وزارة التربية والتعليم العالي المديريّة العامّة للتربية دائرة الامتحانات الرسميّة	امتحانات الشهادة الثانوية العامة فرع: الاجتماع والاقتصاد	دورة العام ٢٠١٩ الاستثنائيّة السبت في ٣ آب ٢٠١٩
عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

The table below shows the population (y_i) of a certain village from the year 1990 till the year 2015, and the rank of the corresponding year (x_i).

Year	1990	1995	2000	2005	2010	2015
Rank of the year: x_i	0	5	10	15	20	25
Population: y_i	5 445	5 940	6 285	6 695	7 085	7 550

Part A

1) Calculate $\overline{X}$ and $\overline{Y}$, the respective means of the two variables x_i and y_i .
2) Calculate the percentage increase of the population from 1990 to 2015.
3) Determine the coefficient of correlation r. Interpret the obtained value.
4) Determine the equation of the regression line, of y in terms of x, $(D_{y/x})$: $y = mx + n$, where m and n are two real numbers (Round m and n to the nearest 10^{-1}).

Part B

Suppose that the preceding model remains valid till the year 2024.

1) Determine the year in which the population of this village exceeds 8 250 for the first time.
2) In this village, the number of people who used the internet in the year 2018 was 2 000. Assume that this number increases by 100 people per year.
   a- Calculate the number of people in the village who will use the internet in the year 2024.
   b- In 2024, suppose that two people are randomly and successively interviewed from this village. Calculate the probability that these two people use the internet.

II- (4 points)

In a sport club:

- 40 % of the members are girls, among which 30 % participate in the national competition
- 80 % of the boys participate in the national competition.

Part A

One member is randomly selected from the club.
Consider the following events:

G: “The selected member is a girl”
B: “The selected member is a boy”
C: “The selected member participates in the national competition”.

1) Calculate the probability $P(G \cap C)$ and verify that $P(C) = \frac{3}{5}$.
2) The selected member did not participate in the national competition. Calculate the probability that this member is a boy.

Part B

This club has 50 members.
The manager of the club decides to select randomly and simultaneously a group of three members to represent the club abroad.

1) Calculate the number of girls and the number of boys in this club.
2) Verify that the probability of selecting a group that consists of two girls and one boy is $\frac{57}{196}$.
3) Calculate the probability of selecting a group that consists of at least one girl and at least one boy.

III- (4 points)

Hadi is an employee at a bank.

In January 2018, Hadi's monthly salary was 1 500 000 LL. Each month his salary increases by 0.2 % with an additional bonus of 48 000 LL. For all natural numbers $n \geq 1$, denote by a_n Hadi's monthly salary, in millions LL, in the nth month. Thus $a_1 = 1.5$.

1) Calculate a_2.
2) For all $n \geq 1$, $a_{n+1} = (1.002)a_n + 0.048$.
   a- Let $V_n = a_n + 24$. Show that (V_n) is a geometric sequence whose 1st term V_1 is to be determined.
   b- Show that $a_n = 25.5 \times (1.002)^{n-1} - 24$ for all $n \geq 1$.
3) Hadi wants to buy a car that costs 25 000 000 LL.
   Starting from the month of January 2018, the bank offered Hadi the following:
   Each month, Hadi deposits 700 000 LL from his monthly salary in a savings account with an annual interest rate of 6 % compounded monthly.
   a- Verify that the amount of money in Hadi's account, after n months, is expressed as $\left[140(1.005)^n - 140\right]$ millions LL for all $n \geq 1$.
   b- Determine the minimum number of months needed for Hadi to be able to buy this car.

IV- (8 points)

Part A

Consider the function f defined over $]0, +\infty[$ as $f(x) = \frac{1}{x} - xe^{x-1}$.

Denote by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Determine $\lim\limits_{x \to 0} f(x)$ and deduce an asymptote to (C).
2) Determine $\lim\limits_{x \to +\infty} f(x)$ and calculate f(2).
3) The adjacent table is the table of variations of the function f.
   a- Copy and complete the given table.
   b- Prove that x = 1 is the unique solution of the equation f(x) = 0.

x	0	+∞
f '(x)		−
f(x)		

4) Draw (C).
5) The area of the domain bounded by (C), the x-axis, and the two lines of equations x = 1 and x = 2 is equal to (e – ln2) units of area. Use this area to calculate the exact value of $\int_1^2 xe^{x-1}dx$.

