This exam is formed of three obligatory exercises in nine pages. The use of non-programmable calculators is recommended.

(إنكليزي)

الاسم: الرقم:

Exercise 1 (7 points)

A mechanical oscillator is formed by :

- a block (S) of mass m
- a spring of negligible mass and spring constant **k**.
- (S) is attached to one end of the spring.
- the other end of the spring is connected to a fixed support **A**.
- (S) can move <u>without friction</u> on a horizontal surface (Doc. 1).



The aim of this exercise is to determine the values of **m** and **k**.

- At equilibrium, the center of mass G of (S) coincides with the origin O of the x- axis.
- (S) is displaced horizontally in the positive direction.

• At the instant $t_0 = 0$, taken as an initial time,

- the abscissa of (S) is \mathbf{x}_0 and (S) is launched in the negative direction with an initial velocity $\overrightarrow{\mathbf{v}_0} = \mathbf{v}_0 \vec{\mathbf{i}}$ ($\underline{\mathbf{v}_0 < 0}$) where $\vec{\mathbf{i}}$ is the unit vector of the x-axis.
- At an instant **t**,
 - the abscissa of **G** is **x** and the algebraic value of its velocity is $\mathbf{v} = \mathbf{x'} = \frac{d\mathbf{x}}{d\mathbf{t}}$.

The horizontal plane containing **G** is taken as a reference level for gravitational potential energy.

Given :

KE = $\frac{1}{2}$ mv²; **GPE** = mgh; **EPE** = $\frac{1}{2}$ kx²; **ME** = **KE** + **EPE** + **GPE**

- Write, at an instant t, the expression of the mechanical energy of the system (Oscillator, Earth) in terms of x, m, k and v.
- 2) Knowing that (S) moves without friction, **deduce** that the second order differential equation in x that describes the oscillations of (S), is $x'' + \frac{k}{m}x = 0$.
- Deduce the expression of the proper angular frequency ω₀ of the oscillations in terms of m and k.
- 4) The solution of the obtained differential equation is: $\mathbf{x} = \mathbf{X}_{\mathbf{m}} \sin (\omega_0 \mathbf{t} + \mathbf{\phi})$, where $\mathbf{X}_{\mathbf{m}}$, ω_0 and $\mathbf{\phi}$ are constants.

Write the expression of v in terms of X_m , ω_0 , ϕ and t.

5) Write the expression of x_0 in terms of X_m and φ . Write the expression of v_0 in terms of X_m , ω_0 and φ .

6) Deduce that
$$X_m = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}$$
.

Given:

 $sin^2\phi + cos^2\phi = 1$

 An appropriate device traces x and v as functions of time as shown in <u>documents 2 and 3</u> respectively.



Referring to documents (2):

7-1) **specify** the type of the oscillations;

7-2) indicate the values of x_0 and X_m .

Referring to <u>document (3)</u>:

7-3) indicate the values of v_0 and V_m where V_m is the amplitude of v.

8) Using the relations of questions 6
$$(X_m = \sqrt{X_0^2 + \frac{v_0^2}{\omega_0^2}})$$
 :

Show that $\omega_0 = 20$ rad/s.

9) We repeat the same experiment by replacing block (S) of mass m by another block (S') of mass m' = 0.8 kg. The new proper angular frequency is $\omega' = \frac{\omega_0}{2}$

mass $\mathbf{m} = \mathbf{0.0} \ \mathbf{kg}$. The new proper angular frequency is $\boldsymbol{\omega} =$

9-1) Write the expression of ω' in terms of m' and k.

9-2) Deduce the values of **k** and **m**.

The aim of this exercise is to determine the capacitance C of a capacitor. We set-up the series circuit of document 4.



This circuit includes:

- an ideal battery of electromotive force **E** = 10 V;
- a rheostat of resistance **R**;
- a capacitor of capacitance **C**;
- an ammeter (A) of negligible resistance;
- a switch **K**.

Initially the capacitor is uncharged.

- We close the switch **K** at the instant $t_0 = 0$.
- At an instant t, plate B carries a charge q and the circuit carries a current i as shown in document 4.
- 1) Write the expression of **i** in terms of **C** and \mathbf{u}_{C} , where $\mathbf{u}_{C} = \mathbf{u}_{BD}$ is the voltage across the capacitor.
- 2) Prove that the differential equation that governs the variation of u_C is:

 $\mathbf{E} = \mathbf{u}_{\mathrm{C}} + \mathbf{R}\mathbf{C}\frac{\mathrm{d}\mathbf{u}_{\mathrm{C}}}{\mathrm{d}\mathbf{t}} \,.$

