

الاسم:
الرقم:

مسابقة في مادة الفيزياء
المدة: ساعتان

This exam is formed of three obligatory exercises in three pages.
The use of non-programmable calculators is recommended.

Exercise 1 (7 points)

Horizontal elastic pendulum

A mechanical oscillator is formed by a block (S) of mass m and a spring of negligible mass and spring constant k . (S) is attached to one end of the spring, and the other end of the spring is connected to a fixed support A. (S) can move without friction on a horizontal surface (Doc. 1).

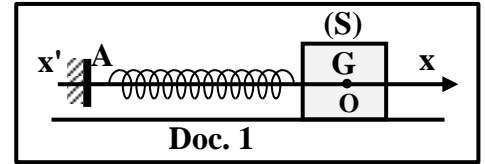
The aim of this exercise is to determine the values of m and k .

At equilibrium, the center of mass G of (S) coincides with the origin O of the x -axis.

(S) is displaced horizontally in the positive direction.

At the instant $t_0 = 0$, the abscissa of G is x_0 and (S) is launched in the

negative direction with an initial velocity $\vec{v}_0 = v_0 \vec{i}$ ($v_0 < 0$) where \vec{i} is the unit vector of the x -axis.



At an instant t , the abscissa of G is x and the algebraic value of its velocity is $v = x' = \frac{dx}{dt}$.

The horizontal plane containing G is taken as a reference level for gravitational potential energy.

- Write, at an instant t , the expression of the mechanical energy of the system (Oscillator, Earth) in terms of x , m , k and v .
- Establish the second order differential equation in x that governs the motion of (S).
- Deduce the expression of the proper angular frequency ω_0 of the oscillations in terms of m and k .
- The solution of the obtained differential equation is:

$$x = X_m \sin(\omega_0 t + \varphi), \text{ where } X_m, \omega_0 \text{ and } \varphi \text{ are constants.}$$

Write the expression of v in terms of X_m , ω_0 , φ and t .

- Write the expressions of x_0 and v_0 in terms of X_m , ω_0 and φ .

- Deduce that: $X_m = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}$.

- An appropriate device traces x and v as functions of time as shown in documents 2 and 3 respectively.

Referring to documents (2) and (3):

7-1) specify the type of the oscillations;

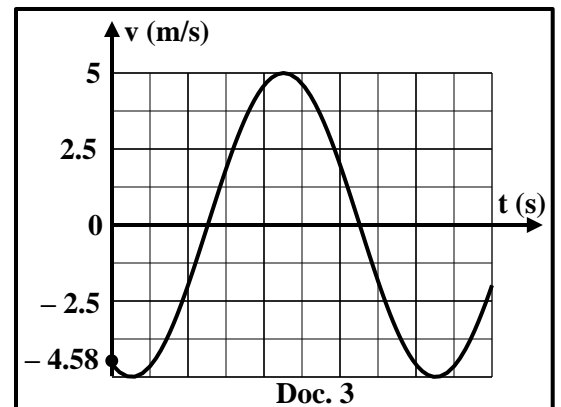
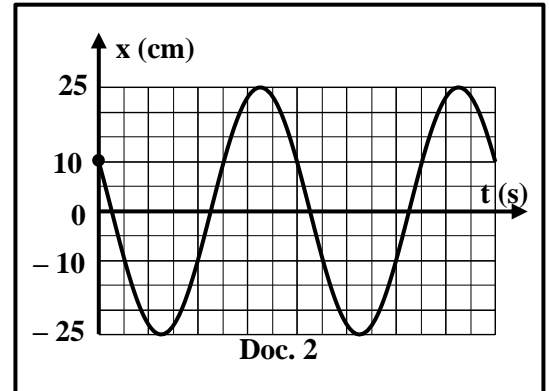
7-2) indicate the values of x_0 , v_0 , X_m and V_m , where V_m is the amplitude of v .

- Deduce that ω_0 is approximately equal to 20 rad/s.
- We repeat the same experiment by replacing the block (S) of mass m by another block (S') of mass $m' = 0.8$ kg.

The new proper angular frequency is $\omega' = \frac{\omega_0}{2}$.

9-1) Write the expression of ω' in terms of m' and k .

