وزارة التربية والتعليم العالي
المديريّة العامّة للتربية
دائرة الامتحانات الرسميّة

امتحانات الشهادة الثانوية العامّة
فرع العلوم العامّة

دورة العام ٢٠١٩ العاديّـة
السبت ٢٢ حزيران ٢٠١٩

عدد المسائل: ست	مسابقة في مادة الرياضيات المدة: أربع ساعات	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (2 points)

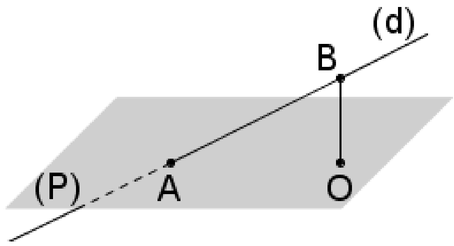
In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, the answer corresponding to it.

Nº	Questions	Answers		
		a	b	c
1	z is a complex number. One of the roots of the equation $z^4+z^2-i\sqrt{3}=0$ is	$e^{i\frac{\pi}{6}}$	i	$e^{-i\frac{\pi}{6}}$
2	Denote by f the function defined on $\mathbb{R}$ as f(x) = $\frac{1}{x^2+4x+8}$. An antiderivative of f is	$\frac{1}{2}\arctan\left(\frac{x+2}{2}\right)$	$\frac{1}{4}\arctan\left(\frac{x+2}{2}\right)$	$\arctan(x+2)$
3	m is a real number (m > 1). If $J=\int_m^{m+1}\frac{1}{x}\,dx$, then J belongs to the interval	$[m\ ,\ m+1]$	$\left]0,\frac{1}{m+1}\right[$	$\left[\frac{1}{m+1},\frac{1}{m}\right]$
4	z is a complex number. If $z=1+\cos\theta+i\sin\theta$, with $\pi<\theta<2\pi$, then $\lvert z\rvert=$	$2\cos\left(\frac{\theta}{2}\right)$	$-2\cos\left(\frac{\theta}{2}\right)$	$\sqrt{2}$

II- (2 points)

In the space referred to an orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the plane (P) with equation x + z = 0

and the line (d) with parametric equations $\begin{cases} x=-t+1 \\ y=-2t \\ z=3t+1 \end{cases}$ (t ∈ $\mathbb{R}$).



1) Calculate the coordinates of point A, the intersection of (d) and (P).
2) The point B(1 , 0 , 1) is on (d).
   a- Show that O is the orthogonal projection of B on (P).
   b- Deduce a system of parametric equations of the line (Δ), the orthogonal projection of (d) on (P).
3) The point J(–5 , 2 , 5) is on (P).
   Calculate the volume of the tetrahedron OBJA.
4) In the plane (P), consider the hyperbola (H) with foci O and A and eccentricity e = 3.
   a- Verify that the point I(1 , 1 , –1) is the center of (H).
   b- Calculate the coordinates of S and G, the two vertices of (H).

III- (3 points)
Consider two fair cubic dice. The faces of each die are numbered from 1 to 6.
The two dice are rolled.
Denote by X the random variable that is defined as follows:

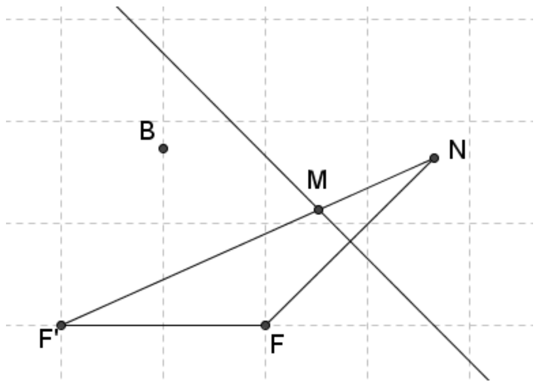
- if the two numbers shown on the dice are different, then X is equal to the greater between them;
- if the two numbers shown on the dice are equal, then X is equal to one them.

