This exam is formed of three obligatory exercises in nine pages. The use of non-programmable calculators is recommended.

المدة: ساعتان
(إنكليزي)


## Exercise 1 (7 points) Horizontal elastic pendulum

A mechanical oscillator is formed by :

- a block ( $\mathbf{S}$ ) of mass $\mathbf{m}$
- a spring of negligible mass and spring constant $\mathbf{k}$.
- (S) is attached to one end of the spring.
- the other end of the spring is connected to a fixed support A.
- (S) can move without friction on a horizontal surface (Doc. 1).


The aim of this exercise is to determine the values of $\mathbf{m}$ and $\mathbf{k}$.

- At equilibrium, the center of mass $\mathbf{G}$ of ( $\mathbf{S}$ ) coincides with the origin $\mathbf{O}$ of the $\mathbf{x}$ - axis.
- (S) is displaced horizontally in the positive direction.
- At the instant $\mathbf{t}_{\mathbf{0}}=\mathbf{0}$, taken as an initial time,
- the abscissa of $(\mathbf{S})$ is $\mathbf{x}_{0}$ and $(\mathbf{S})$ is launched in the negative direction with an initial velocity $\overrightarrow{\mathbf{v}_{\mathbf{0}}}=\mathbf{v}_{\mathbf{0}} \overrightarrow{\mathbf{1}} \underline{\left(\mathbf{v}_{0}<\mathbf{0}\right)}$ where $\overrightarrow{\mathbf{1}}$ is the unit vector of the $\mathbf{x}$-axis.
- At an instant $\mathbf{t}$,
- the abscissa of $\mathbf{G}$ is $\mathbf{x}$ and the algebraic value of its velocity is $\mathbf{v}=\mathbf{x}^{\prime}=\frac{\mathbf{d x}}{\mathbf{d t}}$.

The horizontal plane containing $\mathbf{G}$ is taken as a reference level for gravitational potential energy.

Given :

$$
\mathrm{KE}=1 / 2 \mathrm{mv}^{2} ; \quad \text { GPE }=\mathbf{m g h} ; \quad \mathrm{EPE}=1 / 2 \mathbf{k x}^{2} ; \quad \mathrm{ME}=\mathrm{KE}+\mathrm{EPE}+\mathrm{GPE}
$$

1) Write, at an instant $\mathbf{t}$, the expression of the mechanical energy of the system (Oscillator, Earth) in terms of $\mathbf{x}, \mathbf{m}, \mathbf{k}$ and $\mathbf{v}$.
2) Knowing that (S) moves without friction, deduce that the second order differential equation in $\mathbf{x}$ that describes the oscillations of (S), is $\mathbf{x}^{\prime \prime}+\frac{\mathbf{k}}{\mathbf{m}} \mathbf{x}=\mathbf{0}$.
3) Deduce the expression of the proper angular frequency $\omega_{0}$ of the oscillations in terms of $\mathbf{m}$ and $\mathbf{k}$.
4) The solution of the obtained differential equation is:
$\mathbf{x}=\mathbf{X}_{\mathrm{m}} \sin \left(\omega_{0} \mathbf{t}+\varphi\right)$, where $\mathbf{X}_{\mathrm{m}}, \omega_{0}$ and $\varphi$ are constants.

Write the expression of $\mathbf{v}$ in terms of $\mathbf{X}_{\mathrm{m}}, \omega_{0}, \varphi$ and $\mathbf{t}$.
5) Write the expression of $\mathbf{x}_{\boldsymbol{0}}$ in terms of $\mathbf{X}_{\mathrm{m}}$ and $\boldsymbol{\varphi}$.

Write the expression of $\mathbf{v}_{\mathbf{0}}$ in terms of $\mathbf{X}_{\mathbf{m}}, \omega_{0}$ and $\varphi$.
6) Deduce that $X_{m}=\sqrt{X_{0}^{2}+\frac{v_{0}^{2}}{\omega_{0}^{2}}}$.

Given:

$$
\sin ^{2} \varphi+\cos ^{2} \varphi=1
$$

7) An appropriate device traces $\mathbf{x}$ and $\mathbf{v}$ as functions of time as shown in documents 2 and 3 respectively.



