This exam is formed of three exercises in 9 pages.
The use of a non-programmable calculator is recommended.

مسابقة في مادة الفيزياء

المدة: ساعتان

(اللغة الإنكليزية)

الاسم:....................................

الرقم:....................................

Exercise 1 (7 points)

Characteristics of a coil and a capacitor

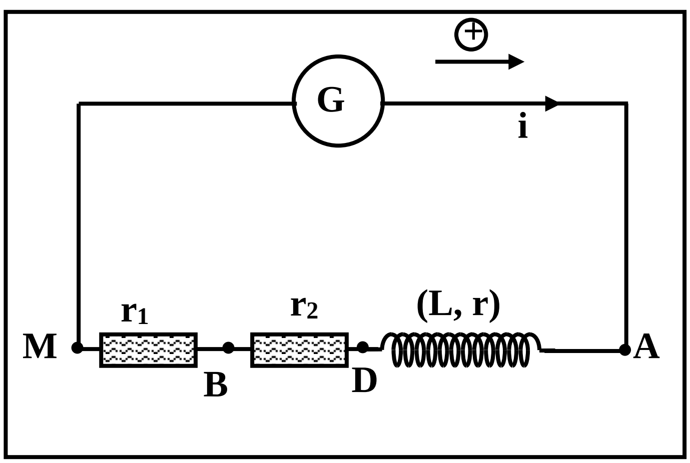
Consider:

- a generator G delivering an alternating sinusoidal voltage :
  $u_{AM} = u_G = U_m \cos(\omega t)$ (SI units);
- a coil of inductance **L** and resistance **r**;
- a capacitor of capacitance **C**;
- two resistors of resistances $r_1 = 10\ \Omega$ and $r_2 = 32\ \Omega$;
- an oscilloscope;
- connecting wires.

The aim of this exercise is to **determine L, r and C**.

1) Experiment 1

We set-up the circuit of document 1.



Doc. 1

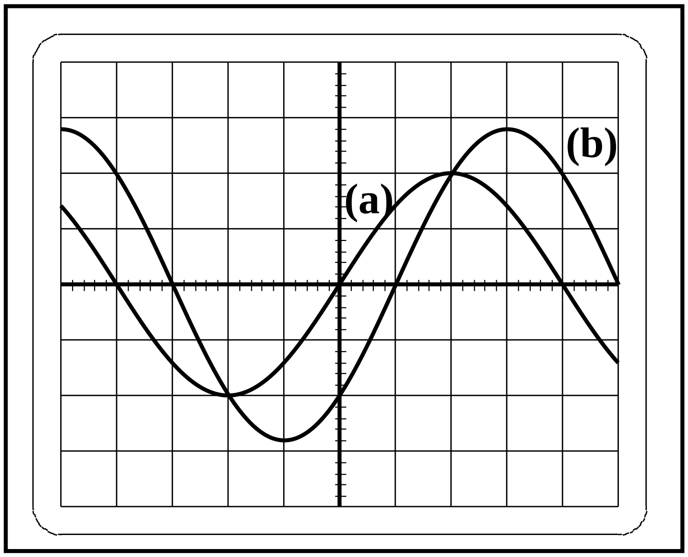
The circuit thus carries an alternating sinusoidal current **i**.

The oscilloscope, conveniently connected, allows us to display the voltage u_{AM} across the generator on channel (Y_1) and the voltage $u_{BM} = u_{r_1}$ across the resistor r_1 on channel (Y_2).

The obtained waveforms are shown in document 2.

The adjustments of the oscilloscope are:

- vertical sensitivity on **(Y_1): S_{V1} = 5 V/div;**
- vertical sensitivity on **(Y_2): S_{V2} = 0.5 V/div;**
- horizontal sensitivity: **S_h = 2.5 ms/div.**



Doc. 2

1-1) Redraw the circuit of document 1 and **show on it** the connections of the oscilloscope.

1-2) Justify that the waveform (**a**) represents $\mathbf{u_{AM}}$.

1-3) Referring to document 2:

1-3-1) determine the angular frequency **ω** of the voltage u_{AM};

1-3-2) determine the amplitudes $\mathbf{U_m}$ of the voltage u_{AM}.

determine the amplitudes $\mathbf{U_{m1}}$ of the voltages u_{BM}.

1-3-3) determine the phase difference **φ** between u_{AM} and u_{BM}.

