

الاسم:  
الرقم:

مسابقة في مادة الرياضيات  
المدّة: ساعة

عدد المسائل: ثلاث

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.  
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

### I- (5 points)

1) Solve the following system: 
$$\begin{cases} 3x + 2y = 6500 \\ 2x + 3y = 6000 \end{cases}$$

2) Nabil and Sami buy croissants from the same bakery.

Nabil paid 65 000 LL for 30 chocolate croissants and 20 cheese croissants.

Sami paid 60 000 LL for 20 chocolate croissants and 30 cheese croissants.

a- Calculate the price of one chocolate croissant and the price of one cheese croissant.

b- The bakery offers a 20% discount on the price of the chocolate croissants only.  
Nabil has 100 000 LL.

Can he buy 50 chocolate croissants and 45 cheese croissants? Justify.

### II- (5 points)

A survey is done on a population of 70 men and 30 women about their preferred mean of transportation.

The person should select only one mean of transportation among: car, bike or bus.

The results are shown in the following table:

	Car	Bike	Bus	Total
Men	35	10		70
Women			10	
Total		15		100

1) Copy and complete the table above.

2) A person is selected randomly from this population.

Consider the following events:

C: "The selected person prefers the car"

B: "The selected person prefers the bike"

M: "The selected person is a man".

a- Determine the following probabilities:  $P(M)$ ,  $P(C \cap M)$  and  $P(\bar{C} \cap \bar{B})$ .

b- Verify that  $P(B \cup M) = \frac{3}{4}$ .

c- The selected person prefers the car. Calculate the probability that this person is a woman.

**III- (10 points)**

Consider the function  $f$  defined over  $]-\infty, -1[ \cup ]-1, +\infty[$  as  $f(x) = \frac{2x^2 + 5x + 2}{x + 1}$ .

Denote by (C) the representative curve of  $f$  in an orthonormal system  $(O; \vec{i}, \vec{j})$ .

- 1) Determine  $\lim_{\substack{x \rightarrow -1 \\ x < -1}} f(x)$  and  $\lim_{\substack{x \rightarrow -1 \\ x > -1}} f(x)$ . Deduce an equation of an asymptote (d) to (C).
- 2) a- Determine  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow +\infty} f(x)$ .  
b- Verify that  $f(x) = 2x + 3 - \frac{1}{x + 1}$ .  
c- Show that the line (D) with equation  $y = 2x + 3$  is an asymptote to (C).
- 3) Calculate  $f'(x)$  and verify that  $f'(x) > 0$ .
- 4) Set up the table of variations of  $f$ .
- 5) a- Copy and complete the following table:

x	-3	-1.5	0	1
f(x)				

- b- Show that the curve (C) intersects the x-axis in two points whose abscissas are to be determined.
- c- Draw (d), (D) and (C).
- d- Solve graphically:  $f(x) < 2$ .

QI	Answer key	° pts
1	$x = 1500$ LL and $y = 1000$ LL	1
2a	Let $x$ be the price of one chocolate croissant Let $y$ be the price of one cheese croissant $\begin{cases} 30x + 20y = 65000 \\ 20x + 30y = 60000 \end{cases}$ The price of 1 chocolate croissant is 1 500 LL The price of 1 cheese croissant is 1 000 LL	2
2b	$0.8 \times 1500 = 1200$ No, because $50 \times 1200 + 45 \times 1000 = 105000 > 100000$	2

QII	Answer key	5 pts																				
1	<table border="1"> <thead> <tr> <th></th> <th>Car</th> <th>Bike</th> <th>Bus</th> <th>Total</th> </tr> </thead> <tbody> <tr> <th>Men</th> <td>35</td> <td>10</td> <td><b>25</b></td> <td>70</td> </tr> <tr> <th>Women</th> <td><b>15</b></td> <td><b>5</b></td> <td>10</td> <td><b>30</b></td> </tr> <tr> <th>Total</th> <td><b>50</b></td> <td>15</td> <td><b>35</b></td> <td>100</td> </tr> </tbody> </table>		Car	Bike	Bus	Total	Men	35	10	<b>25</b>	70	Women	<b>15</b>	<b>5</b>	10	<b>30</b>	Total	<b>50</b>	15	<b>35</b>	100	1 ½
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2a	$P(M) = 0.7$ ; $P(C \cap M) = 0.35$ ; $P(\bar{C} \cap \bar{B}) = 0.35$	1 ½																				
2b	$P(B \cup M) = \frac{70+5}{100} = \frac{3}{4}$	1																				
2c	$P(\bar{M}/C) = \frac{15}{50} = 0.3$	1																				

QIII	Answer key	5 pts
1	$\lim_{\substack{x \rightarrow -1 \\ x < -1}} f(x) = +\infty$ ; $\lim_{\substack{x \rightarrow -1 \\ x > -1}} f(x) = -\infty$ (d) : $x = -1$	1 ½
2a	$\lim_{x \rightarrow -\infty} f(x) = -\infty$ ; $\lim_{x \rightarrow +\infty} f(x) = +\infty$	1
2b	$2x + 3 - \frac{1}{x+1} = \frac{(2x+3)(x+1)-1}{x+1} = \frac{2x^2 + 5x + 2}{x+1}$	1
2c	$\lim_{x \rightarrow -\infty} [f(x) - y_d] = 0$	1
3	$f'(x) = 2 + \frac{1}{(x+1)^2} > 0$	1

4)	<table style="border-collapse: collapse; margin: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>-\infty</math></td> <td style="padding: 5px;"><math>-1</math></td> <td style="padding: 5px;"><math>+\infty</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>f'</math></td> <td style="padding: 5px; text-align: center;">+</td> <td style="background-color: #cccccc; padding: 5px;"></td> <td style="padding: 5px; text-align: center;">+</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>f</math></td> <td style="padding: 5px; text-align: center;"><math>-\infty</math></td> <td style="background-color: #cccccc; padding: 5px;"></td> <td style="padding: 5px; text-align: center;"><math>+\infty</math></td> </tr> </table> <p style="text-align: center; margin-top: 10px;"> </p>	$x$	$-\infty$	$-1$	$+\infty$	$f'$	+		+	$f$	$-\infty$		$+\infty$	$1\frac{1}{2}$
$x$	$-\infty$	$-1$	$+\infty$											
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$x$	$-3$	$-1,5$	$0$	$1$										
$f(x)$	$-2,5$	$2$	$2$	$4,5$										
5b	$f(x)=0 ; x= -2 ; x=-0,5$	$\frac{1}{2}$												
5c		$1\frac{1}{2}$												
5d	$] -\infty ; -1,5[ \cup ] 0 ; +\infty [$	$\frac{1}{2}$												