Part B

A factory produces a certain liquid detergent.

The marginal cost M_C of production of this factory is modeled as $M_C(x) = (x+1)e^{x-1}$, in millions LL, where x is the quantity produced of this factory, in thousands of liters ; $x \in [0, 5]$.

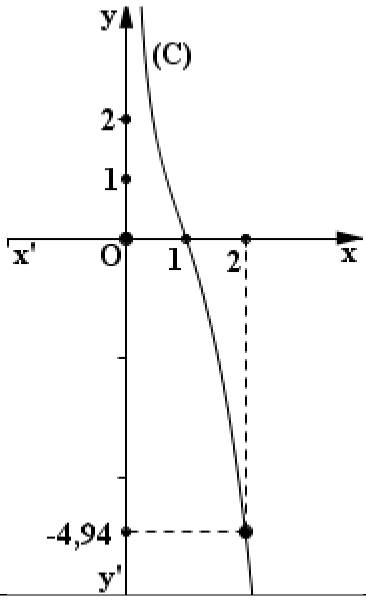
1) Knowing that the fixed cost of this factory is 1 000 000 LL, show that the total cost C_T of production of this factory is modeled as $C_T(x) = xe^{x-1} + 1$ in millions LL.
2) Denote by $\overline{C}$ the average cost of production of this factory.
   a- Verify that $\overline{C}(x) - M_C(x) = f(x)$ where $x \in]0, 5]$ and $\overline{C}(x)$ in millions LL.
   b- In this part, we admit that the average cost is minimum if it is equal to the marginal cost. Determine, in liters, the quantity to be produced of this detergent for the average cost to be minimum.
3) a- For a certain reason, the factory sold 60 % of its production for 5 000 LL the liter and the remaining 40 % for 2 500 LL the liter. Knowing that the entire quantity produced of this detergent is sold, determine the revenue R(x) in millions LL.
   b- This factory produced 1 800 liters of this detergent and sold 75 % of this production. Does the realized revenue cover the cost of this production? Justify.

امتحانات الشهادة الثانوية العامة
فرع: الاجتماع والاقتصاد

دورة العام ٢٠١٩ الاستثنائية
السبت ٣ آب ٢٠١٩

أسس تصحيح مسابقة الرياضيات

Q.I	Answers	7 pts
A1	$\bar{x} = 12.5$; $\bar{y} = 6500$.	1
A2	% of increase $= \frac{7550 - 5445}{5445} \times 100 = 38.6$	1
A3	r = 0.99, hence there is a strong positive correlation between the two variables x_i and y_i.	0.5
A4	y = 82.1x + 5473.6	1
B1	y > 8250 ; 82.1x + 5473.6 > 8250 ; x > 33.8 ; x = 34 ; the population of the village exceeds 8 250 for the first time in 2024	1.5
B2a	The number of persons in the village who will use the internet in year 2024 is 2000 + 100×6 = 2600	0.5
B2b	x = 34 ; y = 82.1(34) + 5473.6 = 8265 $P = \frac{2600}{8265} \times \frac{2599}{8264} = 0.099$	1.5
Q.II	**Answers**	**7 pts**
A1	$P(G \cap C) = P(G) \times P(C/G) = 0.4 \times 0.3 = 0.12$ $P(C) = P(G \cap C) + P(B \cap C) = 0.12 + P(B) \times P(C/B) = 0.12 + 0.6 \times 0.8 = \frac{3}{5}$.	2
A2	$P(B/\bar{C}) = \frac{P(B \cap \bar{C})}{P(\bar{C})} = \frac{P(B) \times P(\bar{C}/B)}{1 - 0.6} = \frac{0.6 \times 0.2}{0.4} = 0.3$	1.5
B1	The number of girls is 0.4×50 = 20 and the number of boys is 50 - 20 = 30	0.5
B2	$P(2G \& 1B) = \frac{C_{20}^{2} \times C_{30}^{1}}{C_{50}^{3}} = \frac{57}{196}$	1.5
B3	P(at least one girl and at least one boy) = P(2B&1G) + P(2G&1B) $= \frac{C_{30}^{2} \times C_{20}^{1}}{C_{50}^{3}} + \frac{57}{196} = \frac{36}{49}$	1.5
Q.III	**Answers**	**7 pts**
1	$a_2 = 1.5 + 1.5 \times 0.002 + 0.048 = 1.551$ millions LL.	1
2a	$V_{n+1} = a_{n+1} + 24 = 1.002\, a_n + 24.048 = 1.002(a_n + 24) = 1.002\, V_n$ Then (V_n) is a geometric sequence of common ratio is 1.002 and first term $V_1 = 25.5$	1.5
2b	$V_n = 25.5(1.002)^{n-1}$; $a_n = V_n - 24 = 25.5(1.002)^{n-1} - 24$	1
3a	$S = \frac{R\left[(1+i)^n - 1\right]}{i} = \frac{0.7\left[\left(1+\frac{1}{200}\right)^n - 1\right]}{\frac{1}{200}} = 140\left(1.005^n - 1\right) = 140(1.005)^n - 140$ millions LL OR $S = \frac{700000\left[\left(1+\frac{1}{200}\right)^n - 1\right]}{\frac{1}{200}} = 140000000\left(1.005^n - 1\right)$ LL; $S = 140(1.005)^n - 140$ millions LL	2
3b	$140(1,005)^n - 140 \geq 25$; $(1,005)^n \geq \frac{33}{28}$; $n \geq 32.9$ then $n = 33$ after 33 months	1.5