1-4) Deduce the expression of the voltage $\mathbf{u_{BM}}$ as a function of time knowing that $\mathbf{u_{BM}}$ lags behind $\mathbf{u_{AM}}$.

1-5) Deduce the expression of the current **i** as a function of time knowing that $\mathbf{i = \frac{u_{BM}}{r_1}}$.

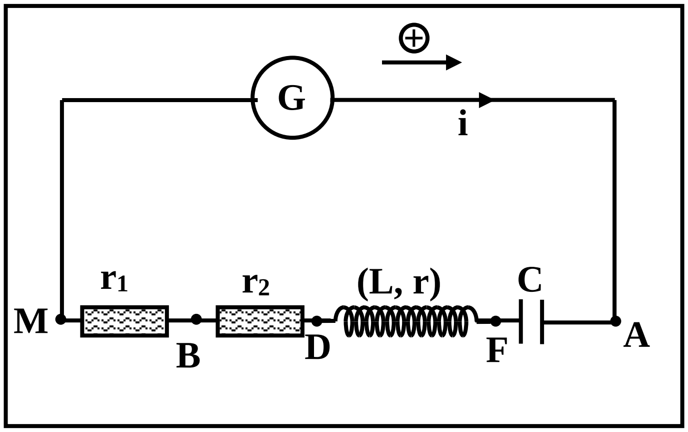
1-6) Determine the values of **L** and **r** by applying the law of addition of voltages ($\mathbf{u_{AM} = u_{AD} + u_{DB} + u_{BM}}$) and by giving **(ωt)** two particular values:

$\mathbf{\omega t = \frac{\pi}{4}}$ and $\mathbf{\omega t = 0}$

2) Experiment 2

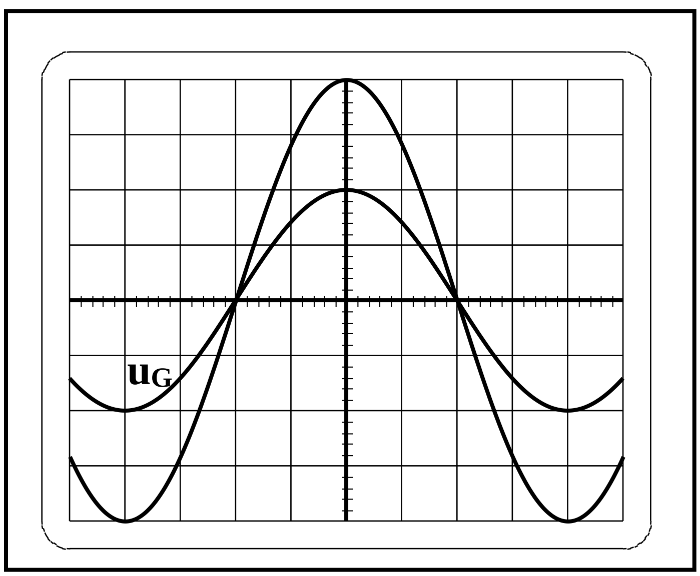
In the circuit of document 1 of the experiment 1, we connect the in series with the electric components, we obtain the circuit of the document 3.

The oscilloscope, conveniently connected, allows us to display the voltage $\mathbf{u_{AM}}$ on channel (**Y_1**) and the voltage $\mathbf{u_{BM}}$ on channel (**Y_2**).



Doc. 3

The obtained waveforms are represented in document 4.



Doc. 4

2- 1) Justify that the circuit is the seat of current resonance..

2- 2) In case of current resonance, the angular frequency $\boldsymbol{\omega}$ of the generator is equal to the proper angular frequency $\boldsymbol{\omega_0}$ of the circuit. **($\boldsymbol{\omega = \omega_0}$)**

Choose, from the statements below, the one that describes correctly the proper angular frequency $\boldsymbol{\omega_0}$ of the circuit of doc.3:

Statement 1	**Statement 2**	**Statement 3**
The proper angular frequency of the circuit is the angular frequency of G such that the current i and the voltage across the coil are in phase.	The proper angular frequency of the circuit is the angular frequency of G such that the amplitude I_m of the current i attains a maximum value.	The proper angular frequency of the circuit is the angular frequency of G such that the amplitude of the voltage across the coil attains a maximum value.

2- 3) **Write** the relation among L, C and ω_0.

Calculate C.