9-2) Deduce the values of k and m .



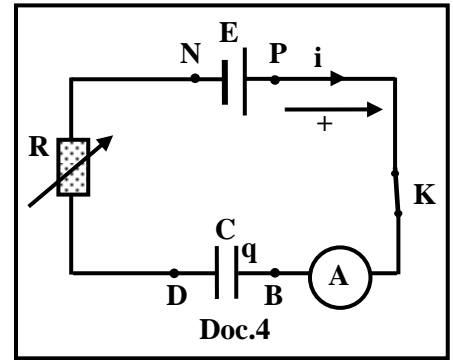
Exercise 2 (7 points) Capacitance of a capacitor

The aim of this exercise is to determine the capacitance C of a capacitor. We set-up the series circuit of document 4.

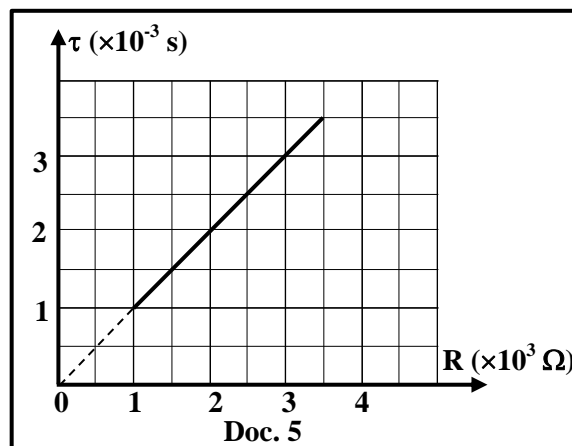
This circuit includes:

- an ideal battery of electromotive force $E = 10 \text{ V}$;
- a rheostat of resistance R ;
- a capacitor of capacitance C ;
- an ammeter (A) of negligible resistance;
- a switch K .

Initially the capacitor is uncharged. We close the switch K at the instant $t_0 = 0$. At an instant t , plate B of the capacitor carries a charge q and the circuit carries a current i as shown in document 4.



- 1) Write the expression of i in terms of C and u_C , where $u_C = u_{BD}$ is the voltage across the capacitor.
- 2) Establish the differential equation that governs the variation of u_C .
- 3) The solution of this differential equation is of the form: $u_C = a + b e^{-\frac{t}{\tau}}$. Determine the expressions of the constants a , b and τ in terms of E , R and C .
- 4) Deduce that the expression of the current is: $i = \frac{E}{R} e^{-\frac{t}{RC}}$.
- 5) The ammeter (A) indicates a value $I_0 = 5 \text{ mA}$ at $t_0 = 0$. Deduce the value of R .
- 6) Write the expression of $u_R = u_{DN}$ in terms of E , R , C and t .
- 7) At an instant $t = t_1$, the voltage across the capacitor is $u_C = u_R$.
 - 7-1) Show that $t_1 = RC \ln 2$.
 - 7-2) Calculate the value of C knowing that $t_1 = 1.4 \text{ ms}$.
- 8) In order to verify the value of C , we vary the value of R . Document 5 represents τ as a function of R .
 - 8-1) Show that the shape of the curve in document 5 is in agreement with the expression of τ obtained in part 3.
 - 8-2) Using the curve of document 5, determine again the value of C .



Exercise 3 (6 points)

Aspects of Light

The aim of this exercise is to show evidence of the two aspects of light.

1) First aspect

Consider Young's double-slit experiment. The two thin parallel horizontal slits S_1 and S_2 are separated by a distance $a = 0.5$ mm.

The screen (E) is placed parallel to the plane of the slits at a distance $D = 2$ m.

A laser source illuminates the two slits by a monochromatic light of wavelength $\lambda = 600$ nm in air, under normal incidence.

O is the point of intersection between the perpendicular bisector of $[S_1S_2]$ and the screen (E).

P is a point on the screen having an abscissa

$$x_P = \overline{OP} = 9.6 \text{ mm (Doc. 6).}$$

1-1) Calculate the inter-fringe distance i .

1-2) Specify the nature and the order of the fringe whose center is point P.

