وزارة التربية والتعليم العالي
المديريّة العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة المتوسطة

دورة العام ٢٠١٩ الاستثنائية
الخميس ٢٥ تموز ٢٠١٩

عدد المسائل: خمس — مسابقة في مادة الرياضيات — الاسم:
المدة: ساعتان — الرقم:

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- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I - (2 points)

In what follows, show all the steps of calculation:

Given $A = \sqrt{80} - \sqrt{20} + \sqrt{5}$.

1) Write A in the form of $m\sqrt{5}$ where m is an integer.

2) Let $B = 5\sqrt{5}$.

a. Show that the adjacent table is a proportionality table.

b. Write $\frac{20}{B-5}$ in the form of $p + \sqrt{5}$ where p is an integer.

A	$2\sqrt{19}+1$
$2\sqrt{19}-1$	B

II - (3 points)

A box F contains **twelve** red and black balls.

1) If **one** red ball is removed and **one** black ball is added, then the number of red balls becomes double that of black balls.

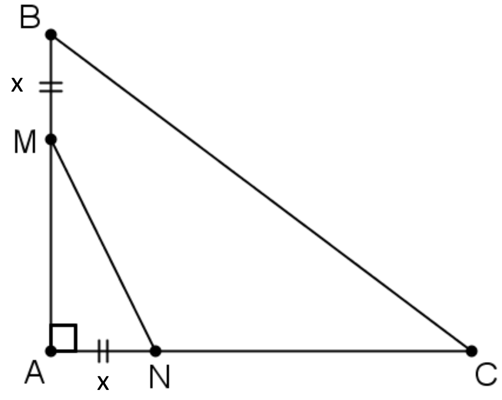
a. Prove that the previous information is modeled by the following system: $\begin{cases} x + y = 12 \\ x - 2y = 3 \end{cases}$

b. Solve the previous system and determine the number of red balls and that of black balls.

2) In what follows, the box F contains **nine** red balls and **three** black balls.
Five red balls and **eight** black balls are added to this box.
Calculate the percentage of red balls in this box.

III - (3 points)

In the adjacent figure:



ABC is a right angled triangle at A; $AB = 6$ and $AC = 8$.
M is a point of [AB] and N a point of [AC] so that:
$AN = BM = x \ (0 < x < 6)$
Denote by S the area of triangle ABC and S' that of triangle AMN.

1) Calculate S.

2) Calculate AM in terms of x and show that $S' = \frac{6x - x^2}{2}$.

3) a. Verify that: $3(x-2)(x-4) = 3x^2 - 18x + 24$.

b. Calculate x so that $S = 6S'$.

4) a. Show that $S' - \frac{9}{2} = \frac{-1}{2}(x-3)^2$.

b. Deduce that the area of triangle AMN is less than or equal to $\frac{9}{2}$.

IV - (6.5 points)

In an orthonormal system of axes x'O x and y'O y, given the points $A(3;3)$, $B(6;0)$ and $E(0;-6)$.

Let (d) be the line with equation $y=-x+6$.

1) **a.** Plot the points A, B and E.

b. Verify that A and B are two points on (d). Draw (d).

2) The line (d) intersects y'O y at F.

Determine the coordinates of F, then verify that A is the midpoint of [BF].

3) **a.** Verify that an equation of the line (AE) is $y=3x-6$.

b. The line (AE) intersects x'O x at $C(2;0)$. What does point C represent for triangle EBF?

c. The lines (CF) and (BE) intersect at M. Prove that M is the midpoint of [BE].

4) Prove that $OBF = OBE = 45°$.

5) The parallel through C to (EB) intersects [FB] at K.

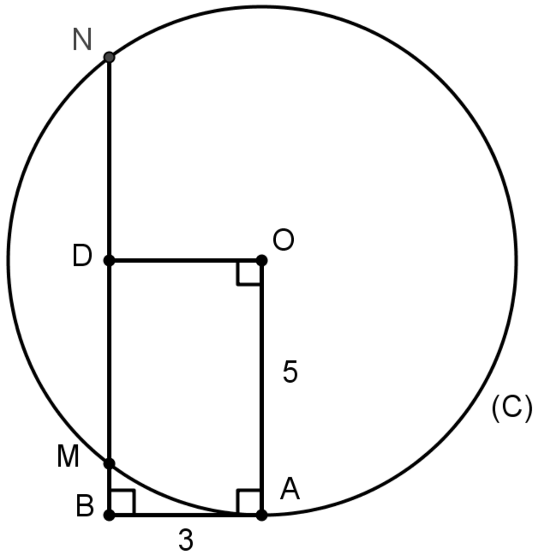
a. Show that the triangle CKB is right isosceles of vertex K.

b. Deduce that $CK = 2\sqrt{2}$.

6) Calculate the ratio $\frac{FC}{FM}$.

V - (5.5 points)

In the adjacent figure:



- OABD is a rectangle so that OA = 5 and AB = 3
- (C) is the circle of center O passing through point A
- The line (BD) intersects the circle (C) at M and N.

1) Draw the figure.

