This exam is formed of four obligatory exercises in thirteen pages.
The use of non-programmable calculators is recommended.

مسابقة في مادة الفيزياء

المدة: ثلاث ساعات

(انكليزي)

الاسم:..................................

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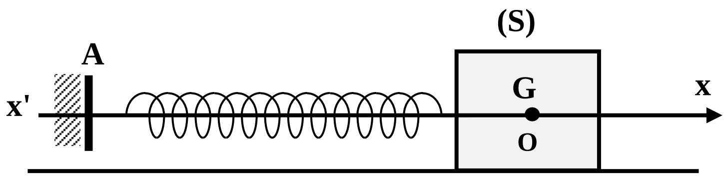
Exercise 1 (8 points) **Free damped mechanical oscillations**

Consider a mechanical oscillator formed by a rigid object **(S)** of mass **m** and a horizontal spring of spring constant **k** and of negligible mass.



Doc. 1

- **(S)** is attached to one end of the spring, and the other end is fixed to a support **A**.
- **(S)** may move on a horizontal surface with its center of mass **G** being on a horizontal **x-axis** (Doc. 1)
- At equilibrium, **G** coincides with the origin **O** of the **x-axis**.
- **(S)** is shifted horizontally in the positive direction from its equilibrium position.
- At the instant $t_0 = 0$, the abscissa of **G** is X_m and **(S)** is released without initial velocity.
- At an instant **t**, the abscissa of **G** is $x = \overline{OG}$ and the algebraic value of its velocity $v = x' = \frac{dx}{dt}$.
- During its motion, **(S)** is subjected to several forces including:
- The tension force $\vec{F} = -k\,x\,\vec{i}$ of the spring and
- The friction force $\vec{f} = -h\vec{v}$, where **h** is a positive constant called the damping coefficient.

Take the horizontal plane containing **G** as a reference level for gravitational potential energy.

The aim of this exercise is to study the effect of friction on the oscillations and to determine the value of **h**.

1) Theoretical study

1-1) Show by applying Newton's second law: $\sum \vec{F}_{ext} = m \frac{d\vec{v}}{dt}$

that $\mathbf{m \frac{dv}{dt} + k\,x = -\,hv}$

1-2) Write, at an instant t, the expression of the mechanical energy ME of the system (Oscillator - Earth) in terms of m, k, x and v.

1-3) Deduce that $\frac{dME}{dt} = -\,hv^2$.

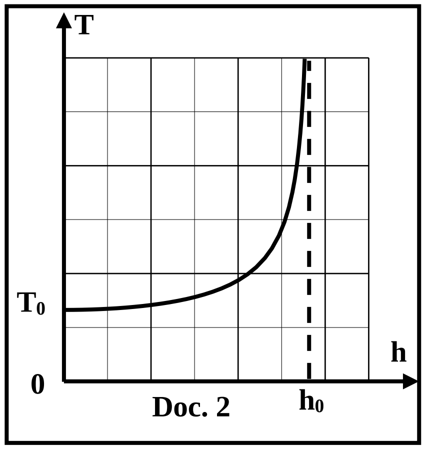
1-4) Establish the differential equation, second order of x, of the motion of **(G).**

1-5) The center of mass **G** oscillates with an angular frequency

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{h}{2m}\right)^2}\ .$$

Deduce the expression of the pseudo-period T.

1-6) For different values of **h**, we obtain the curve of document 2 which represents **T** as a function of **h**, for $\mathbf{0 \leq h < h_0}$.



Doc. 2

1-6-1) **How does T** vary for $\mathbf{0 \leq h < h_0}$?

1-6-2) $\mathbf{T_0}$ represents the proper period of oscillation of G.

Justify by referring to document 2.

1-7) **Deduce** the expression of $\mathbf{T_0}$ in terms of **m** and **k**.