1-3) Slits S_1 and S_2 are replaced by a horizontal slit S of width $a_1 = 0.1$ mm. O is the center of the central bright fringe and $\alpha = 2\theta_1$ where α is the angular width of the central bright fringe (θ_1 is a small angle) (Doc. 7).

1-3-1) Name the phenomenon that takes place at the slit S.

1-3-2) Show that the width L of the central bright fringe is given by the

$$\text{expression: } L = \frac{2\lambda D}{a_1}.$$

1-3-3) Deduce the distance d between O and the center of the first dark fringe.

1-3-4) Deduce that P is neither the center of a bright fringe nor the center of a dark fringe.

1-4) The previous two experiments show evidence of an aspect of light. Name this aspect.

2) Second aspect

The monochromatic radiation of wavelength $\lambda = 600$ nm in air, emitted by the laser source, illuminates now the surface of a lithium metal of work function $W_0 = 2.39$ eV.

Given:

Planck's constant $h = 6.6 \times 10^{-34}$ J.s ; $1 \text{ eV} = 1.6 \times 10^{-19}$ J

Take: the speed of light in air $c = 3 \times 10^8$ m/s.

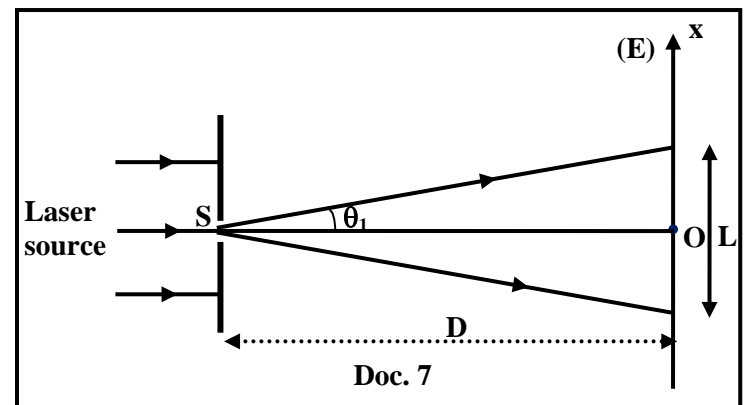
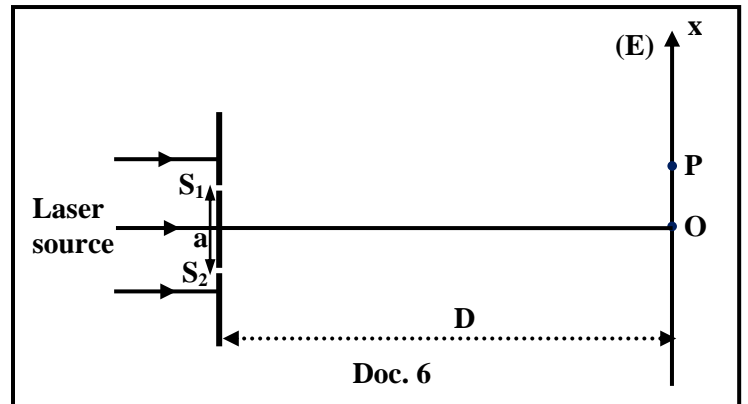
2-1) Define the work function (extraction energy) of a metal.

2-2) Calculate, in eV, the energy of a photon in this radiation.

2-3) Deduce that there is no photoelectric emission from the surface of the lithium metal.

2-4) In order to extract electrons from the surface of the lithium metal, the laser source is replaced by another one emitting a radiation of wavelength $\lambda' = 500$ nm in air. Determine, in eV, the maximum kinetic energy of the liberated electrons.

2-5) This experiment shows evidence of an aspect of light. Name this aspect.