2) **a.** What is the nature of triangle ONA? Justify.

b. Show that [NA) is the bisector of angle BNO.

3) Show that DN = 4 and calculate BN.

4) The two lines (NA) and (OD) intersect at L.

a. Show that the two triangles BAN and OLA are similiar.

b. Deduce that $BN \times LO = 15$, then calculate LO.

5) The perpendicular through A to (OB) intersects circle (C) at F.

Show that (BF) is tangent to the circle (C).

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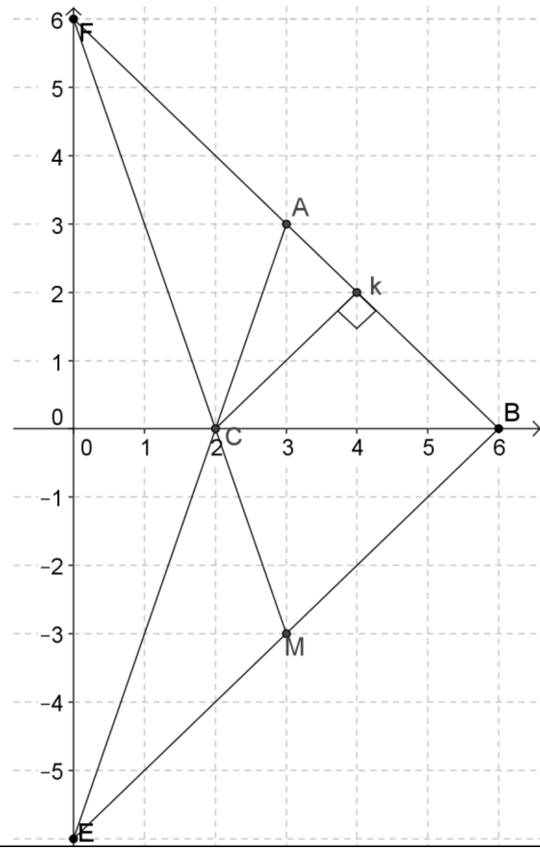
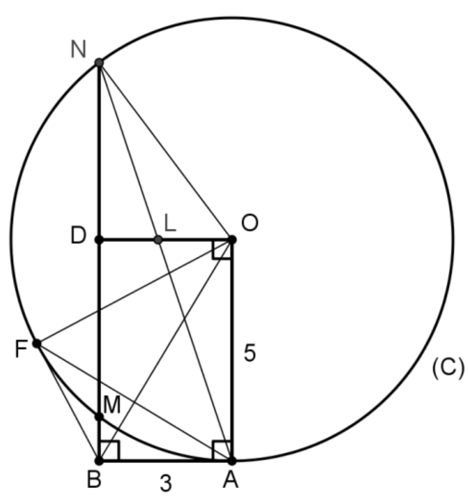
عدد المسائل: خمس	مسابقة في مادة الرياضيات	مشروع اسس التصحيح - انكليزي

Part	Answer Key	Grade
Question I		
1	$A = \sqrt{80} - \sqrt{20} + \sqrt{5}$ $A = 4\sqrt{5} - 2\sqrt{5} + \sqrt{5}$ **0.25 + 0.25** $A = 3\sqrt{5}$ **0.25**	**0.75**
2a	$A \times B = 3\sqrt{5} \times 5\sqrt{5} = 75$ **0.25** $(2\sqrt{19} - 1)(2\sqrt{19} + 1) = 76 - 1 = 75$ **0.25** Then: $A \times B = (2\sqrt{19} - 1)(2\sqrt{19} + 1)$. The given table is a proportionality table **0.25**	**0.75**
2b	$\frac{20}{5\sqrt{5}-5} = \frac{20}{5(\sqrt{5}-1)} = \frac{4}{\sqrt{5}-1} = \frac{4}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = 1 + \sqrt{5}$ **(0.25 conjugate + 0.25 result)**	**0.5**
Question II		
1a	Let x be the number of red balls and y that of black balls. **0.25** $x + y = 12$ **0.25** $x - 1 = 2(y + 1)$ **0.25** Then: $x - 2y = 3$ **0.25**	**1**
1b	Using the calculator : $x = 9 \; et \; y = 3$ **0.75 + 0.25**	**1**
2	14 red balls and 11 black balls. **0.25** $\frac{14 \times 100}{25} = 56\ \%$ **0.5 + 0.25**	**1**
Question III		
1	$S = \frac{b \times h}{2} = \frac{6 \times 8}{2} = 24$ **0.25 + 0.25**	**0.5**
2	$AM = AB - BM = 6 - x$ **0.25** $S' = \frac{b \times h}{2} = \frac{(6-x)x}{2} = \frac{6x - x^2}{2}$ **0.25**	**0.5**
3a	$3(x-2)(x-4) = 3(x^2 - 4x - 2x + 8) = 3x^2 - 18x + 24$ **0.5**	**0.5**
3b	$S = 6S'$ $24 = 6\left(\frac{6x - x^2}{2}\right)$ $24 = 18x - 3x^2$ **0.25** $3x^2 - 18x + 24 = 0$ $3(x-2)(x-4) = 0$ gives $x = 2$ or $x = 4$. **0.25**	**0.5**
4a	$S' - \frac{9}{2} = \frac{6x - x^2}{2} - \frac{9}{2} = \frac{6x - x^2 - 9}{2} = \frac{-1}{2}(x-3)^2$. **0.25 + 0.25** or expanding both sides. **0.25 + 0.25**	**0.5**
4b	$S' - \frac{9}{2} = \frac{-1}{2}(x-3)^2 \leq 0$. **0.25** then : $S' \leq \frac{9}{2}$ **0.25**	**0.5**