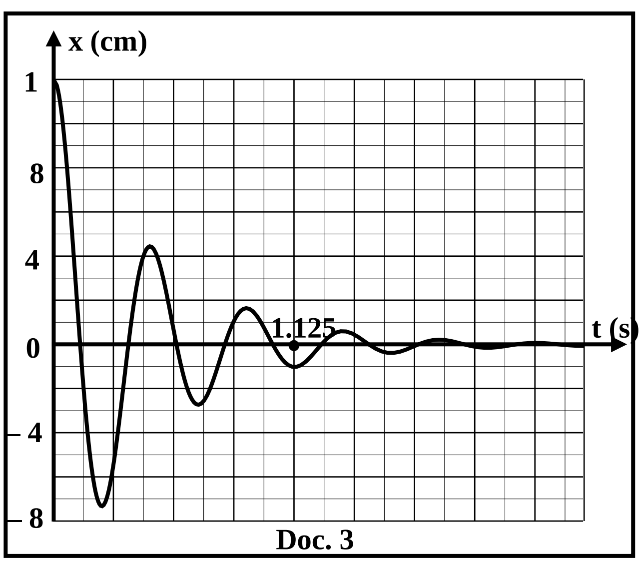
2) Experimental study

In the experimental study, we take **m = 0.5 kg and k = 100 N/m.**

2-1) **Calculate** the value of $\mathbf{T_0}$**.**

2-2) The curve of document (3) represents **x** as a function of time **t**.

Use document 3 to:



Doc. 3

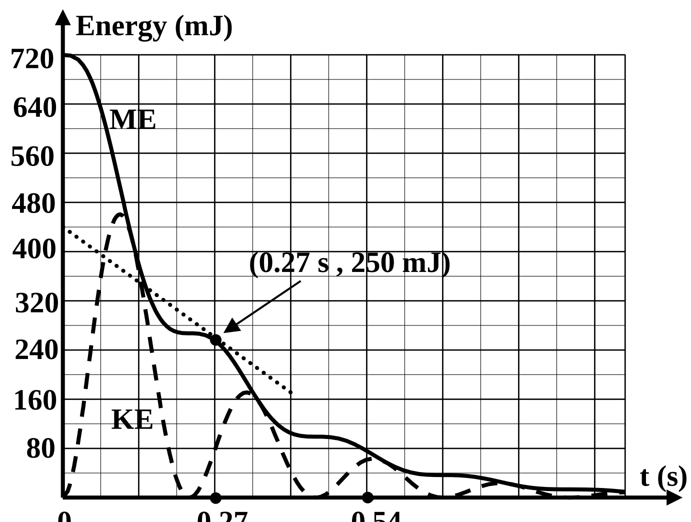
2-2-1) Determine the pseudo-period **T**

2-2-2) Give two indicators showing that **(S)** is submitted to a friction force.

2-3) Calculate h.

2-4) In order to determine again the value of **h**, an appropriate device is used to trace the curves of **ME** of the system (oscillator, earth) and the kinetic energy **KE** of **(S)** as functions of time.

Document 4 represents the curves of **ME** and **KE** as a function of time, and also the tangent to the curve of **ME at t = 0.27 s**



Doc. 4

2-4-1) Determine the speed of **G** at **t = 0.27 s (**by using the dotted curve) representing **KE**.

2-4-2) Determine $\frac{dME}{dt}$ at **t = 0.27 s**. (use the dotted line)

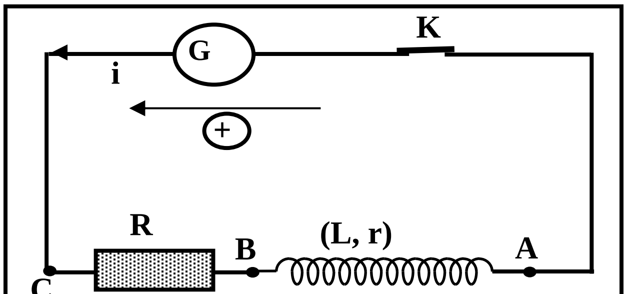
2-4-3) Deduce again the value of **h**.