For example,
– if the two numbers shown on the dice are 2 and 3, then X = 3
– if the two numbers shown on the dice are 4 and 4, then X = 4

1) a- Calculate the probabilities $P(X = 1)$ and $P(X = 2)$.
   b- Prove that $P(X \leq 3) = \frac{1}{4}$.
2) In this part, consider an urn U that contains 6 balls: 4 red and 2 blue.
   The two dice are rolled:
   - if $X \leq 3$, then 3 balls are randomly and simultaneously selected from U;
   - if $X > 3$, then 3 balls are selected randomly and successively with replacement from U.

   Consider the following events:
   A: “$X \leq 3$”
   S: “The 3 selected balls have the same color”
   a- Calculate $P\left(S/A\right)$ and $P(A \cap S)$.
   b- Verify that $P(\overline{A} \cap S) = \frac{1}{4}$ and calculate $P(S)$.
   c- Knowing that $X > 3$, calculate the probability that the three selected balls do not have the same color.

IV- (3 points)
In the adjacent figure,



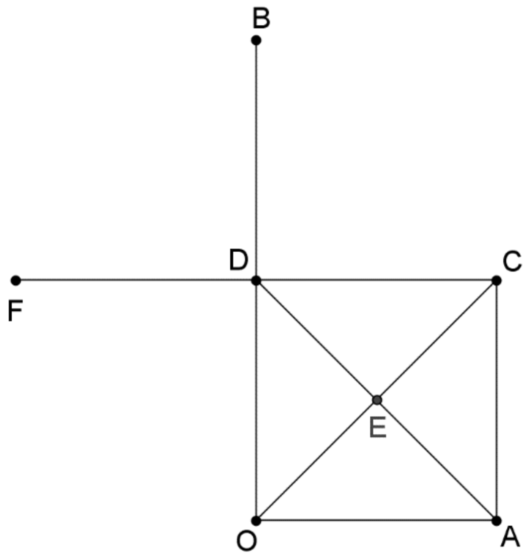
- F and F' are two fixed points so that FF' = 2.
- N is a variable point on the circle with center F' and radius 4.
- The perpendicular bisector of [NF] intersects [F'N] at M.
- B is a fixed point so that BF'F is an equilateral triangle.

1) a- Show that MF + MF' = 4.
   b- Deduce that M moves on a conic (E) whose nature, foci and center O are to be determined.
2) Let A be the symmetric of O with respect to F.
   a- Show that A is one of the vertices of (E).
   b- Determine the non-focal axis of (E) and verify that B is one of the vertices of (E).
   c- Draw (E).
3) The plane is referred to a direct orthonormal system $(O; \vec{i}, \vec{j})$ with $\vec{i} = \overrightarrow{OF}$.
   a- Verify that $\frac{x^2}{4} + \frac{y^2}{3} = 1$ is an equation of (E).
   b- Write an equation of (d) the directrix of (E) associated to F.
   c- Let $L(\alpha, \beta)$ be a point on (E) where α and β are two real numbers with $\beta \neq 0$.
   Write an equation of the tangent (T) to (E) at L.

V- (3 points)

In the figure below,

- OACD is a direct square with center E and side 2.
- F is the symmetric of C with respect to D.
- B is the symmetric of O with respect to D.



Denote by S the direct plane similitude of center O that maps A onto B.

Part A

1) a- Calculate the ratio k and an angle α of S.

   b- Verify that S(E) = F.

   c- Show that the triangle OBF is right isosceles.

2) Consider the direct plane similitude $S'\left(E,\, 2, \frac{\pi}{2}\right)$ and the transformation $h = S \circ S'$.

   Denote by W the center of h. Show that $\overrightarrow{WF} = -4\overrightarrow{WE}$.

Part B

The plane is referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$ with $\vec{u} = \frac{1}{2}\overrightarrow{OA}$.

1) Show that the complex form of h is $z' = -4z + 2 + 6i$ and deduce the affix of W.
2) For all $n \in \mathbb{N}$, consider the numerical sequence (d_n) defined as $d_n = WE_n$ where $E_0 = E$ and $E_{n+1} = h(E_n)$.

   a- Verify that $d_0 = \frac{\sqrt{10}}{5}$.

   b- Show that (d_n) is a geometric sequence of common ratio 4.

   c- Determine the number of points E_n such that $d_n < 2019$.

