عدد المسائل: أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

The table below represents the number of passengers (in millions) in an airport, for each year from 2014 to 2018.

Year	2014	2015	2016	2017	2018
Rank of the year: x_i	4	5	6	7	8
Number of passengers (in millions): y_i	15	18	22	24	25

1) Represent the scatter plot of points (x_i , y_i) in a rectangular system.
2) Calculate the coordinates of the center of gravity $G(\bar{x} , \bar{y})$ and plot G in the preceding system.
3) Calculate the correlation coefficient r and give an interpretation of the value thus obtained.
4) Determine an equation of the regression line $(D_{y/x})$, of y in terms x, and draw $(D_{y/x})$ in the preceding system.
5) Assume that the above model remains valid till the year 2023.
   a- Estimate the number of passengers in 2021.
   b- Suppose that the percentage increase in the number of passengers from the year 2018 to a year to be determined is 45.6 %. Determine this year.

II- (4 points)

A cafeteria sells dessert and coffee only.
A customer can buy one dessert, one cup of coffee, both or none.
In this cafeteria:

- 70 % of the customers buy dessert, among which 40 % buy coffee,
- among the customers who do not buy dessert, 35 % do not buy coffee.

One customer of this cafeteria is randomly selected and interviewed.
Consider the following events:
D: "The interviewed customer buys a dessert",
C: "The interviewed customer buys a cup of coffee".

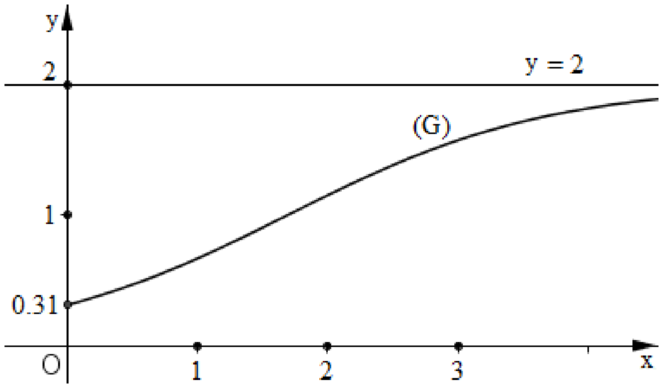
1) a- Calculate the probabilities $P(C \cap D)$ and $P(C \cap \bar{D})$.
   b- Deduce that $P(C) = 0.475$.
2) A customer does not buy a cup of coffee. Calculate the probability that this customer does not buy a dessert.
3) In this cafeteria, the price of a dessert is 7 000 LL and the price of a cup of coffee is 3 000 LL. Denote by X the random variable equal to the sum paid by a customer.
   a- Justify that $P(X = 0) = 0.105$.
   b- Determine the probability distribution of X.
   c- During a certain day, 500 customers entered the cafeteria. Estimate the total revenue during that day.

III- (4 points)
A company produces a certain type of objects.
At the end of January 2018, the monthly total cost was 9 million LL.
At the end of each month, the monthly total cost increases by 21% with additional expenses of 840 000 LL, and so on for the following months.
For all natural numbers n > 0, denote by C_1 the monthly total cost, in millions LL, at the end of January 2018 and by C_n the monthly total cost, in millions LL, at the end of the nth month.
Thus $C_1 = 9$ and $C_{n+1} = 1.21C_n + 0.84$.

1) Calculate C_3. Interpret the result obtained.
2) Consider the sequence (V_n) defined by $V_n = C_n + 4$.
   a- Show that (V_n) is a geometric sequence whose common ratio r and first term V_1 are to be determined.
   b- Show that $C_n = 13\times(1.21)^{n-1} - 4$.
   c- Show that the sequence (C_n) is strictly increasing.
   d- After how many months will the monthly total cost exceed 300 million LL for the first time? Justify.
3) The monthly revenue in millions LL, at the end of the nth month, for this company is modeled as $R_n = 100\times(1.07)^{n-1}$ for all natural numbers n > 0.
   Does the company achieve profit at the end of June 2019? Justify.

IV- (8 points)
Consider the function f defined over $[0, +\infty[$ as $f(x) = \dfrac{2}{2+e^{x-1}}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

Part A
1) a- Calculate f(0) and f(2) to the nearest 10^{-3}.
   b- Determine $\lim\limits_{x\to+\infty} f(x)$. Deduce an asymptote to (C).
2) a- Show that f is strictly decreasing.
   b- Set up the table of variations of f.
3) Consider the function g defined over $[0, +\infty[$ as
   $g(x) = \dfrac{2e^{x-1}}{2+e^{x-1}}$.
   The representative curve (G) of the function g and its asymptote are given in the adjacent figure.
   a- Solve the equation f(x) = g(x).
   b- Copy (G) and draw (C) in the same system.