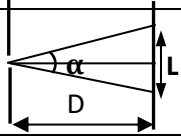
Exercise 1 (7 points) Horizontal elastic pendulum

| Partie | Answer | Mark |
|--------|---|------------------------|
| 1 | $ME = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ | 0.5 |
| 2 | There is no friction therefore the mechanical energy is conserved. ME = constant , then $\frac{dME}{dt} = 0$, hence $m v v' + k x x' = 0$ with $v = x'$ and $v' = x''$ $x' (mv + kx) = 0$, but $x' = 0$ is rejected ; therefore, $x'' + \frac{k}{m}x = 0$ | 1 |
| 3 | The differential equation is of the form: $x'' + \omega_0^2 x = 0$ then : $\omega_0 = \sqrt{\frac{k}{m}}$ | 0.5 |
| 4 | $v = X_m \omega_0 \cos(\omega_0 t + \varphi)$ | 0.5 |
| 5 | $x_0 = X_m \sin\varphi$ $v_0 = \omega_0 X_m \cos\varphi$ | 0.25 0.25 |
| 6 | $\sin\varphi = \frac{x_0}{X_m}$ and $\cos\varphi = \frac{v_0}{\omega_0 X_m}$ $\sin^2\varphi + \cos^2\varphi = 1$ $\frac{x_0^2}{X_m^2} + \frac{v_0^2}{\omega_0^2 X_m^2} = 1$, so $X_m^2 = x_0^2 + \frac{v_0^2}{\omega_0^2}$ Therefore, $X_m = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}$ | 1 |
| 7 | 7.1 Free undamped mechanical oscillations | 0.25 |
| | 7.2 $x_0 = 10 \text{ cm}$; $v_0 = -4.58 \text{ m/s}$ $X_m = 25 \text{ cm}$; $V_m = 5 \text{ m/s}$ | 0.25 0.25 0.25 0.25 |
| 8 | Substituting the values of x_0 , v_0 and X_m into the expression of X_m gives : $0.25 = \sqrt{0.1^2 + \frac{-4.58^2}{\omega_0^2}}$, then $\omega_0 = 19.98 \cong 20 \text{ rad/s}$ <u>Or :</u> $V_m = \omega_0 X_m$, then $\omega_0 = \frac{V_m}{X_m} = \frac{5}{0.25} = 20 \text{ rad/s}$ | 0.5 |
| 9 | 9.1 $\omega' = \sqrt{\frac{k}{m'}}$ | 0.25 |
| | 9.2 $\omega' = 10 \text{ rad/s}$ $k = m' \times \omega'^2 = 0.8 \times 10^2 = 80 \text{ N/m}$ $m = \frac{k}{\omega_0'^2} = \frac{80}{400} = 0.2 \text{ kg}$ | 0.5 0.5 |

Exercise 2 (7 points) Capacitance of a capacitor

| Part | Answer | notes |
|------|--|-------|
| 1 | $i = \frac{dq}{dt}$, but $q = C \times u_C$, then $i = C \frac{du_C}{dt}$ | 0.5 |
| 2 | $E = u_{AB} + u_{BN} = u_C + Ri$, but $i = C \frac{du_C}{dt}$; therefore, $E = u_C + R C \frac{du_C}{dt}$ | 0.75 |
| 3 | <p>$\frac{du_C}{dt} = -\frac{b}{\tau} e^{-\frac{t}{\tau}}$; Substituting u_C and $\frac{du_C}{dt}$ in the differential equation gives :</p> <p>$E = a + b e^{-\frac{t}{\tau}} + R C (-\frac{b}{\tau} e^{-\frac{t}{\tau}})$, so $E = a + b e^{-\frac{t}{\tau}} (1 - \frac{RC}{\tau})$</p> <p>By comparison we obtain :</p> <p>$a = E$ and $b e^{-\frac{t}{\tau}} (1 - \frac{RC}{\tau}) = 0$, but $b e^{-\frac{t}{\tau}} = 0$ is rejected ,then $1 - \frac{RC}{\tau} = 0$</p> <p>Therefore, $\tau = R C$</p> <p>At $t_0 = 0$, the charge is $q_0 = 0$, then $u_{C0} = 0$.</p> <p>Substituting $u_{C0} = 0$ into the expression of u_C gives: $0 = a + b$, so $b = -a = -E$</p> | 2 |
| 4 | $i = C \frac{du_C}{dt} = C \frac{E}{\tau} e^{-\frac{t}{\tau}} = \frac{E}{R} e^{-\frac{t}{\tau}}$ | 0.5 |
| 5 | A $t_0 = 0 : i = I_0 = \frac{E}{R} e^0$, then $I_0 = \frac{E}{R}$, thus $R = \frac{E}{I_0} = \frac{10}{5 \times 10^{-3}} = 2 \times 10^3 \Omega$ | 0.5 |
| 6 | $u_R = Ri = R C \frac{du_C}{dt} = RC \frac{E}{\tau} e^{-\frac{t}{\tau}}$, then $u_R = E e^{-\frac{t}{\tau}}$ | 0.5 |
| 7 | <p>7.1 $u_C = u_R$</p> <p>$E - E e^{-\frac{t_1}{\tau}} = E e^{-\frac{t_1}{\tau}}$, so $E = 2 E e^{-\frac{t_1}{\tau}}$, then $\frac{1}{2} = e^{-\frac{t_1}{\tau}}$, hence $-\ln 2 = -\frac{t_1}{\tau}$</p> <p>Then, $t_1 = \tau \ln 2$; therefore, $t_1 = RC \ln 2$</p> | 0.75 |
| | <p>7.2 $C = \frac{t_1}{R \ln 2} = \frac{1.4 \times 10^{-3}}{2 \times 10^3 \times \ln 2} = 1 \times 10^{-6} F$</p> | 0.5 |
| 8 | <p>8.1 The curve is a straight line passing through the origin with a positive slope, then it is in agreement with the expression $\tau = RC$.</p> | 0.5 |
| | <p>8.2 Slope = $C = \frac{\Delta \tau}{\Delta R} = \frac{3 \times 10^{-3}}{3 \times 10^3} = 1 \times 10^{-6} F$</p> | 0.5 |