VI- (7 points)

Part A

Consider the differential equation (E): $y'' - 2y' + y = e^{2x}$ and let $y = z + e^{2x}$.

1) Write a differential equation (E') satisfied by z.
2) Determine the general solution of (E).
3) Determine the particular solution of (E) whose representative curve, in an orthonormal system, has at the point A(0 , –2) a tangent parallel to the x-axis.

Part B

Let f be the function defined on ℝ as $f(x) = e^{2x} + (x-3)e^{x}$.

Denote by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to+\infty} \dfrac{f(x)}{x}$.
2) Determine $\lim\limits_{x\to-\infty} f(x)$ and deduce an asymptote to (C).
3) Let g be the function defined on ℝ as $g(x) = x - 2 + 2e^{x}$.
   a- Set up the table of variations of the function g.
   b- Calculate $g(0)$ then deduce, according to the values of x, the sign of $g(x)$.
4) Verify that $f'(x) = e^{x}g(x)$ and set up the table of variations of the function f.
5) Show that the equation $f(x) = 0$ has, on ℝ, a unique root α. Verify that $0.7 < \alpha < 0.8$.
6) Draw the curve (C).
   a- Prove that f has, over [0 , + ∞[, an inverse function f^{-1} and determine its domain of definition.
   b- Draw the representative curve (C') of f^{-1} in the same system $(O;\vec{i},\vec{j})$.
   c- Calculate, in terms of α, the area of the region bounded by (C'), the x-axis and the y-axis.

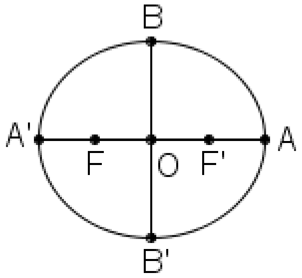
Part C

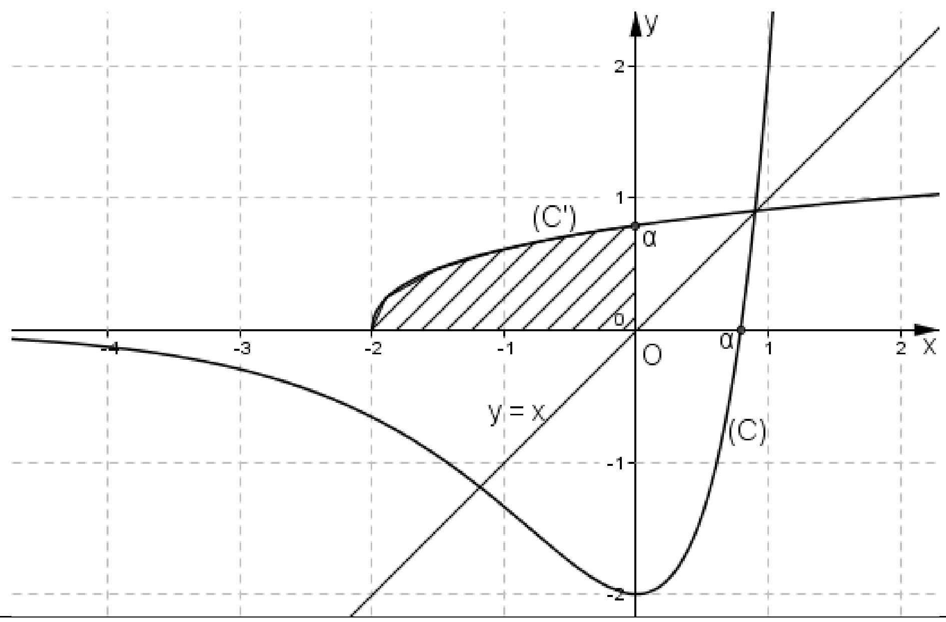
Let h be the function given by $h(x) = \arcsin\left(f^{-1}(x)\right)$.

Show that the domain of definition of the function h is $[-2 , e^{2} - 2e]$.