Part B
A factory produces a certain type of objects.
The demand function and the supply function are respectively modeled as
$f(x) = \dfrac{2}{2+e^{x-1}}$ and $g(x) = \dfrac{2e^{x-1}}{2+e^{x-1}}$; where x is the unit price expressed in millions LL.
f(x) and g(x) are expressed in thousands of objects with $x \in]0, 5]$.

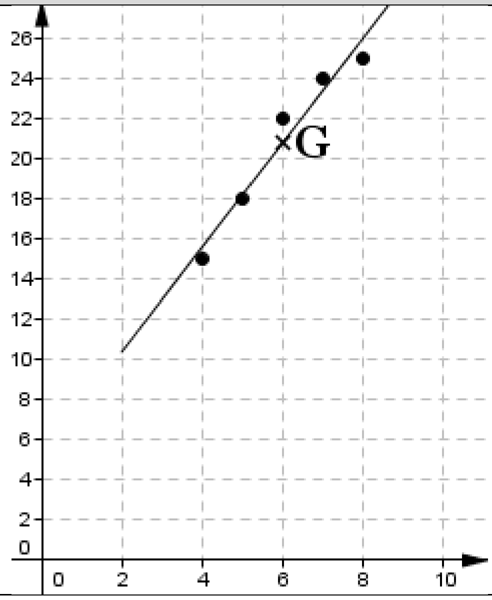
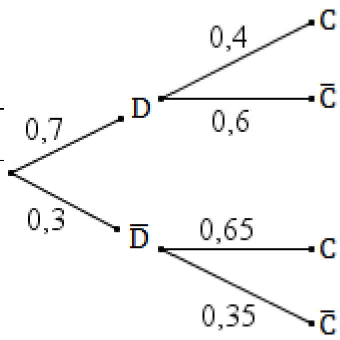
1) Calculate the demanded number of objects for a unit price of 2 million LL.
2) Calculate the unit price for a supply of 1000 objects.
3) Determine the equilibrium price.
4) Denote by E(x) the elasticity of the demand with respect to the unit price x.
   a- Show that $E(x) = \dfrac{xe^{x-1}}{2+e^{x-1}}$ and calculate E(2).
   b- If the unit price of 2 million LL increases by 1 %, then calculate the number of demanded objects.

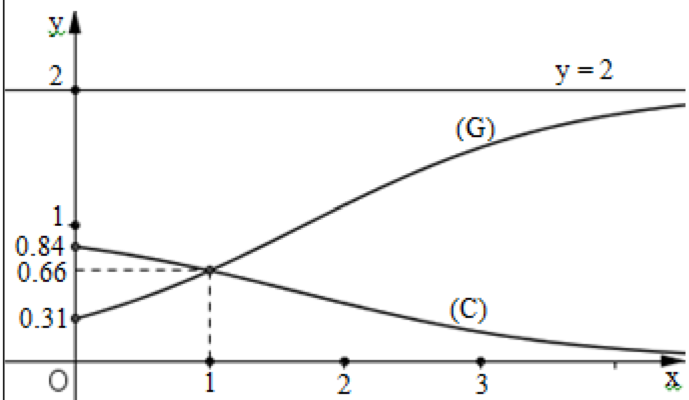
وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة الثانوية العامة
فرع: الاجتماع والاقتصاد

دورة العام ٢٠١٩ العاديّة
الاثنين ٢٤ حزيران ٢٠١٩

أسس تصحيح مسابقة الرياضيات

Q.I	Answers	4pts
1		1
2	$\overline{x} = 6$ and $\overline{y} = 20.8$ then G(6 , 20.8). Figure	1
3	r = 0.977. Strong positive correlation	1
4	y = bx+a = 2.6x + 5.2 Figure	1.5
5a	x = 11, y = 2.6(11) + 5.2 = 33.8 in millions of passengers. Then 33 800 000 passengers	1
5b	$\frac{y-25}{25} = 0.456$; y = 25(0.456) + 25 = 36.4. 2.6x + 5.2 = 36.4 gives x = 12. In the year 2022 **OR** In 2018, y = 25 so y = $25\left(\frac{45.6}{100}\right)$ + 25 = 36.4 then 2.6x + 5.2 = 36.4 gives x = 12. In the year 2022	1.5
Q.II	**Answers**	**4pts**
1a	$P(C \cap D) = P(D).P\left(C/D\right)$ = 0.4 × 0.7 = 0.28 $P\left(C \cap \overline{D}\right) = P(\overline{D}).P\left(C/\overline{D}\right)$ = 0.3 × 0.65 = 0.195 	1
1b	$P(C) = P(C \cap D) + P\left(C \cap \overline{D}\right) = 0.475$	1
2	$P\left(\overline{D}/\overline{C}\right) = \frac{P\left(\overline{C} \cap \overline{D}\right)}{P\left(\overline{C}\right)} = \frac{P\left(\overline{D}\right) - P\left(\overline{D} \cap C\right)}{1 - 0.475} = \frac{0.3 - 0.195}{0.525} = 0.2$ **OR** $P\left(\overline{D}/\overline{C}\right) = \frac{P\left(\overline{C} \cap \overline{D}\right)}{P\left(\overline{C}\right)} = \frac{P\left(\overline{D}\right) \times P\left(\overline{C}/\overline{D}\right)}{1 - P(C)} = \frac{0.3 \times 0.35}{0.525} = 0.2$	1.5
3a	$P(X = 0) = P\left(\overline{C} \cap \overline{D}\right) = 0.3 \times 0.35 = 0.105$	0.5
3b	The possible values of X are: 0, 3000, 7000, 10 000 P(X = 0) = 0.105 $P(X = 3000) = P\left(C \cap \overline{D}\right) = 0.195$ $P(X = 7000) = P\left(D \cap \overline{C}\right) = 0.7 \times 0.6 = 0.42$ $P(X = 10\,000) = P(C \cap D) = 0.28$	2
3c	E(X) = 6325 LL ; 500 × 6325 = 3 162 500 LL Then the total revenue is about 3 162 500 LL	1