Referring to documents (2) :
7-1) specify the type of the oscillations;
7-2) indicate the values of $\mathbf{x}_{0}$ and $\mathbf{X}_{\mathrm{m}}$.
Referring to document (3):
7-3) indicate the values of $\mathbf{v}_{\mathbf{0}}$ and $\mathbf{V}_{\mathbf{m}}$ where $\mathbf{V}_{\mathbf{m}}$ is the amplitude of $\mathbf{v}$.
8) Using the relations of questions $6 \quad\left(X_{m}=\sqrt{X_{0}^{2}+\frac{v_{0}^{2}}{\omega_{0}^{2}}}\right)$ :

Show that $\omega_{0}=20 \mathrm{rad} / \mathrm{s}$.
9) We repeat the same experiment by replacing block ( $\mathbf{S}$ ) of mass $m$ by another block ( $\mathbf{S}^{\prime}$ ) of mass $\mathbf{m}^{\prime}=\mathbf{0 . 8} \mathbf{~ k g}$. The new proper angular frequency is $\boldsymbol{\omega}^{\prime}=\frac{\boldsymbol{\omega}_{\mathbf{0}}}{\mathbf{2}}$

9-1) Write the expression of $\boldsymbol{\omega}^{\prime}$ in terms of $\mathbf{m}^{\prime}$ and $\mathbf{k}$.
9-2) Deduce the values of $\mathbf{k}$ and $\mathbf{m}$.

## Exercise 2 ( 7 points) Capacitance of a capacitor

The aim of this exercise is to determine the capacitance $\mathbf{C}$ of a capacitor.
We set-up the series circuit of document 4.


This circuit includes:

- an ideal battery of electromotive force $\mathbf{E}=\mathbf{1 0} \mathbf{V}$;
- a rheostat of resistance $\mathbf{R}$;
- a capacitor of capacitance $\mathbf{C}$;
- an ammeter (A) of negligible resistance;
- a switch $\mathbf{K}$.

Initially the capacitor is uncharged.

- We close the switch $\mathbf{K}$ at the instant $\mathbf{t}_{\mathbf{0}}=\mathbf{0}$.
- At an instant $\mathbf{t}$, plate $\mathbf{B}$ carries a charge $\mathbf{q}$ and the circuit carries a current $\mathbf{i}$ as shown in document 4.

1) Write the expression of $\mathbf{i}$ in terms of $\mathbf{C}$ and $\mathbf{u}_{\mathbf{C}}$, where $\mathbf{u}_{\mathbf{C}}=\mathbf{u}_{\mathbf{B D}}$ is the voltage across the capacitor.
2) Prove that the differential equation that governs the variation of $\mathbf{u}_{\mathrm{C}}$ is:

$$
\mathbf{E}=\mathbf{u}_{\mathrm{C}}+\mathbf{R C} \frac{\mathrm{du}_{\mathrm{C}}}{\mathrm{dt}} .
$$

3) The solution of this differential equation is of the form: $\mathbf{u}_{\mathbf{C}}=\mathbf{a}+\mathbf{b} \mathbf{e}^{-\frac{\mathbf{t}}{\tau}}$. Determine the expressions of the constants $\mathbf{a}, \mathbf{b}$ and $\tau$ in terms of $\mathbf{E}, \mathbf{R}$ and $\mathbf{C}$.
4) Deduce that the expression of the current is: $\mathbf{i}=\frac{\mathbf{E}}{\mathbf{R}} \mathbf{e}^{-\frac{\mathbf{t}}{\mathbf{R C}}}$.
5) The ammeter $(\mathbf{A})$ reads a value $\mathbf{I}_{\mathbf{0}}=\mathbf{5} \times \mathbf{1 0}^{-\mathbf{3}}$ A at $\mathbf{t}_{\mathbf{0}}=\mathbf{0}$.

Deduce the value of $\mathbf{R}$.
6) Write, using Ohm's law, the expression of $\mathbf{u}_{\mathbf{R}}=\mathbf{u}_{\mathrm{DN}}$ in terms of $\mathbf{E}, \mathbf{R}, \mathbf{C}$ and $\mathbf{t}$.
7) At an instant $\mathbf{t}=\mathbf{t}_{\mathbf{1}}$, the voltage across the capacitor is $\mathbf{u}_{\mathbf{C}}=\mathbf{u}_{\mathbf{R}}$.

## 7-1) Show that $t_{1}=R C \ln 2$.

7-2) Calculate the value of $\mathbf{C}$ knowing that $\mathbf{t}_{\mathbf{1}}=\mathbf{1 . 4} \times \mathbf{1 0}^{-\mathbf{3}} \mathrm{s}$.
8) In order to verify the value of $\mathbf{C}$, we change the value of $\mathbf{R}$.

Document 5 represents $\tau$ as a function of $\mathbf{R}$.


8-1) Show that the shape of the curve in document 5 is in agreement with the expression of $\tau$ obtained in question 3.
8-2) Using the curve of document 5, determine again the value of $\mathbf{C}$.

The aim of this exercise is to show evidence of the two aspects of light.

