ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

مسابقة في مادة الرّياضيات

المدّة: ساعتان

باللّغة الانكليزية

الاسم : ....

الرقم : ....

I- (4 points)

The table below shows **the population (y_i) of a certain village from the year 1990 till the year 2015, and the rank of the corresponding year (x_i).**

Year	**1990**	**1995**	**2000**	**2005**	**2010**	**2015**
Rank of the year: x_i	0	5	10	15	20	25
Population: y_i	5 445	5 940	6 285	6 695	7 085	7 550

Part A

1) **Calculate** $\overline{X}$ and $\overline{Y}$, the respective means of the two variables x_i and y_i .
2) **Calculate** the percentage increase of the population from 1990 to 2015.
3) **Determine** the coefficient of correlation **r**.

   Interpret the obtained value.
4) **Determine** the equation of the regression line, of y in terms of x,

   $\left(D_{y/x}\right)$: **y = mx + n,**

   where m and n are two real numbers (Round m and n to the nearest 10^{-1}).

Part B

Suppose that the preceding model remains valid till the year 2024.

1) **Solve** the inequality **y > 8250**.

   Determine the year in which the population of this village exceeds 8 250 for the first time.
2) In this village, **the number of people** who used the internet in the year 2018 **was 2 000**.

   Assume that this number **increases by 100 people** per year.

   a- **Calculate** the number of people in the village who will use the internet in the year 2024.

   b- In 2024, suppose that two people are randomly and successively interviewed from this village. **Calculate** the probability that these two people use the internet.

II- (4 points)

In a sport club:

- **40 %** of the members are girls, **among** which **30 %** participate in the national competition
- **60 %** of the members are boys, **among** which **80 %** participate in the national competition.

Part A

One member is randomly selected from the club.

Consider the following events:

G: "The selected member is a girl"

B: "The selected member is a boy"

C: "The selected member participates in the national competition".

1) **Calculate** the probability $P(G \cap C)$

Verify that $P(C) = \frac{3}{5}$.

2) The selected member did not participate in the national competition.

Calculate the probability that this member is a boy.

Part B

This club has **50** members.

The manager of the club decides to select **randomly** and **simultaneously** a group of three members to represent the club abroad.

1) **Verify** that the number of girls in this club is **20**.

Determine the number of boys in this club.

2) **Verify** that the probability of selecting a group that consists of two girls and one boy is $\frac{57}{196}$.

3) **Verify** that the probability of selecting a group that consists of at least one girl and at least one boy is $\frac{36}{49}$.

III- (4 points)

Hadi is an employee at a bank.

In January 2018, Hadi's monthly salary was **1 500 000 LL**.

Each month his salary increases by **0.2 %** with an additional bonus of **48000 LL**.

For all natural numbers $\mathbf{n \geq 1}$, denote by $\mathbf{a_n}$ Hadi's monthly salary, in millions LL, in the nth month. Thus $\mathbf{a_1 = 1.5}$.

1) **Calculate $\mathbf{a_2}$.**

2) For all $\mathbf{n \geq 1}$, $\mathbf{a_{n+1} = (1.002)a_n + 0.048}$.

   a- Let $\mathbf{V_n = a_n + 24}$.

   Show that $(\mathbf{V_n})$ is a geometric sequence whose common ratio is **1.002** then **determine** its 1st term $\mathbf{V_1}$.

   b- Express $\mathbf{V_n}$ in terms of $\mathbf{n}$.

   Show that $\mathbf{a_n = 25.5 \times (1.002)^{n-1} - 24}$ for all $\mathbf{n \geq 1}$.

3) Hadi wants to buy a **car** that costs **25 000 000 LL**.

   Starting from the month of January 2018, the bank offered Hadi the following:

   Each month, Hadi deposits **700 000 LL** from his monthly salary in a savings account with an annual interest rate of **6 %** compounded monthly.

   a- **Verify** that the amount of money in Hadi's account, after **n** months, is expressed as

   $\left[\mathbf{140(1.005)^n - 140}\right]$ millions LL for all $\mathbf{n \geq 1}$.

   b- **Solve the inequality** $\mathbf{140(1.005)^n - 140 > 25}$

   Determine the minimum number of months needed for Hadi to be able to buy this car.

IV- (8 points)

Part A

Consider the function **f** defined over $\left]0,+\infty\right[$ as $\mathbf{f(x)=\frac{1}{x}-xe^{x-1}}$.

Denote by (**C**) the representative curve of **f** in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) **Determine** $\lim\limits_{x\to 0} f(x)$

Deduce an asymptote to (**C**).

2) **Determine** $\lim\limits_{x\to +\infty} f(x)$ and **calculate f(2).**

3) The adjacent table is the table of variations of the function f.

x	0		$+\infty$
f '(x)		$-$	
f(x)			

a- **Copy** and **complete** the given table.

b- **Prove** that **x = 1** is the unique solution of the equation **f(x) = 0**.

4) **Draw** (C).

5) The area of the domain bounded by (C), the x-axis, and the two lines of equations **x = 1** and **x = 2** is equal to **(e – ln2)** units of area.

- **Calculate** $\displaystyle\int_1^2 \frac{1}{x}dx$

- **Use** this area to calculate the exact value of $\displaystyle\int_1^2 xe^{x-1}dx$.

Part B

A factory produces a certain liquid detergent.

The marginal cost $\mathbf{M_C}$ of production of this factory is modeled as

$\mathbf{M_C(x) = (x+1)e^{x-1}}$, in millions LL,

where **x** is the quantity produced of this factory, in thousands of liters ; $\mathbf{x \in [0;5]}$.

1) Knowing that the fixed cost of this factory is **1 000 000 LL**, show that the total cost $\mathbf{C_T}$ of production of this factory is modeled as $\mathbf{C_T(x) = xe^{x-1} + 1}$ in millions LL.

2) Denote by $\mathbf{\overline{C}}$ the average cost of production of this factory.

a- Verify that $\mathbf{\overline{C}(x) - M_C(x) = f(x)}$ where $\mathbf{x \in]0;5]}$ and $\mathbf{\overline{C}(x)}$ in millions LL.

b- In this part, we admit that the average cost is minimum if it is equal to the marginal cost: $\mathbf{\overline{C}(x) = M_C(x)}$.

Determine, in <u>liters, the quantity to be produced of this detergent for the average cost to be minimum.</u>

3) a- For a certain reason, the factory sold **60 %** of its production for **5 000 LL** the liter and the remaining **40 %** for **2 500 LL** the liter.

Knowing that the entire quantity produced of this detergent is sold,

Verify that the revenue R(x) in millions LL is **R(x) = 4x.**

b- This factory produced **1 800** liters of this detergent and sold **75 %** of this production.

Calculate the number of liters sold by this factory.

Does the realized revenue cover the cost of this production? **Justify**.