عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة واحدة	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (5 points)

1) Solve the system: $\begin{cases} x + y = 650\,000 \\ 2x + 3y = 1\,350\,000 \end{cases}$

2) **Shop A** sells musical instruments.
The price of one guitar and one flute is 650 000 LL.
The price of two guitars and three flutes is 1 350 000 LL.
Calculate the price of one guitar and that of one flute.

3) Lynn deposited a sum of 5 000 000 LL in a savings account for a period of 4 years at an annual interest rate of 5% compounded annually.
   a- Calculate the sum in Lynn's account at the end of the fourth year.
   b- At the end of the fourth year, Lynn drew 6 000 000 LL from her savings account to buy 8 guitars and a certain number of flutes from **Shop A**.
   Determine the maximum number of flutes that Lynn can buy.

II- (5 points)

A bag contains 13 balls distributed as shown in the following table:

Color of the ball / Number on the ball	Green	Red	White
Odd	4	1	2
Even	2	3	1

1) One ball is randomly selected from the bag.
Consider the following events:
G: "The selected ball is green"
R: "The selected ball is red"
O: "The selected ball holds an odd number".
   a- Calculate the following probabilities: P(G), P(O), $P(G \cap O)$ and $P(G \cup O)$.
   b- Knowing that the selected ball is not red, calculate the probability that this ball holds an odd number.

2) **Two** balls are randomly selected from the bag, one after the other and without replacement.
Let S be the event: "The sum of the two numbers on the selected balls is odd".
Calculate P(S).

III- (10 points)

Consider the function f defined, over $]1, +\infty[$, as $f(x) = -x + 1 + \frac{m}{x-1}$, where m is a non-zero real number. Denote by (C) the representative curve of f in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) Calculate m so that point A(2 , −5) is on the curve (C).

2) In what follows, m = − 4 and $f(x) = -x + 1 - \frac{4}{x-1}$.

   a- Determine $\lim\limits_{\substack{x\to 1\\ x>1}} f(x)$ and deduce an equation of an asymptote (D) to (C).

   b- Determine $\lim\limits_{x\to+\infty} f(x)$ and show that the line (d) with equation y = –x +1 is an asymptote to (C).

   c- Show that $f'(x) = \frac{(3-x)(x+1)}{(x-1)^2}$.

   d- Copy and complete the table of variations of f:

x	1		3		+∞
f '(x)	‖		0		
f(x)	‖				

   e- Show that the equation f(x) = 0 has no real roots.

   f- Let (L) be the line with equation y = –5.
   Determine the abscissas of the points of intersection of (L) and (C).

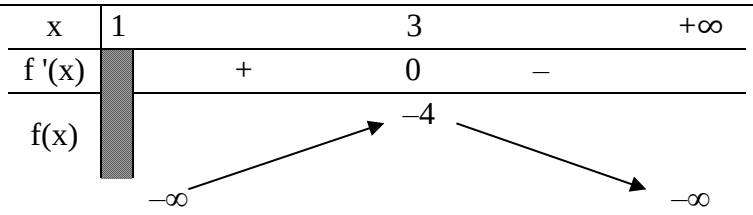
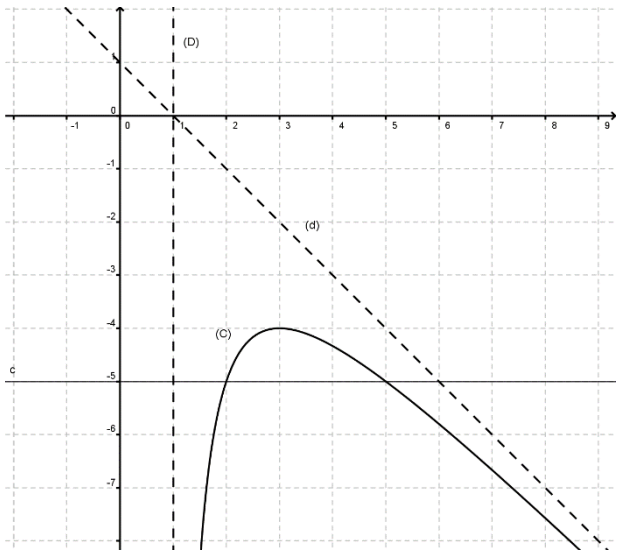
   g- Draw (d), (D), (C) and (L).

   h- Solve graphically the inequality f(x) > – 5.

عدد المسائل: ثلاث — أسس التصحيح - مادة الرياضيات

QI	Correction	Note
1	x = 600 000 and y = 50 000	**1**
2	Let x be the price of one guitar and y be the price of one flute $\begin{cases} x + y = 650\,000 \\ 2x + 3y = 1\,350\,000 \end{cases}$ x = 600 000 and y = 50 000 The price of one guitar is 600 000LL and the price of one flute is 50 000LL.	**1** **1**
3a	$F = P(1 + i)^n = 5\,000\,000(1 + 0.05)^4 = 6\,077\,531.25$ LL	**1**
3b	6 000 000 – 8×600 000 = 1 200 000 LL 1 200 000 ÷ 50 000 = 24 The maximum number of flutes that Lynn can buy is 24.	**1**

QII	Correction	Note
1a	$P(G) = \frac{6}{13}$ $P(O) = \frac{7}{13}$ $P(G \cap O) = \frac{4}{13}$ $P(G \cup O) = \frac{9}{13}$	¾ ¾ ¾ ¾
1b	$P(O / \bar{R}) = \frac{6}{9} = \frac{2}{3}$	**1**
2	$p(S) = P(O,E) + P(E,O) = \frac{6}{13} \times \frac{7}{12} \times 2 = \frac{7}{13}$	**1**

QIII	Correction	Note
1	f(2) = –5 –1 + m = –5 then m = –4	**1**
2a	$\lim\limits_{\substack{x\to 1\\ x>1}} f(x) = -\infty$ x = 1 is an asymptote	**1**
2b	$\lim\limits_{x\to+\infty} f(x) = -\infty$ $\lim\limits_{x\to+\infty}[f(x)-(-x+1)] = \lim\limits_{x\to+\infty}\left(-\frac{4}{x-1}\right) = 0$ Then the line (d) with equation y = –x +1 is an asymptote to (C).	**1**
2c	$f'(x) = -1 + \frac{4}{(x-1)^2} = \frac{4-(x-1)^2}{(x-1)^2} = \frac{(3-x)(x+1)}{(x-1)^2}$.	**1**
2d		**1**
2e	<u>First method</u>: According to the table of variations, for $x \in]1, +\infty[$ the maximum value of f(x) is –4 < 0 then f(x) < 0 So f(x) = 0 has no real roots. <u>Second method</u>: f(x) = 0 ; $-x+1-\frac{4}{x-1} = 0$; $(x-1)^2 = -4$ impossible So f(x) = 0 has no real roots.	**1**
2f	f(x) = −5 ; $-x+1-\frac{4}{x-1} = -5$; $-x^2+7x-10 = 0$ then x = 2 , x = 5.	**1**
2g		**2**
2h	f(x) > – 5 for $x \in]2 ; 5[$	**1**