أسس تصحيح مسابقة الرياضيات

Q.I	Answers	4pts
1	$z = e^{i\frac{\pi}{6}}$. **Then a**	1
2	$\int \frac{dx}{x^2+4x+8} = \int \frac{1}{2^2+(x+2)^2}dx = \frac{1}{4}\int \frac{1}{1+(\frac{x+2}{2})^2} = \frac{1}{2}\arctan(\frac{x+2}{2}) + C$. **Then a** **OR** find the derivative of $\frac{1}{2}\arctan(\frac{x+2}{2})$	1
3	$m \le x \le m+1$, $\frac{1}{m+1} \le \frac{1}{x} \le \frac{1}{m}$ then $\frac{1}{m+1}\int_m^{m+1} dx \le \int_m^{m+1}\frac{1}{x}dx \le \frac{1}{m}\int_m^{m+1} dx$ so $\frac{1}{m+1} \le J \le \frac{1}{m}$. **Then c**	1
4	$z = 2\cos^2\left(\frac{\theta}{2}\right) + 2\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right).i = -2\cos\left(\frac{\theta}{2}\right)\left[-\cos\left(\frac{\theta}{2}\right) - i.\sin\left(\frac{\theta}{2}\right)\right] = -2\cos\left(\frac{\theta}{2}\right)e^{i\left(\pi+\frac{\theta}{2}\right)}$ $\left(\pi < \theta < 2\pi \text{ donc } \frac{\pi}{2} < \frac{\theta}{2} < \pi ;\ \cos\left(\frac{\theta}{2}\right) < 0\right)$ alors $\lvert z\rvert = -2\cos\left(\frac{\theta}{2}\right)$. **Then b** **OR** $\lvert z\rvert = \sqrt{(1+\cos\theta)^2 + (\sin\theta)^2} = \sqrt{2+2\cos\theta} = \sqrt{4(\cos\frac{\theta}{2})^2} = \left\lvert 2\cos(\frac{\theta}{2})\right\rvert$ $= -2\cos(\frac{\theta}{2})$ car $\frac{\pi}{2} < \frac{\theta}{2} < \pi$. **OR** take a particular value satisfied for one solution only.	1
Q.II	**Answers**	**4pts**
1	$A \in (d)$ then $A(-t+1 ; -2t ; 3t+1)$ and $A \in (P)$, $x_A + z_A = 0$, $t = -1$ so $A(2 ; 2 ; -2)$.	0.5
2a	O is a point on (P) and $\overrightarrow{OB}(1;0;1) = \overrightarrow{N_P}$	0.5
2b	$(\Delta) \equiv (OA)$: $\begin{cases} x = 2m \\ y = 2m \\ z = -2m \end{cases}$ $(m \in \mathbb{R})$ **OR** $\begin{cases} x = 2m+2 \\ y = 2m+2 \\ z = -2m-2 \end{cases}$ $(m \in \mathbb{R})$ **OR** $\begin{cases} x = m \\ y = m \\ z = -m \end{cases}$ $(m \in \mathbb{R})$	0.5
3	$V_{OBJA} = \frac{1}{6}\left\lvert \det\left(\overrightarrow{OB}, \overrightarrow{OA}, \overrightarrow{OJ}\right)\right\rvert = \frac{28}{6} = \frac{14}{3}$ volume units.	1
4a	The center of (H) is the midpoint of [OA] then $I(1 ; 1 ; -1)$.	0.5
4b	$S \in (OA)$ then $S(m ; m ; -m)$. $e = \frac{c}{a} = 3$ and $c = OI = \sqrt{3}$ so $a = \frac{1}{\sqrt{3}}$ $IS = a$ thus $3(m-1)^2 = \frac{1}{3}$; $m = \frac{4}{3}$ or $m = \frac{2}{3}$. Hence, $S\left(\frac{4}{3} ; \frac{4}{3} ; -\frac{4}{3}\right)$ and $G\left(\frac{2}{3} ; \frac{2}{3} ; -\frac{2}{3}\right)$	1
Q.III	**Answers**	**6pts**
1a	$P(X=1) = \frac{1}{36}$; $P(X=2) = \frac{3}{36} = \frac{1}{12}$	1
1b	$P(X \le 3) = P(X=1) + P(X=2) + P(X=3) = \frac{9}{36} = \frac{1}{4}$	1
2a	$P(S/A) = \frac{C_4^3}{C_6^3} = \frac{1}{5}$ $P(A \cap S) = P(A) \times P(S/A) = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$	1
2b	$P(\bar{A} \cap S) = P(\bar{A}) \times P(S/\bar{A}) = \frac{3}{4} \times \left(\frac{4^3}{6^3} + \frac{2^3}{6^3}\right) = \frac{1}{4}$ $P(S) = P(\bar{A} \cap S) + P(A \cap S) = \frac{1}{4} + \frac{1}{20} = \frac{3}{10}$	1.5
2c	$P(\bar{S}/\bar{A}) = 1 - P(S/\bar{A}) = 1 - \frac{P(S \cap \bar{A})}{P(\bar{A})} = 1 - \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{2}{3}$ **OR** $P(\bar{S}/\bar{A}) = \frac{P(\bar{S} \cap \bar{A})}{P(\bar{A})} = \frac{\frac{3}{4} \times \frac{2}{3}}{1 - \frac{1}{4}} = \frac{2}{3}$	1.5

