ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

مسابقة في مادة الرياضيات

المدة: ساعتان

(باللغة الإنكليزية)

الاسم: ....

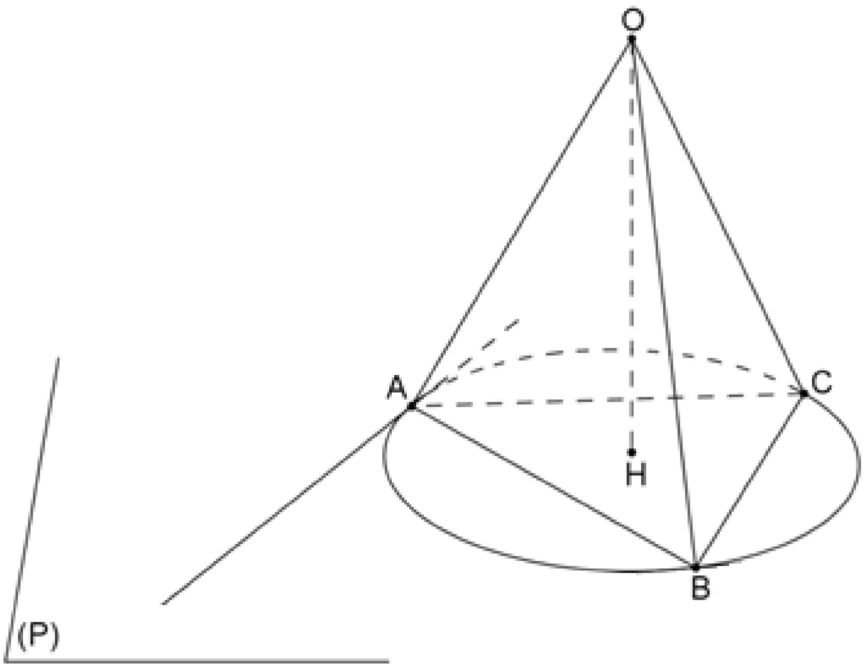
الرقم: ....

I- (4 points)

The space is referred to a direct orthonormal system $\left(O; \vec{i}, \vec{j}, \vec{k}\right)$.

Consider the points **A**(6 , 0 , 0), **B**(0 , 6 , 0) and **C**(0 , 0 , 6).

Let **(Ω)** be the circle circumscribed about triangle **ABC**.



1) Show that triangle **ABC** is equilateral.

2) Write a cartesian equation of the plane **(P)** determined by the points **A**, **B** and **C**.

3) a- Show that point **H**(2 , 2 , 2) is the orthogonal projection of point **O** on **(P)**.

b- Verify that H is the center of **(Ω)**.

c- Show that the volume of tetrahedron **OABC** is triple the volume of tetrahedron **OAHB**.

4) Consider the line **(D)** with parametric equations: $\begin{cases} x = 6 \\ y = -m \\ z = m \end{cases}$; where $m \in \mathbb{R}$.

Show that (D) is tangent to **(Ω)** at **A**.

II- (4 points)

A survey conducted on a group of patients showed that these patients either have a heart disease only or a lung disease only or both diseases.

It was noted that:

- **60 %** of the patients are men.
- Among the men: **20 %** have a heart disease only and **50%** have a lung disease only.
- Among the women: **25%** have a heart disease only and **40%** have both diseases.

Part A

One patient is selected at random.

Consider the following events:

- **M**: "The selected patient is a man"
- **H**: "The selected patient has a heart disease only"
- **L**: "The selected patient has a lung disease only"
- **B**: "The selected patient has both diseases".

1) **a- Calculate** the probability:
   - $P(M \cap H)$
   - $P(M \cap L)$
   - $P(M \cap B)$.

2) **Calculate** :
   - $P(H)$
   - $P(L)$.

   Verify that $P(B) = \frac{17}{50}$

3) **Show that** $P(H \cup L) = \frac{33}{50}$.

4) Knowing that the selected patient has only one disease, **calculate** the probability that this patient has a heart disease.

Part B

The group consists of **500** patients.

The names of three patients were randomly and simultaneously selected to get each a free insurance policy.

Knowing that the three selected patients have both diseases, **calculate** the probability that they are men.

III- (4 points)

The complex plane is referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$.

Consider the points **A** and **B** with affixes **1** and **–2** respectively.

M and M' are two points with respective affixes z and z' such that $z' = \dfrac{\overline{z}+2}{\overline{z}}$ with $z \neq 0$.

1) Write z in <u>exponential form</u> in the case where $z' = 1+i$.

2) a- Show that :

- $\left|\overline{z}+2\right| = BM$
- $OM' = \dfrac{BM}{OM}$.

b- If $\left|z'\right| = 1$, **show that** M is <u>on a straight line</u> to be **determined**.

3) a- For all $z \neq 0$, **show that** $\overline{z}(z'-1) = 2$.

b- For all $z \neq 0$, **verify that** $\arg\left(z'-1\right) = \arg\left(z\right) + 2k\pi$ with $k \in \mathbf{Z}$.

c- In this part, suppose that $z' = 1 + e^{i\theta}$ with $\theta \in \left]-\pi,\pi\right]$.

Show that $\overrightarrow{OM} = 2\overrightarrow{AM'}$.

IV- (8 points)

Part A

Consider the differential equation $(E): y' - y = -2x.$

Let y = **z** + 2x + 2.

1) **Form** the <u>differential equation</u> **(E')** satisfied by **z**.
2) **Solve (E').**

   Deduce the <u>particular solution</u> of **(E)** satisfying y(0) = 0.

Part B

Consider the function **f** defined over $]-\infty,+\infty[$ as $f(x) = 2x + 2 - 2e^{x}$.

Denote by (C) the representative curve of **f** in an orthonormal system $(O; \vec{i}, \vec{j})$.

Let (Δ) be the straight line with equation $y = 2x + 2$.

1) a- **Determine** $\lim\limits_{x\to-\infty} f(x)$.

   b- Show that (C) is below (Δ) for all x.

   c- Show that (Δ) is an <u>asymptote</u> to (C).

2) **Determine** $\lim\limits_{x\to+\infty} f(x)$.

   Calculate $f(1)$ and $f(1.5)$.

3) **Calculate** $f'(x)$.

   Set up the <u>table of variations</u> of **f** .

4) **Draw** (Δ) and (C).

5) a- **Show that f** has, over $]0,+\infty[$, an <u>inverse function</u> **g** .

   Determine the <u>domain of definition</u> of **g**.

   b- Denote by **(G)** the representative curve of **g**.

   Denote by **(T)** the tangent to **(C)** at the point L with abscissa $\ln\left(\frac{3}{2}\right)$.

   Show that (T) is <u>tangent</u> to **(G)** at a point **L'** whose abscissa is $2\ln\left(\frac{3}{2}\right)-1$.