وزارة التربية والتعليم العالي المديريّة العامة للتربية دائرة الامتحانـات الرسميّة	امتحانات الشهادة المتوسطة	دورة العام ٢٠٢١ العاديّة الخميس ٢٩ تموز ٢٠٢١
عدد المسائل: خمس	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الإلتزام بترتيب المسائل الواردة في المسابقة.

I- (3 points)

In the table below, only one of the proposed answers to each question is correct.
Write down the number of the question and give, with justification, its corresponding answer.

N^0	Questions	Proposed answers		
		a	b	c
1	If $x = -2$, then the value of the expression $x^2 + 3x - 2$ is	-12	-2	-4
2	$\dfrac{(\sqrt{3}+1)(\sqrt{3}-1)}{2\sqrt{3}} =$	$\dfrac{1}{2\sqrt{3}}$	$\dfrac{\sqrt{3}}{3}$	$\dfrac{1}{3}$
3	$(\sqrt{2}+1)^2 + (\sqrt{2}-1)^2 =$	6	4	$4\sqrt{2}$
4	If a is a nonzero real number, then $\frac{a}{3} - \frac{a}{3} \times 4 =$	4	$-a$	0

II- (3.5 points)

1) Solve the following system : $\begin{cases} x + y = 12 \\ 3x + 5y = 52. \end{cases}$

2) A box « B » contains 12 pens of two colors, red and green.
The price of this box is 52 000 LL. The price of a red pen is 3 000 LL, and that of a green pen is 5 000 LL.
Denote by x the number of red pens and by y that of green pens in « B ».
a. Prove that the previous text is modeled by the system in question **1)**.
b. Determine the number of pens of each color in « B ».

3) Jad bought some boxes « B » and he paid 208 000 LL.
Find the number of red pens that Jad has.

III- (3.5 points)

Given : $A(x) = (x+2)^2 - 3(x+4)(x+2)$ and $B(x) = x^2 + 3x + 2$.

1) Show that $A(x) = -2(x+5)(x+2)$.

2) Verify that $B(x) = (x+2)(x+1)$.

3) Solve the equation $B(x) = 0$.

4) Given : $F(x) = \dfrac{-2\,(x+5)\,(x+2)}{(x+1)(x+2)}$.

a. For what values of x, $F(x)$ is defined ?

b. Simplify $F(x)$.

c. Can you find x so that $F(x) = -2$? Justify.

IV- (5.5 points)

In a plane referred to an orthonormal system of axes $(x'Ox\ ;\ y'Oy)$, consider the points A(1 ; 4) and B(5 ; 2). Let (d) be the line with equation $y = 2x + 2$.

1) Plot the points A and B.

2) Verify that A is on (d), then draw (d).

3) Let (d') be the line through B and perpendicular to (d).

a. Prove that y = $\frac{-1}{2}x + \frac{9}{2}$ is the equation of (d').

b. Verify that (d) and (d') intersect at A.

4) The line (d) intersects $(x'Ox)$ at E.

a. Calculate the coordinates of E.

b. Prove that the triangle AEB is right isosceles at A.

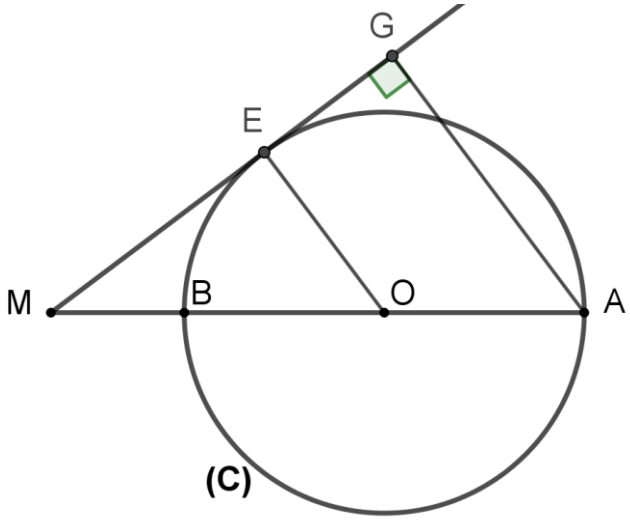
5) Let J be the midpoint of [EB]. Denote by F the symmetric of A with respect to J.

Prove that AEFB is a square.

V- (4.5 points)

In the next figure:

- (C) is a circle with center O and radius 3
- [AB] is a diameter of (C)
- M is a point on (AB) such that OM = 5
- The line (ME) is tangent to (C) at E
- The line (AG) is perpendicular to (ME) at G.



1) Draw the figure.

2) **a.** Verify that ME = 4.

b. Show that $\frac{ME}{MG} = \frac{5}{8}$, then deduce MG and AG.

3) Let F be the orthogonal projection of E on (AM).

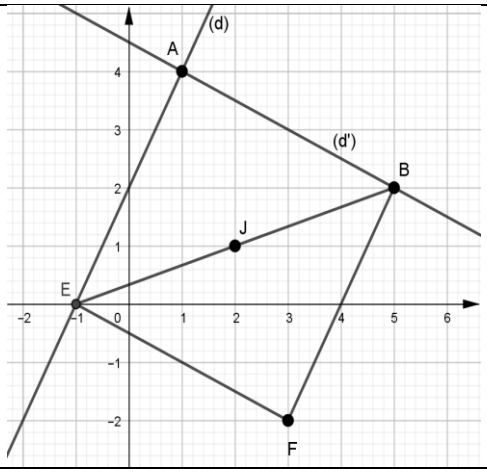
a. Prove that the four points F, E, G and A are on the same circle (C').