## 1) First aspect

- Consider Young's double-slit experiment.
- The two thin parallel horizontal slits $\mathbf{S}_{\mathbf{1}}$ and $\mathbf{S}_{\mathbf{2}}$ are separated by a distance $\mathbf{a}=\mathbf{0 . 5} \times \mathbf{1 0 ^ { - 3 }} \mathbf{m}$.
- The screen (E) is placed parallel to the plane of the slits at a distance $\mathbf{D}=\mathbf{2} \mathbf{~ m}$.
- A laser source illuminates the two slits by a monochromatic light of wavelength $\lambda=\mathbf{6 0 0} \mathbf{~ n m}\left(\mathbf{1} \mathbf{n m}=\mathbf{1} \times \mathbf{1 0}^{-9} \mathbf{~ m}\right)$, in air, under normal incidence.
- $\mathbf{O}$ is the point of intersection between the perpendicular bisector of $\left[\mathrm{S}_{1} \mathrm{~S}_{2}\right]$ and the screen ( $\mathbf{E}$ ).
$-\mathbf{P}$ is a point on the screen having an abscissa $\mathbf{x}_{\mathbf{P}}=\overline{\mathbf{0 P}}=\mathbf{9 . 6} \times \mathbf{1 0}^{-\mathbf{3}} \mathbf{m}$ (Doc. 6).


1-1) Prove that the value of the inter-fringe distance $\mathbf{i}$ is $2.4 \times 10^{-3} \mathrm{~m}$.

1-2) $\mathbf{P}$ is the center of a bright fringe, why?

1-3) Slits $\mathbf{S}_{\mathbf{1}}$ and $\mathbf{S}_{\mathbf{2}}$ are replaced by a horizontal slit $\mathbf{S}$ of width $\mathbf{a}_{\mathbf{1}}=\mathbf{0 . 1} \times \mathbf{1 0} \mathbf{1 0}^{-\mathbf{3}} \mathbf{m}$.
$\mathbf{O}$ is the center of the central bright fringe and $\boldsymbol{\alpha}=\mathbf{2} \theta_{1}$ where $\boldsymbol{\alpha}$ is the angular width of the central bright fringe ( $\theta_{1}$ is a small angle) Doc. 7 .


## 1-3-1) Choose the right answer:

The phenomenon that takes place at the slit $\mathbf{S}$ is:
a- Interference
b- Diffraction
c- Refraction

1-3-2) Show, using document 7, that the width $\mathbf{L}$ of the central bright fringe is given by the expression: $\mathbf{L}=\frac{2 \lambda \mathbf{D}}{\mathbf{a}_{\mathbf{1}}}$.

1-3-3) Deduce the distance $\mathbf{d}$ between $\mathbf{O}$ and the center of the first dark fringe.
1-3-4) Knowing that $\mathbf{x}_{\mathbf{P}}=\mathbf{9 . 6} \times \mathbf{1 0} 0^{-\mathbf{3}} \mathbf{~ m}$, deduce that $\mathbf{P}$ is neither the center of a bright fringe, nor the center of a dark fringe.

1-4) The previous two experiments show evidence of an aspect of light.
Name this aspect.

## 2) Second aspect

The monochromatic radiation, of wavelength $\lambda=\mathbf{6 0 0} \mathbf{~ n m}$ in air, emitted by the laser source, illuminates now the surface of a lithium metal of work function $W_{\mathbf{0}}=\mathbf{2 . 3 9} \mathbf{e V}$.
Given:

- Planck's constant $\mathbf{h}=\mathbf{6 . 6 \times 1 0} 0^{-34} \mathrm{~J} . \mathbf{s}$;
- $\mathbf{1 ~ e V}=\mathbf{1 . 6 \times 1 0} \mathbf{1 0}^{-19} \mathrm{~J}$;

Take: The speed of light in air $\mathbf{c}=\mathbf{3} \times \mathbf{1 0}^{\mathbf{8}} \mathbf{~ m} / \mathbf{s}$.

2-1) Complete the following sentence:
The work function (extraction energy) of a metal is the of a
$\ldots \ldots \ldots .$. to extract an electron from the surface of a metal.

2-2) Write, the expression of the energy of the photon in terms of $\mathbf{h}, \mathbf{c}$ and $\lambda$.
Prove that its value is $\mathbf{E}_{\mathrm{ph}}=\mathbf{2 . 0 6 2 5} \mathbf{e V}$.

## 2-3) Compare $\mathrm{E}_{\mathrm{ph}}$ and $\mathrm{W}_{0}$.

Deduce that there is no photoelectric emission from the surface of the lithium metal.

2-4) In order to extract electrons from the surface of the lithium metal, the laser source is replaced by another one emitting a radiation of wavelength $\boldsymbol{\lambda}^{\prime}=\mathbf{5 0 0} \mathbf{~} \mathbf{~ m}$ in air.

Determine, using the relation $\mathbf{E}_{\mathbf{p h}}=\mathbf{W}_{\mathbf{0}}+\mathbf{K E}$, the maximum kinetic energy of the liberated electrons.

2-5) This experiment shows evidence of an aspect of light.
Name this aspect.