<u>Exercise 2</u> (6.5 points)

Mechanical oscillator

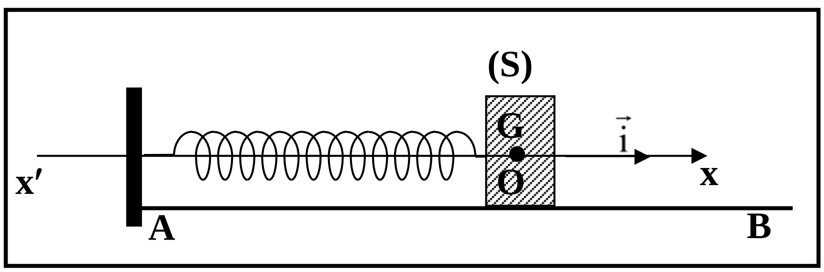
Consider a mechanical oscillator formed of a spring, of negligible mass and <u>spring constant</u> **k**, and an <u>object (**S**) of mass</u> **m**.

The aim of this exercise is to determine **k** and **m**.

The spring is placed horizontally, fixed from one of its extremities to a fixed support.

(**S**) is attached to the other extremity of the spring and it may slide without friction on a horizontal rail **AB** and its center of mass **G** can move along a horizontal **x-axis**.

At equilibrium, **G** coincides with the origin **O** of **the x-axis** (Doc. 5).



Doc. 5

(**S**) is shifted from its equilibrium position **O** and then released without initial velocity at the instant $t_0 = 0$.

Thus (S) performs mechanical oscillations.

At <u>an instant t</u>, the abscissa of **G is** $\mathbf{x} = \overline{\mathbf{OG}}$ and the algebraic value of its velocity is $\mathbf{v} = \frac{\mathbf{dx}}{\mathbf{dt}} = x'$.

The horizontal plane containing **G** is considered as a <u>reference level for gravitational potential energy (GPE = 0)</u>.

1) The differential equation that describes the motion of **G** is: $\mathbf{2x'' + 200x = 0}$ (SI units).

<u>Use this differential equation</u> to:

1-1) **show** that the motion of **G** is <u>simple harmonic</u>;

1-2) **calculate** the <u>value of the proper angular frequency ω_0</u> of oscillations.

2) The time equation of the motion of **G** is of the form: $\mathbf{x = X_m \cos(\omega_0 t)}$, where $\mathbf{X_m}$ is the amplitude of **x**.

2-1) Write the expression of **v** in terms of $\mathbf{X_m}$, ω_0 and **t.**

2-2) Given: $\mathbf{\sin^2\omega_0 t + \cos^2\omega_0 t = 1}$ and using the expressions of x and v:

show that $\boldsymbol{\omega_0^2 = \frac{v^2}{X_m^2 - x^2}}$.

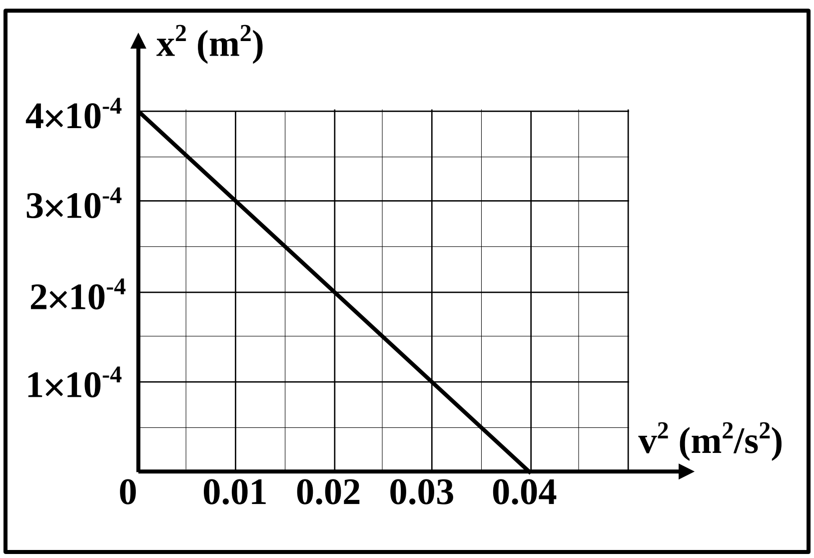
3) Applying the principle of conservation of mechanical energy «ME» of the system [(S), spring, Earth],

Show that $\mathbf{x^2 = a\, v^2 + b}$; where «**a**» and «**b**» are two constants.

terms of k, m and ME.

Deduce that $\mathbf{a = -\frac{m}{k}}$ and $\mathbf{b = \frac{2ME}{k}}$.