<table>
<tr><th>Q.III</th><th>Answers</th><th>4pts</th></tr>
<tr><td>1</td><td>$C_2 = C_1(1.21) + 0.84 = 11.73$
$C_3 = C_2(1.21) + 0.84 = 15.0333$ then the cost at the end of March is 15 033 300 LL</td><td>1</td></tr>
<tr><td>2a</td><td>$V_{n+1} = C_{n+1} + 4 = 1.21(C_n + 4) = 1.21V_n$ then (V_n) is a geometric sequence whose common ratio $r = 1.21$ and the first term $V_1 = C_1 + 4 = 13$</td><td>1</td></tr>
<tr><td>2b</td><td>$V_n = V_1 \times q^{n-1} = 13\times(1.21)^{n-1}$ then $C_n = V_n - 4 = 13\times(1.21)^{n-1} - 4$</td><td>1</td></tr>
<tr><td>2c</td><td>$C_{n+1} - C_n = ... = 13\times(1.21)^{n-1}(1.21-1) = 2.73\times(1.21)^{n-1} > 0$
Then (C_n) is strictly increasing</td><td>1</td></tr>
<tr><td>2d</td><td>$C_n > 300$ then $13\times(1.21)^{n-1} - 4 > 300$; $n - 1 > \frac{\ln\left(\frac{304}{13}\right)}{\ln(1.21)}$; $n > 17.53$ so n = 18, then 18 months.</td><td>1</td></tr>
<tr><td>3</td><td>$R_n = 100(1.07)^{n-1}$ then $R_{18} = 100(1.07)^{17} = 315.88$
$C_{18} = 13(1.21)^{17} - 4 = 328.119$
Profit = $R_{18} - C_{18} = -12.239 < 0$ then the company does not achieve a profit.</td><td>2</td></tr>
<tr><th>Q.IV</th><th>Answers</th><th>8pts</th></tr>
<tr><td>A1a</td><td>f(0) = 0.845 ; f(2) = 0.423</td><td>1</td></tr>
<tr><td>A1b</td><td>$\lim\limits_{x\to+\infty} f(x) = 0$ then y = 0 HA</td><td>1</td></tr>
<tr><td>A2a</td><td>$f'(x) = \frac{-2e^{x-1}}{(e^{x-1}+2)^2} < 0$ then f is strictly decreasing.</td><td>1</td></tr>
<tr><td>A2b</td><td><table><tr><td>x</td><td>0 $+\infty$</td></tr><tr><td>f '(x)</td><td>–</td></tr><tr><td>f(x)</td><td>0.845 ↘ 0</td></tr></table></td><td>1.5</td></tr>
<tr><td>A3a</td><td>f(x) = g(x) then $2 = 2e^{x-1}$ so x = 1.</td><td>1</td></tr>
<tr><td>A3b</td><td>
</td><td>2</td></tr>
<tr><td>B1</td><td>f(2) = 0,423 in thousands of objects then 423 objects.</td><td>1</td></tr>
<tr><td>B2</td><td>g(x) = 1 then $2e^{x-1} = e^{x-1} + 2$ so $x = 1 + \ln 2 \cong 1.693$ in million LL.</td><td>1.5</td></tr>
<tr><td>B3</td><td>f(x) = g(x) then x = 1 so 1 000 000 LL.</td><td>1</td></tr>
<tr><td>B4a</td><td>$e(x) = x\,\frac{f'(x)}{f(x)} = \frac{-xe^{x-1}}{e^{x-1}+2}$
e(2) = –1.15</td><td>1.5</td></tr>
<tr><td>B4b</td><td>f(2) – (0.0115) × f(2) = 0.423 – (0.0115) × 0.423 ≅ 0.418 then 418 objects
OR f(2.02) ≅ 0.419 then 419 objects</td><td>1.5</td></tr>
<tr><td></td><td></td><td></td></tr>
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