Exercise 2 (8 points) Characteristics of a coil

The aim of this exercise is to determine the characteristics of a coil by two methods.

We connect in series:

A generator **(G)**, a switch **K**, a resistor of resistance **R = 90 Ω** and a coil of inductance **L** and resistance **r** (Doc. 5).



Doc. 5

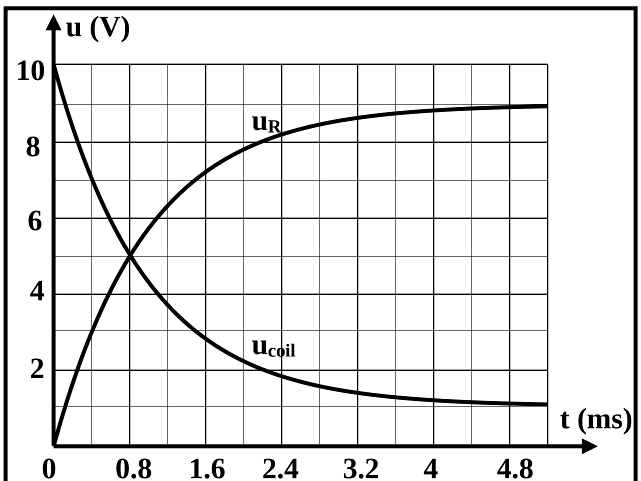
We close the switch **K** at the instant $t_0 = 0$.

At an instant **t**, the circuit carries a current **i**.

1) First method

(G) is a generator providing a constant voltage $u_{CA} = E$.

An appropriate device traces the curves of $u_{CB} = u_R$ **and** $u_{BA} = u_{coil}$ as functions of time (Doc. 6).



Doc. 6

1-1) Using the curves of document 6:

1-1-1) **Determine** the value of **E; (knowing that $u_{CA} = u_{CB} + u_{BA}$);**

1-1-2) **Determine** the value of the current **I_0** at the steady state; (using the the curve u_R);

1-1-3) **Show** that **r = 10 Ω.**

1-2) **Establish** the first order differential equation in i **by applying the law of addition of voltages**.

1-3) The solution of this differential equation is $i = I_0 \left(1 - e^{\frac{-(R+r)}{L}t}\right)$.

Deduce the expressions of **u_R** and **u_{coil}** in terms of **R, r, L, I_0** and **t**.

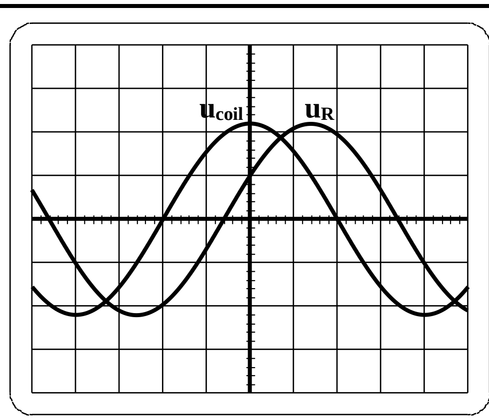
1-4) At an instant t_1, $u_{coil} = u_R$. **Show** that $t_1 = -\frac{L}{R+r}\ell n\left(\frac{R-r}{2R}\right)$.

1-5) **Deduce** the value of **L** using (Doc.6).

2) Second method

The generator **(G)** provides now an alternating sinusoidal voltage of angular frequency ω.

An oscilloscope is connected conveniently in the circuit in order to display **$u_{CB} = u_R$** on channel 1 and **$u_{BA} = u_{coil}$** on channel 2 (Doc. 7).



Doc. 7

The adjustments of the oscilloscope:

Horizontal sensitivity: **S_h = 4 ms/div**

Vertical sensitivity: For Ch_1: **S_{V1} = 4 V/div;**

For Ch_2: **S_{V2} = 1 V/div**

2-1) The circuit carries an alternating sinusoidal current **$i = I_m \sin(\omega t)$,(SI).**

Determine the expression of u_{coil} in terms of L, ω, I_m , r and t.