Exercise 3 (6 points) Aspects of Light

| Part | | Answer | Mark | |
|-------|--|---|---|------|
| 1 | 1.1 | $i = \frac{\lambda D}{a} = \frac{600 \times 10^{-9} \times 2}{0.5 \times 10^{-3}} = 24 \times 10^{-4} \text{ m} = 2.4 \text{ mm}$ | 0.5 | |
| | 1.2 | $x_P = 9.6 \text{ mm} = 4 i$, then P is the center of the 4 th bright fringe. <u>Or:</u> P is the center of a bright fringe if $x_P = \frac{k \lambda D}{a}$ with $k \in \mathbb{Z}$. $x_P = \frac{k \lambda D}{a}$, then $k = \frac{a x_P}{\lambda D} = \frac{0.5 \times 10^{-3} \times 9.6 \times 10^{-3}}{600 \times 10^{-9} \times 2} = 4 \in \mathbb{Z}$, then P is the center of the 4 th bright fringe. | 1 | |
| | 1.3.1 | Diffraction of light | 0.25 | |
| | 1.3.2 | From the figure: $\tan \frac{\alpha}{2} = \frac{L/2}{D}$, but α is small then $\tan \alpha \cong \alpha$ So $\frac{\alpha}{2} = \frac{L}{2D}$ But $\alpha = \frac{2\lambda}{a_1}$; therefore, $L = \frac{2\lambda D}{a_1}$ |  | 0.75 |
| | 1.3.3 | $d = \frac{L}{2} = \frac{2 \times 600 \times 10^{-9} \times 2}{2 \times 0.1 \times 10^{-3}} = 0.012 \text{ m} = 12 \text{ mm}$ | 0.5 | |
| 1.3.4 | $x_P < d = \frac{L}{2}$, then it is neither the center of a bright nor the center of a dark fringe. | 0.25 | | |
| 1.4 | Wave aspect of light | 0.25 | | |
| 2 | 2.1 | W_0 is the minimum energy needed to extract an electron from the surface of a metal. | 0.5 | |
| | 2.2 | $E_{ph} = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}} = 3.3 \times 10^{-19} \text{ J}$ $E_{ph} = \frac{3.3 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.0625 \text{ eV}$ | 0.75 | |
| | 2.3 | $E_{ph} < W_0$, then there is no photoelectric emission. | 0.25 | |
| | 2.4 | $E'_{ph} = W_0 + KE_{max}$, then $KE_{max} = E'_{ph} - W_0 = \frac{hc}{\lambda'} - W_0$ $KE_{max} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9} \times 1.6 \times 10^{-19}} - 2.39 = 0.085 \text{ eV}$ | 0.75 | |
| | 2.5 | Corpuscular (particle) aspect of time | 0.25 | |