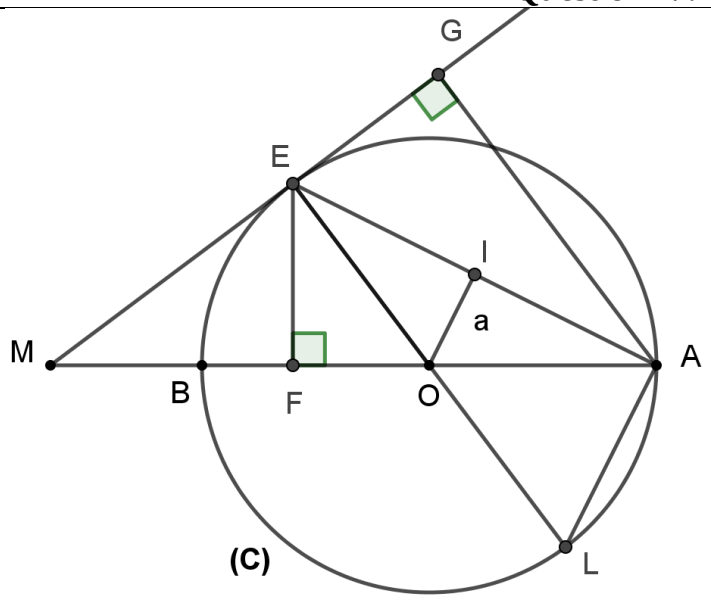
b. Determine the center I of the circle (C').

4) The parallel through A to (OI) intersects (OE) at L.

Prove that L is on (C).

Part of Q	Question I	grades
1	$x^2 + 3x - 2 = 4 - 6 - 2 = -4$ (c)	**0.75**
2	$\frac{(\sqrt{3}+1)(\sqrt{3}-1)}{2\sqrt{3}} = \frac{3-1}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ (b)	**0.75**
3	$(\sqrt{2}+1)^2 + (\sqrt{2}-1)^2 = 2 + 1 + 2\sqrt{2} + 2 + 1 - 2\sqrt{2} = 6$ (a)	**0.75**
4	$\frac{a}{3} - \frac{a}{3} \times 4 = \frac{a}{3} - \frac{4a}{3} = -a$ (b)	**0.75**
	Question II	
1	Using calculator : $x = 4$ and $y = 8$	**1**
2a	The system $\begin{cases} x + y = 12 \\ 3000x + 5000y = 52\,000 \end{cases}$	**1**
2b	Refer to 1) : $x = 4$ *and* $y = 8$	**0.5**
3	$208000 \div 52000 = 4$; $4 \times 4 = 16$	**1**
	Question III	
1	$A(x) = (x+2)^2 - 3(x+4)(x+2)$ $A(x) = (x+2)[(x+2) - 3(x+4)]$ $A(x) = (x+2)(x+2-3x-12)$ $A(x) = (x+2)(-10-2x)$ $A(x) = -2(x+2)(5+x)$	**0.75**
2	$(x+1)(x+2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$ Then $B(x) = (x+1)(x+2)$	**0.75**
3	$B(x) = 0$ then : $x = -2$ or $x = -1$	**0.5**
4a	$F(x) = \frac{-2\,(x+5)\,(x+2)}{(x+1)\,(x+2)}$ $F(x)$ is defined if : $(x+1)\,(x+2) \neq 0$ then : $x \neq -1$ and $x \neq -2$	**0.5**
4b	$F(x) = \frac{-2\,(x+5)}{x+1}$	**0.25**
4c	$\frac{-2\,(x+5)}{x+1} = -2$; then $-2x - 10 = -2x - 2$; $ox = -8$ impossible	**0.75**

Question IV		
1		**0.5**

2	$y_A = 2x_A + 2$;then A is on (d). (0; 2) and A are on (d) see the figure	**1**
3a	Let m be the slope of (d') $m \times 2 = -1$ then $m = -\frac{1}{2}$; $y = \frac{-1}{2}x + \frac{9}{2}$.	**1**
3b	A is on (d') and A is on (d).	**0. 5**
4a	E is on $(x'ox)$ then $y_E = 0$ and E(-1,0)	**0.75**
4b	(d) $\perp$ (d') then: $\widehat{EAB} = 90^0$ And $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{20} = 2\sqrt{5}$ $AE = \sqrt{(x_E - x_A)^2 + (y_E - y_A)^2} = \sqrt{(-2)^2 + (4)^2} = \sqrt{20} = 2\sqrt{5}$ Then ABE is a right isosceles triangle with vertex A.	**1**
5	[AF] and [EB] bisect each other Then the quadrilateral AEFB is a parallelogram. $\widehat{EAB} = 90^0$ hence it's a rectangle. And $AE = AB$. Therefore AEFB is a square.	**0.75**
	Question V.	
1		**0.5**
2 a	According to Phytagoras $ME = \sqrt{16} = 4$.	**0.5**
2 b	According to Thales theorem $\frac{ME}{MG} = \frac{MO}{MA} = \frac{5}{8}$ then $MG = \frac{32}{5}$ $\frac{MO}{MA} = \frac{EO}{AG}$ then $\frac{5}{8} = \frac{3}{AG}$ then $AG = \frac{24}{5}$. **Or** $AG^2 = MA^2 - MG^2$	**1.5**
3a	$\widehat{EFA} = \widehat{EGA} = 90°$, then the four points F,E, G, and A are on the same circle (C') with diameter [EA].	**0.75**
3b	The center I of (C') is the midpoint of [AE].	**0.5**
4	(AL) parallel to (OI) and I midpoint of [AE], then O midpoint of [EL]. Hence L is on (C)	**0.75**