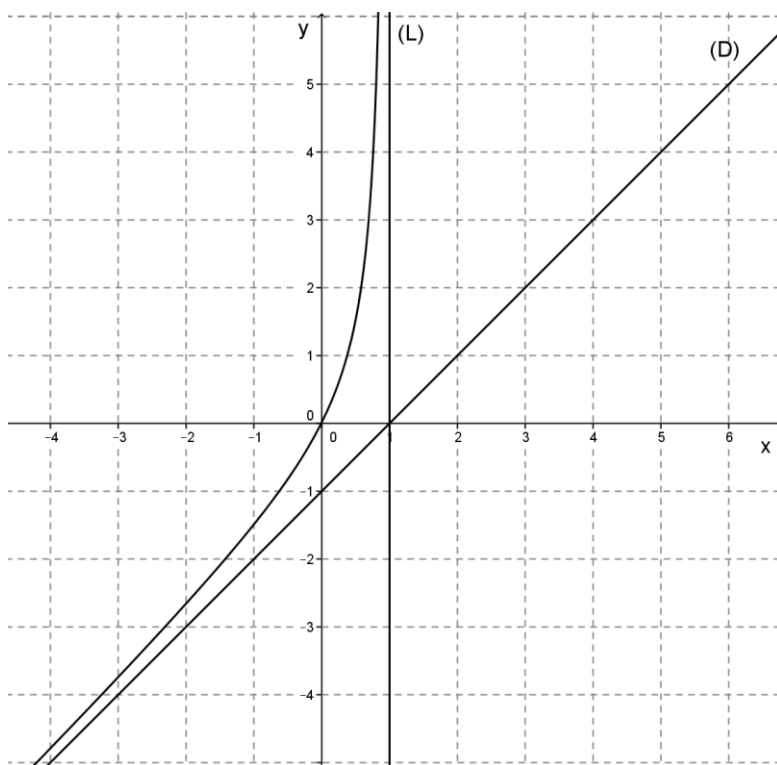
Copy the histogram, then estimate the mode. Interpret the result.

### III- (10 points)

Consider the function  $f$  defined on  $]-\infty, 1[ \cup ]1, +\infty[$  as  $f(x) = \frac{x^2 - 2x}{x-1}$ .

Denote by (C) the representative curve of  $f$  in an orthonormal system  $(O; \vec{i}, \vec{j})$ .

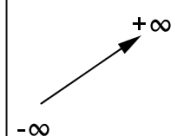
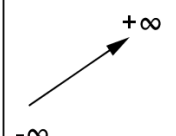
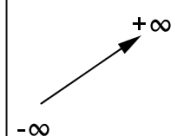
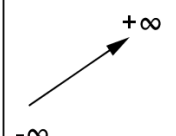
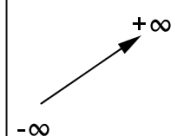
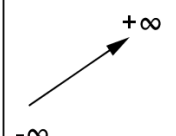
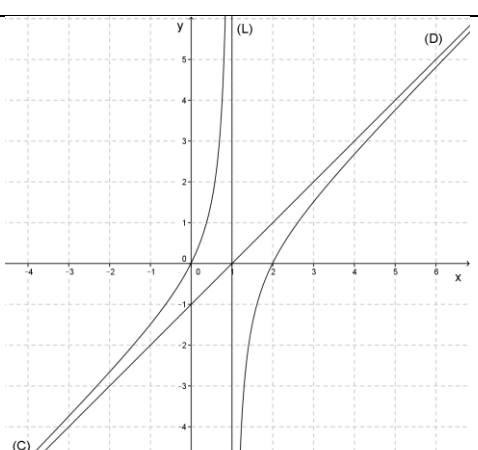
- 1) Show that  $f(x) = x - 1 - \frac{1}{x-1}$ .
- 2) a- Determine  $\lim_{x \rightarrow 1^-} f(x)$  and  $\lim_{x \rightarrow 1^+} f(x)$ .  
b- Deduce an equation of an asymptote (L) to (C).
- 3) a- Determine  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow +\infty} f(x)$ .  
b- Show that the line (D) with equation  $y = x - 1$  is an oblique asymptote to (C).
- 4) a- Show that  $f'(x) = \frac{x^2 - 2x + 2}{(x-1)^2}$ .  
b- Show that  $f'(x) > 0$  on  $]-\infty, 1[ \cup ]1, +\infty[$  and set up the table of variations of  $f$ .
- 5) Show that (C) intersects the x-axis in two points  $O(0, 0)$  and  $A(2, 0)$ .
- 6) The following graph represents the curve (C) on  $]-\infty, 1[$  as well as its asymptotes.  
Copy the graph and draw (C) on  $]1, +\infty[$ .



دورة العام ٢٠٢١ العادية السبت في ٣١ تموز ٢٠٢١	امتحانات الشهادة الثانوية العامة فرع: الآداب والانسانيات	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسميّة عدد المسائل: ثلاث مسائل
	أسس تصحيح مسابقة الرياضيات	

QI	Answer key	Mark
1a	$P(M) = 0.6 ; P(A \cap M) = 0.4$	2
1b	$P(A / M) = \frac{40}{60} = \frac{2}{3}$	1
1c	$P(A \cup M) = 0.65$	1
2	$P(MM) = \frac{60}{100} \times \frac{59}{99} = \frac{59}{165}$	1

QII	Answer key	Mark					
1a	Duration (in hours)	[0 ; 2[	[2 ; 4[	[4 ; 6[	[6 ; 8]	Total	1.5
	Frequency	10	20	40	30	100	
	Increasing cumulative frequency	10	30	70	100		
1b	$40 + 30 = 70$ students	1					
2	<p>Frequency</p> <p>Duration</p>	2.5					
	<p>Frequency</p> <p>Duration</p> <p>About 5.3 hours</p>						

QIII	Question	Mark													
1)	$x - 1 - \frac{1}{x-1} = \frac{x^2 - 2x + 1 - 1}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$	0.5													
2a	$\lim_{\substack{x \rightarrow -1 \\ x < 1}} f(x) = +\infty$ and $\lim_{\substack{x \rightarrow -1 \\ x > 1}} f(x) = -\infty$	1													
2b	(L) : $x = 1$	0.5													
3a	$\lim_{x \rightarrow -\infty} f(x) = -\infty$ and $\lim_{x \rightarrow +\infty} f(x) = +\infty$	1													
3b	$\lim_{x \rightarrow -\infty} [f(x) - y_{(D)}] = \lim_{x \rightarrow -\infty} \left[-\frac{1}{x-1}\right] = 0$	1													
4a	$f'(x) = 1 + \frac{1}{(x-1)^2} = \frac{x^2 - 2x + 2}{(x+1)^2} > 0$	1													
4b	$f'(x) = 1 + \frac{1}{(x-1)^2} > 0$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;"><math>-\infty</math></td> <td style="padding: 5px;">1</td> <td style="padding: 5px;"><math>+\infty</math></td> </tr> <tr> <td style="padding: 5px;">f'(x)</td> <td style="padding: 5px;">+</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">+</td> </tr> <tr> <td style="padding: 5px;">f(x)</td> <td colspan="2" style="padding: 5px; text-align: center;">  </td> <td colspan="2" style="padding: 5px; text-align: center;">  </td> </tr> </table>	x	$-\infty$	1	$+\infty$	f'(x)	+		+	f(x)					2
x	$-\infty$	1	$+\infty$												
f'(x)	+		+												
f(x)															
5	$f(x) = 0; x^2 - 2x = 0; x = 0$ and $x = 2; (0, 0)$ and $(2, 0)$	1													
6		2													