4) Document 6 shows x^2 as a function of v^2.



Doc. 6

Using document 6:

4-1) Indicate X_m^2, then **calculate** $\mathbf{X_m}$.

4-2) Calculate again the value of ω_0 referring to the part 2.2. and by choosing a particular point from doc.6.

5) Determine the **values** of **k** and **m** knowing that the **ME = 0.04 J.**

Exercise 3 (**6.5 points**)

Dating of a volcanic rock

Some of the volcanic rocks contain the radioactive isotope of potassium $^{40}_{19}\mathbf{K}$ of half-life **T** and radioactive constant λ.

A small proportion of this isotope decays into argon $^{40}_{18}\mathbf{Ar}$.

The aim of this exercise is to determine the age of a volcanic rock.

1) **Indicate** the composition (number of protons and neutrons) of the potassium $^{40}_{19}\mathbf{K}$ nucleus.

2) The decay equation of potassium-40 into argon-40 is:

$$^{40}_{19}\mathrm{K} \rightarrow \, ^{40}_{18}\mathrm{Ar} + \, ^{A}_{Z}\mathrm{X}$$

2-1) **Determine** **Z** and **A**;

Indicate the two laws used.

2-2) **Name** the emitted particle $^{A}_{Z}\mathbf{X}$.

3) A sample of a volcanic rock contains at the instant of its formation, $\mathbf{t_0 = 0}$,

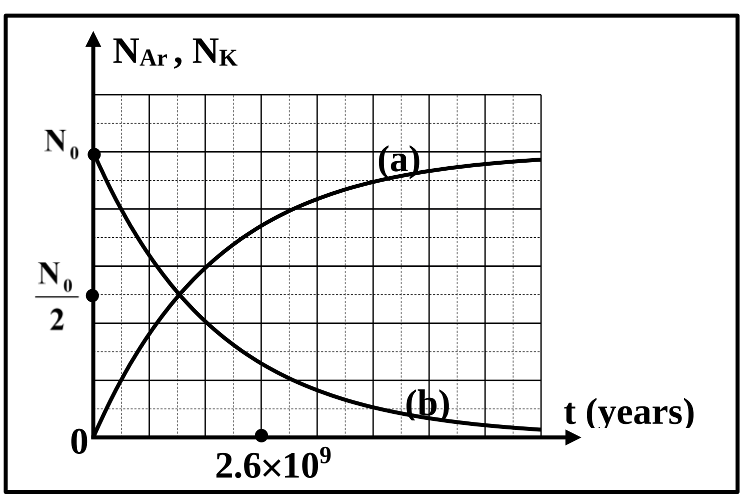
$\mathbf{N_0}$ nuclei of potassium-40 that decay into **argon-40**.

3-1) Write the expression of the remaining number N_K of potassium-40 nuclei in terms of $\mathbf{N_0}$, **λ** and **t**.

3-2) **Deduce** that the number of the formed argon-40 nuclei is: $\mathbf{N_{Ar} = N_0 (1 - e^{-\lambda t})}$.

3-3) **Determine**, in terms of λ, the expression of t when $\mathbf{N_{Ar} = N_K}$.

4) The curves (**a**) and (**b**) of document 7 represent $\mathbf{N_K}$ and $\mathbf{N_{Ar}}$ as functions of time.



Doc. 7

4-1) Specify the curve that represents $\mathbf{N_K}$.

4-2) Determine graphically the half-life T of potassium-40.

4-3) Verify that the value of $\boldsymbol{\lambda = 0.533\times10^{-9}}$ **year**$^{-1}$

5) The sample of the volcanic rock contains at the instant of its formation, $\mathbf{t_0 = 0}$, $\mathbf{N_0}$ nuclei of potassium-40 that decay into argon-40.

At this instant the sample does not contain any argon-40 nucleus.

At an instant **t**:

- $\mathbf{N_K}$ is the remaining number of nuclei of N_0 of potassium-40;
- $\mathbf{N_{Ar}}$ is the formed number of the argon-40 nuclei.

A geologist analyzes this sample to determine the age of the volcanic rock.

He finds that the number $\mathbf{N_{Ar}}$ of argon-40 nuclei is 3 times the number $\mathbf{N_K}$ of potassium-40 nuclei ($\mathbf{N_{Ar} = 3\ N_K}$**).**

5-1) Show that $\frac{N_0}{N_K} = 4$.

5-2) Deduce that the age of the rock is $\mathbf{2.6\times10^9}$ **years.**