Q.IV	Answers	6pts
1a	MF + MF' = MN + MF' = 4	**0.5**
1b	MF + MF' = 4 = 2a which is a constant greater than FF' = 2. So, M belongs an ellipse (E) with foci F and F'. The center O is the midpoint of [FF'].	**1**
2a	$AF + AF' = 1 + 3 = 4$ then $A \in (E)$ and A is a point on the focal axis (FF '). Then A is a principal vertex of (E).	**0.5**
2b	The non-focal axis is the line (OB) passing through O and perpendicular to the focal axis (FF') $a = \frac{4}{2} = 2 , c = \frac{FF'}{2} = 1$ et $b = \sqrt{a^2 - c^2} = \sqrt{3}$ B is a point of the non-focal axis of (E). OBF is a semi-equilateral triangle and $OB = \frac{BF\times\sqrt{3}}{2} = \frac{2\times\sqrt{3}}{2} = \sqrt{3}$ Thus, B is a vertex of (E). **OR** BF + BF ' = 4 and B is a point of the non-focal axis of (E).	**1**
2c	The second vertex on the non-focal axis of (E) other than B is the symmetric of B with respect to O. The second vertex on the focal axis of (E) other than A is the symmetric of A with respect to O. 	**1**
3a	(E) is an ellipse of center O(0 ; 0) and focal axis **(FF')** $\equiv (x'x)$ Then (E) : $\frac{(x-0)^2}{2^2} + \frac{(y-0)^2}{\sqrt{3}^2} = 1$ So $\frac{x^2}{4} + \frac{y^2}{3} = 1$	**1**
3b	$(d) : x = \frac{a^2}{c}$ then $(d) : x = 4$	**0.5**
3c	$(T) : \frac{\alpha x}{4} + \frac{\beta y}{3} = 1$; $y = \frac{-3\alpha}{4\beta}x + \frac{3}{\beta}$	**0.5**
Q.V	**Answers**	**6pts**
A1a	The ratio $k = \frac{OB}{OA} = 2$ The angle $\alpha = (\overrightarrow{OA} ; \overrightarrow{OB})(2\pi) = \frac{\pi}{2}(2\pi)$	**1**
A1b	$\frac{OF}{OE} = 2 = k$ $(\overrightarrow{OE} ; \overrightarrow{OF}) = \frac{\pi}{4} + \frac{\pi}{4}(2\pi) = \frac{\pi}{2}(2\pi) = \alpha$ Then S(E) = F.	**0.5**
A1c	S(OAE) = OBF and OAE is right isosceles triangle. Then OBF is isosceles right triangle.	**0.5**
A2	h = sim(W, 4 , π) = hom(W, – 4). h(E) = S(S'(E)) = S(E) = F then $\overrightarrow{WF} = -4\overrightarrow{WE}$	**1**
B1	z' = – 4z + 2 + 6i $Z_W = \frac{2}{5} + \frac{6}{5}i$	**1**
B2a	$d_0 = WE = \frac{\sqrt{10}}{5}$	**0.5**
B2b	$d_{n+1} = 4d_n$ then (d_n) is a geometric sequence of common ratio 4.	**0.5**
B2c	$d_n = d_0 \times 4^n = \frac{\sqrt{10}}{5} \times 4^n$ $d_n < 2019$; $\frac{\sqrt{10}}{5} \times 4^n < 2019$ then n < 5.8 So n = 5 then the number of points is 6.	**1**