2-2) The expression of the voltage across the coil is of the form:

$u_{coil} = A \sin(\omega t) + B \cos(\omega t)$ where A and B are constants.

Determine A and B in terms of r, L, I_m and ω.

2-3) Calculate, by using document 7:

2-3-1) the **values** of **I_m** and **ω;**

2-3-2) the **maximum voltage U_m** of the voltage across the coil;

2-3-3) the phase difference φ between u_{coil} and u_R.

2-4) Determine again the values of **L** and **r** knowing that

$\tan \varphi = \frac{L\omega}{r}$ and $U_m^2 = A^2 + B^2$.

Exercise 3 (7 points) **Decay of radon-219**

The aim of this exercise is to determine **the values of the power and the energy of the electromagnetic radiation** γ emitted in the disintegration of radon-219.

The radionuclide radon $^{219}_{86}Rn$ decays into polonium $^{A}_{Z}Po$ with the emission of an α particle and γ radiation of energy E_γ according to the following equation:

$$^{219}_{86}Rn \rightarrow \ ^{A}_{Z}Po + \alpha + \gamma$$

Given: **m(**$^{219}_{86}Rn$**) = 204007.3316 MeV/c²** **;**

m($^{A}_{Z}Po$**) = 200271.9597 MeV/c²** **;**

m(α) = 3728.4219 MeV/c²

1 MeV = 1.602×10^{-13} **J** **;**

Molar mass of $^{219}_{86}Rn$ **is 219 g/mol** **;**

N_A **= 6.022×**10^{23} **mol⁻¹**

1) **Calculate** A and Z, indicating the used laws.
2) **Verify that** the energy liberated by the decay of one nucleus of radon-219 is: **6.95 MeV**.
3) **Deduce** that the energy of the emitted γ radiation is E_γ **= 0.195 MeV** knowing that:

   The kinetic energy of the emitted α particle **is 6.755 MeV**

   The kinetic energy of the polonium nucleus is negligible et the radon nucleus is at rest.

4) **At $t_0 = 0$**, the initial mass of a radon sample is $m_0 = 8$ g.

Show that the initial number N_0 of radon nuclei present in the sample at

$t_0 = 0$ is $N_0 = 21.998 \times 10^{21}$ nuclei.

5) **Calculate** the number of the radon nuclei $^{219}_{86}Rn$ disintegrates. ($N_d = N_0 - N$)

6) **Calculate**:

- the value <u>of the decay constant λ</u>
- the value <u>of the half-life T of radon-219.</u>

7) **Calculate**, <u>in becquerel, the activity A_1 of the radon sample at the instant $t_1 = 10$ s.</u>

8) The energy of the emitted γ radiation between the instant $t_0 = 0$ and an instant t is

$E = N_d E_\gamma$ where N_d is the number of the decayed nuclei of radon-219 between these two instants.

8-1) **Show** that $E = N_0 E_\gamma (1 - e^{-\lambda t})$ knowing that $N = N_0 e^{-\lambda t}$

8-2) **Deduce** the value of E during the time interval [0, ∞[.

9) **The power p of the emitted γ radiation at an instant t is given by: $p = \frac{dE}{dt}$.**

9-1) **Show** that $p = \lambda N_0 E_\gamma e^{-\lambda t}$.

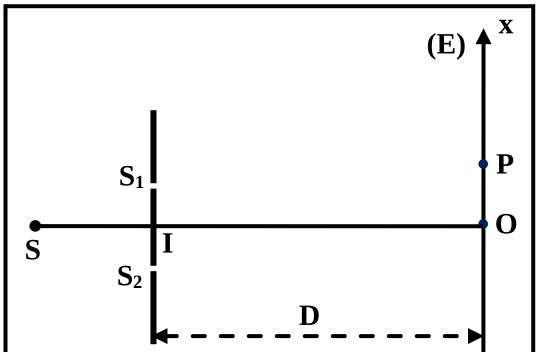
9-2) **Deduce** the maximum power P_{max} of the γ radiation. (By taking a particular value of t)

9-3) **Deduce** the power of the γ radiation as $t \to \infty$.