Question IV		
1a		**0.75**
1b	$y_A = -x_A + 6$ $3 = -3 + 6$ $3 = 3$ Then A is a point of (d). **0.25** $y_B = -x_B + 6$ $0 = -6 + 6$ $0 = 0$ Then B is a point of (d). **0.25** A and B determine the line (d) Therefore to draw (d), we join A and B. **0.25**	**0.75**
2	$F \in y'oy$ then $x_F = 0$ **0.25** $F \in (d)$ then $y_F = -x_F + 6 = -(0) + 6 = 6$ **0.25** So $F(0; 6)$ $x_A = \frac{x_B + x_F}{2}$ $3 = \frac{6 + 0}{2}$ $3 = 3$ **0.25** $y_A = \frac{y_F + y_B}{2}$ $3 = \frac{6+0}{2}$ **0.25** $3 = 3$ Then A is the midpoint of [BF].	**1**

3a	$a_{(AE)} = \frac{3+6}{3-0} = 3$ **0.75** $3 = 9 + b$ $b = 3 - 9 = -6$ **0.25** Then: $(AE): y = 3x - 6$ or by replacing the coordinates of both points A and E. **0.75 + 0.25**	**1**
3b	In triangle BFE we have : [BO] and [EA] are two medians intersecting at C. **0.25** Then C is the center of gravity of this triangle.(Intersection point of the 3 medians) **0.25**	**0.5**
3c	In the triangle BFE: FM is the third median passing through C (the center of gravity) **0.25** Then M is the midpoint of [BE].	**0.25**
4	In the triangle OBF ; we have: OB = OF = 6 and $\widehat{FOB} = 90^0$ **0.25 + 0.25** Then FBO is a right isosceles triangle at O so $OBF = 45°$. **or: using the method of the acute angle formed between (AB) and the x-axis** Similarly we prove $OBE = 45°$. **0.25**	**0.75**
5a	We have: $(CK)//(EB)$ then: $\widehat{EBK} = \widehat{CKF} = 90^0$ (corresponding angles). Then triangle CKB is right at K. **0.25** and: $KBC = 45°$. (Proved) then triangle CKB is also isosceles. **0.25**	**0.5**
5b	CKB is a right isosceles triangle then : CK = $\frac{CB}{\sqrt{2}} = 2\sqrt{2}$ Or $\sin 45^0 = \cdots$	**0.5**
6	$\frac{FC}{FM} = \frac{2}{3}$ since C is the center of gravity of triangle FBE. or : Thales' theorem or : by calculation ….	**0.5**
	Question V	
1		**0.5**
2a	OA = ON (radii of the same circle), then triangle ONA is isosceles of vertex O. **0.5**	**0.5**

2b	$\widehat{OAN} = \widehat{ANB}$ (alt – int) $\widehat{OAN} = \widehat{ONA}$ (base angles of isosceles triangle) Then : $\widehat{ANB} = \widehat{ONA}$ [NA) is the bisector of $\widehat{BNO}$	**0.25** **0.25** **0.25**	**0.75**
3	ODN is a right triangle at D. $ON^2 = OD^2 + DN^2$ (Pythagorean) $DN^2 = ON^2 - DO^2 = 5^2 - 3^2 = 25 - 9 = 16$ $DN = 4$ BN = DN + DB = 4 + 5 = 9	**0.5** **0.25** **0.25**	**1**
4a	In the two triangles BAN and OLA we have : $\widehat{AOL} = \widehat{ABN} = 90^0$ $\widehat{OAN} = \widehat{ANB}$ (alt – int) Then they are similar	**0.5** **0.5**	**1**
4b	$\frac{OA}{BN} = \frac{OL}{AB} = \frac{LA}{AN}$ (ratio of similitude) $OL \times BN = AO \times AB = 15.$ $OL = \frac{15}{BN} = \frac{15}{9} = \frac{5}{3}$	**0.5** **0.25** **0.25**	**1**
5	OAF is an isosceles triangle since OF = OA (radii of the same circle) Then (OB) is a perpendicular bisector. (height in an isosceles triangle is also a perp. Bis) In the two triangles OBF and OBA we have : OF = OA (proved) BF = BA (Any point on the perpendicular bisector of a seg. is equidistant from its extremities) OB = OB (common side) then the two triangles are congruent. Then : $\widehat{OFB} = \widehat{OAB} = 90^0$ (c.p.c.t) (BF) is tangent to the circle (Being perpendicular to the radius at the point of tangency)	**0.25** **0.25** **0.25**	**0.75**