<table>
<tr><th>Q.VI</th><th>Answers</th><th>14pts</th></tr>
<tr><td>A1</td><td>$z'' - 2z' + z = 0$</td><td>0.5</td></tr>
<tr><td>A2</td><td>$r^2 - 2r + 1 = 0$, $r = 1$, $z = (Ax + B)e^x$, $y = z + e^{2x} = (Ax + B)e^x + e^{2x}$</td><td>1</td></tr>
<tr><td>A3</td><td>$y(0) = -2$, $B = -3$;
$y' = Ae^x + (Ax + B)e^x + 2e^{2x}$, $y'(0) = 0$, $A = 1$. Then $y = (x - 3)e^x + e^{2x}$</td><td>1</td></tr>
<tr><td>B1</td><td>$\lim\limits_{x\to+\infty} f(x) = +\infty$; $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = +\infty$</td><td>1</td></tr>
<tr><td>B2</td><td>$\lim\limits_{x\to-\infty} f(x) = 0$, $y = 0$ HA</td><td>1</td></tr>
<tr><td>B3a</td><td>$g'(x) = 1 + 2e^x > 0$
<table>
<tr><td>x</td><td>$-\infty$</td><td>0</td><td>$+\infty$</td></tr>
<tr><td>g'(x)</td><td></td><td>+</td><td></td></tr>
<tr><td>g(x)</td><td>$-\infty$</td><td>↗ 0 ↗</td><td>$+\infty$</td></tr>
</table></td><td>1</td></tr>
<tr><td>B3b</td><td>$g(0) = 0$.
$g(x) < 0$ for $x < 0$, $g(x) = 0$ for $x = 0$, $g(x) > 0$ for $x > 0$.</td><td>1</td></tr>
<tr><td>B4</td><td>$f'(x) = 2e^{2x} + e^x + (x - 3)e^x = e^x g(x)$.
<table>
<tr><td>x</td><td>$-\infty$</td><td>0</td><td>$+\infty$</td></tr>
<tr><td>f '(x)</td><td>–</td><td>0</td><td>+</td></tr>
<tr><td>f(x)</td><td>0 ↘</td><td>–2</td><td>↗ $+\infty$</td></tr>
</table></td><td>1.5</td></tr>
<tr><td>B5</td><td>Over $]-\infty ; 0[$: $f(x) < 0$ then $f(x) = 0$ has no roots.
Over $[0 ; +\infty[$: f is continuous and strictly increasing from $-2 < 0$ to $+\infty$ then the equation $f(x) = 0$ has a unique root α.
But $f(0.7).f(0.8) = (-0.57)(0.05) < 0$ then $0.7 < \alpha < 0.8$</td><td>1.5</td></tr>
<tr><td>B6</td><td>
</td><td>1.5</td></tr>
<tr><td>B7a</td><td>Over $[0 ; +\infty[$: f is continue et strictly increasing then f has an inverse function f^{-1}.
$D_{f^{-1}} = [-2 ; +\infty[$</td><td>0.5</td></tr>
<tr><td>B7b</td><td>(C) and (C') are symmetric with respect to (y = x). Figure</td><td>1</td></tr>
<tr><td>B7c</td><td>Area = $\int_0^{\alpha} -f(x)dx = -\left[\frac{1}{2}e^{2x} + (x-3)e^x - e^x\right]_0^{\alpha} = \left[(4-\alpha)e^{\alpha} - \frac{1}{2}e^{2\alpha}\right] - \left[\frac{7}{2}\right]$ unit of area.
Remark: $f(\alpha) = 0$, $e^{\alpha} = 3 - \alpha$. Then Area = $\frac{1}{2}\alpha^2 - 4\alpha + 4$ unit of area.</td><td>1</td></tr>
<tr><td>C</td><td>$-1 \le f^{-1}(x) \le 1$ and $f^{-1}(x) \ge 0$ then $0 \le f^{-1}(x) \le 1$, $f(0) < x < f(1)$ since f is increasing, then $-2 \le x \le e^2 - 2e$.</td><td>0.5</td></tr>
</table>