<table>
<tr><th>Q.IV</th><th>Answers</th><th>14 pts</th></tr>
<tr><td>A1</td><td>$\lim_{\substack{x\to0\\x>0}} f(x)=+\infty$ then $x=0$ is an asymptote.</td><td>1</td></tr>
<tr><td>A2</td><td>$\lim_{x\to+\infty} f(x)=-\infty$; $f(2)=\frac{1}{2}-2e=-4.93$</td><td>1</td></tr>
<tr><td>A3a</td><td>.
<table>
<tr><td>x</td><td>0</td><td>$+\infty$</td></tr>
<tr><td>f '(x)</td><td></td><td>–</td></tr>
<tr><td>f(x)</td><td></td><td>$+\infty$ ↘ $-\infty$</td></tr>
</table>.</td><td>0.5</td></tr>
<tr><td>A3b</td><td>Over $]0, +\infty[$: f is continuous and strictly decreasing from $+\infty$ to $-\infty$ then f(x) = 0 has a unique solution but f(1) = 0 then 1 is the unique solution of f(x) = 0.</td><td>1</td></tr>
<tr><td>A4</td><td>
</td><td>1.5</td></tr>
<tr><td>A5</td><td>$A=-\int_1^2\left(\frac{1}{x}-xe^{x-1}\right)dx$; $-\int_1^2\frac{1}{x}dx+\int_1^2 xe^{x-1}dx=e-\ln 2$; $-\ln x]_1^2+\int_1^2 xe^{x-1}dx=e-\ln 2$;
$\int_1^2 xe^{x-1}dx=e$</td><td>1.5</td></tr>
<tr><td>B1</td><td>$C_T(0)=1$
$C_T'(x)=e^{x-1}+xe^{x-1}=(x+1)e^{x-1}=M_C(x)$</td><td>1.5</td></tr>
<tr><td>B2a</td><td>$\overline{C}(x)-M_C(x)=e^{x-1}+\frac{1}{x}-(x+1)e^{x-1}=\frac{1}{x}-xe^{x-1}=f(x).$</td><td>1</td></tr>
<tr><td>B2b</td><td>The average cost is minimal when $\overline{C}(x)=M_C(x)$; f(x) = 0 ; x = 1
The average cost is minimum when the production is 1000 liters</td><td>2</td></tr>
<tr><td>B3a</td><td>$R(x)=\left(\frac{5000\times1000}{1000000}\right)(0.6x)+\left(\frac{2500\times1000}{1000000}\right)(0.4x)=4x$ millions LL</td><td>1.5</td></tr>
<tr><td>B3b</td><td>75%(1800) = 1350
R(1.35) = 5.4 millions LL
$C_T(1.8)$ = 5.005 millions LL.
Yes, since R(1.35) > $C_T(1.8)$</td><td>1.5</td></tr>
</table>