- 3) The solution of this differential equation is of the form: $\mathbf{u}_{\mathrm{C}} = \mathbf{a} + \mathbf{b}\mathbf{e}^{-\frac{\mathbf{t}}{\tau}}$. Determine the expressions of the constants \mathbf{a} , \mathbf{b} and τ in terms of \mathbf{E} , \mathbf{R} and \mathbf{C} .
- 4) Deduce that the expression of the current is: $\mathbf{i} = \frac{\mathbf{E}}{\mathbf{R}} \mathbf{e}^{-\frac{\mathbf{t}}{\mathbf{RC}}}$.
- 5) The ammeter (A) reads a value $I_0 = 5 \times 10^{-3}$ A at $t_0 = 0$. Deduce the value of R.
- 6) Write, <u>using Ohm's law</u>, the expression of $u_R = u_{DN}$ in terms of E, R, C and t.
- 7) At an instant $\mathbf{t} = \mathbf{t}_1$, the voltage across the capacitor is $\mathbf{u}_{\rm C} = \mathbf{u}_{\rm R}$.
 - 7-1) Show that $t_1 = R C \ln 2$.
 - 7-2) Calculate the value of C knowing that $t_1 = 1.4 \times 10^{-3}$ s.
- In order to <u>verify the value</u> of C, we change the value of R.
 <u>Document 5</u> represents τ as a function of R.



- 8-1) Show that the shape of the curve in document 5 is in agreement with the expression of τ obtained in question 3.
- 8-2) Using the curve of <u>document 5</u>, determine again the value of C.

Aspects of Light

The aim of this exercise is to show evidence of the two aspects of light.

1) First aspect

- Consider Young's double-slit experiment.
- The two thin parallel horizontal slits S_1 and S_2 are separated by a distance $a = 0.5 \times 10^{-3}$ m.
- The screen (E) is placed parallel to the plane of the slits at a distance D = 2 m.
- A laser source illuminates the two slits by a monochromatic light of wavelength $\lambda = 600 \text{ nm} (1 \text{nm} = 1 \times 10^{-9} \text{ m})$, in air, under normal incidence.
- **O** is the point of intersection between the perpendicular bisector of $[S_1S_2]$ and the screen (**E**).
- $-\mathbf{P}$ is a point on the screen having an abscissa $\mathbf{x}_{\mathbf{P}} = \overline{\mathbf{OP}} = 9.6 \times 10^{-3} \text{ m} (\underline{\text{Doc. 6}}).$



1-1) Prove that the value of the inter-fringe distance i is 2.4×10^{-3} m.

1-2) P is the center of a bright fringe, why?

1-3) Slits S₁ and S₂ are replaced by a horizontal slit S of width $a_1 = 0.1 \times 10^{-3}$ m.

O is the central bright fringe and $\alpha = 2 \theta_1$ where α is the angular width of the central bright fringe (θ_1 is a small angle) <u>Doc. 7</u>.



1-3-1) Choose the right answer:

The phenomenon that takes place at the slit **S** is:

- a- Interference
- **b-** Diffraction
- c- Refraction
- 1-3-2) Show, using document 7, that the width L of the central bright fringe is given by the expression: $L = \frac{2 \lambda D}{a_1}$.
- 1-3-3) Deduce the distance d between O and the center of the first dark fringe.
- **1-3-4**) Knowing that $\mathbf{x}_{\mathbf{P}} = 9.6 \times 10^{-3}$ m, deduce that P is <u>neither the center of a bright</u> <u>fringe</u>, <u>nor the center of a dark fringe</u>.
- 1-4) The previous two experiments show evidence of an aspect of light.Name this aspect.

2) Second aspect

The monochromatic radiation, of wavelength $\lambda = 600$ nm in air, emitted by the laser source, illuminates now the surface of a lithium metal of work function $W_0 = 2.39$ eV. Given:

- Planck's constant $\mathbf{h} = 6.6 \times 10^{-34} \text{ J.s}$;
- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J};$

Take: The speed of light in air $c = 3 \times 10^8 \text{ m/s}$.

2-1) **Complete** the following sentence:

2-2) Write, the expression of the energy of the photon in terms of h, c and λ . Prove that its value is $E_{ph} = 2.0625 \text{ eV}$.

$\label{eq:2-3} \textbf{Compare} \; E_{ph} \; \textbf{and} \; W_0.$

Deduce that there is no photoelectric emission from the surface of the lithium metal.

- 2-4) In order to extract electrons from the surface of the lithium metal, the laser source is replaced by another one emitting a radiation of wavelength $\lambda' = 500$ nm in air. Determine, using the relation $E_{ph} = W_0 + KE$, the maximum kinetic energy of the liberated electrons.
- 2-5) This experiment shows evidence of an aspect of light.Name this aspect.