Exercise 4 (7 points) **Interference of light**

The aim of this exercise is to study the phenomenon of interference of light using Young's double-slit set-up.

Document 8 shows Young's double-slit set-up, which is constituted of two thin parallel and horizontal slits S_1 and S_2 separated by a distance a = 0.5 mm, and a screen (E) placed parallel to the plane of the two slits at a distance D = 2 m.



Doc. 8

A point source S, equidistant from S_1 and S_2, illuminates the two slits by monochromatic radiation of wavelength λ = 600 nm in air. (OI) is the perpendicular bisector of the segment $[S_1S_2]$.

The expression of the optical path difference at point P on the vertical x-axis in the interference pattern is:

$\delta = (SS_2 + S_2P) - (SS_1 + S_1P) = \frac{a\,x}{D}$ **where** $x = \overline{OP}$.

1) Describe the interference pattern on the screen (E).

2) Show that O is the center of the central bright fringe.

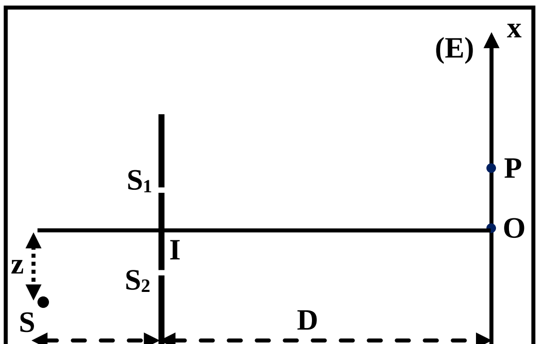
3) Suppose that P is the center of a dark fringe of **order k ($k \in \mathbb{Z}$).**

3-1) Give the expression of the optical path difference δ at **point P in terms of k and λ.**

3-2) **Deduce** the expression of the abscissa x_k of P **in terms of k, λ, D and a.**

3-3) **Determine** the order of the dark fringe at P **knowing that x_k = 6 mm.**

4) The point source S which is placed at a distance d from the plane of the slits, is moved by a displacement z, **in the negative direction**, to the side of S_2 parallel to the x-axis (Doc. 9).



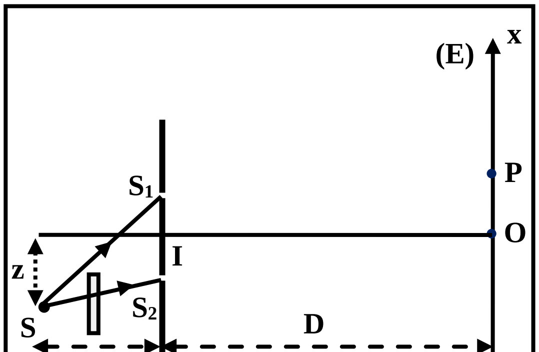
Doc. 9

The optical path difference at point P becomes: $\delta = \frac{a\,z}{d} + \frac{a\,x}{D}$.

4-1) **Determine** the position of the center O' of the central bright fringe in terms of **D, z and d**.

4-2) **Specify** whether the central bright fringe is displaced to the side of S_1 or to the side S_2.

4-3) A thin transparent plate of parallel faces, of thickness e and of refractive index n, is placed in front of S_2 (Doc.10).



Doc. 10

The optical path difference at point P becomes:

$$\delta = \frac{a\,z}{d} + \frac{a\,x}{D} + e\,(n - 1).$$

We adjust **the distance d** in order that the center of the central bright fringe returns back to the point O.

Determine the value of **d knowing that |*z*| = 0.4 cm.**