وزارة التربية والتعليم العالي المديريّــة العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	دورة الـعام 2019 العاديّة الثلاثاء 18 حـزيران 2019

عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

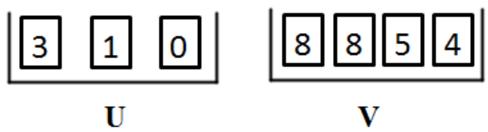
ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O; \vec{i}, \vec{j}, \vec{k}\right)$, given the points A(4 , 1 , 4), B(1 , 0 , 1), E(3 , −1 , 1) and the plane (P) with equation x + 2y + 3z – 4 = 0.

1) Show that the point E is the orthogonal projection of point A on plane (P).
2) a- Determine an equation of the plane (Q) determined by A, B and E.
   b- Verify that the two planes (P) and (Q) are perpendicular.
3) Let (d) be the line of intersection of (P) and (Q).
   Show that a system of parametric equations of (d) is $\begin{cases} x = -2t + 1 \\ y = t \\ z = 1 \end{cases}$ $(t \in \mathbb{R})$.
4) Consider, in plane (P), the circle (C) with center E and radius $\sqrt{5}$.
   Show that the line (d) intersects the circle (C) in two points whose coordinates are to be determined.

II- (4 points)



U and V are two urns such that:

- U contains three cards holding the numbers 3, 1 and 0;
- V contains four cards holding the numbers 8, 8, 5 and 4.

One card is selected randomly from urn U:

- If the selected card from U holds the number 0, then two cards are selected randomly and simultaneously from urn V;
- If the selected card from U does not hold the number 0, then three cards are selected randomly and simultaneously from urn V.

Consider the following events:

A: "The selected card from urn U holds the number 0"
S: "The sum of the numbers held on the selected cards from urn V is even"

1) a- Calculate the probabilities $P(S / A)$ and $P(S \cap A)$.
   b- Verify that $P(S \cap \overline{A}) = \dfrac{1}{6}$ and calculate P(S).
2) The sum of the numbers held on the selected cards from urn V is even. Calculate the probability that the selected card from urn U does not hold the number 0.
3) Let X be the random variable equal to the product of numbers held by the cards selected from the two urns U and V.
   Calculate P(X = 0) and deduce $P(X \leq 160)$.

III- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points M and M' with respective affixes z and z' such that z' = (1 + i) $\bar{z}$.

1) In this part, let z = $e^{i\frac{\pi}{3}}$.
   a- Write z' in exponential form.
   b- Verify that $(z')^6$ is pure imaginary.
2) a- Show that $|z'|=\sqrt{2}|z|$.
   b- Deduce that, when M moves on the circle with center O and radius $\sqrt{2}$, M' moves on a circle whose center and radius are to be determined.
3) Let z = x + iy and z' = x' + iy', where x , y , x' and y' are real numbers.
   a- Express x' and y' in terms of x and y.
   b- For all z ≠ 0, denote by N the point with affix $\bar{z}$.
   Prove that the triangle ONM' is right isosceles with principal vertex N.

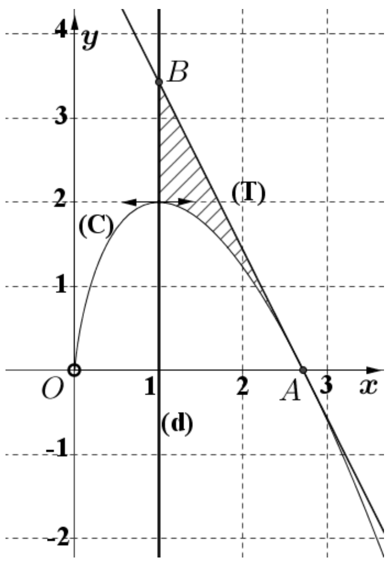
IV- (8 points)

Consider the function f defined over $]0,+\infty[$ as $f(x)=2x(1-\ln x)$. Denote by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) Determine $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.
2) a- Let A be the point of intersection of (C) with the x-axis.
   Determine the coordinates of A.
   b- Show that $f'(x)=-2\ln x$ and set up the table of variations of f.
   c- Determine an equation of the tangent (T) to (C) at A.

In the adjacent figure:

- (C) is the representative curve of f
- (T) is the tangent to (C) at A
- (d) is the line with equation $x=1$
- B(1 , 2e − 2) is the point of intersection of (d) and (T).

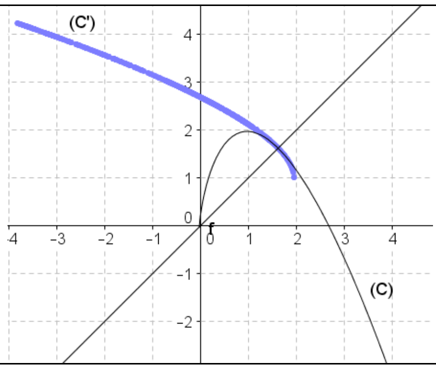


3) a- Show that f has, over $]1,+\infty[$, an inverse function g whose domain of definition is to be determined.
   b- Set up the table of variation of g.
   c- Copy (C), then draw (C'), the representative curve of g in the same system.
4) a- Using integration by parts, determine $\int x\ln(x)\,dx$.
   b- Show that $\int_1^e f(x)dx=\frac{e^2-3}{2}$.
   c- Calculate the area of the shaded region bounded by (C), (T) and (d).

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	دورة العام ٢٠١٩ العاديّة الثلاثاء ١٨ حزيران ٢٠١٩

أسس تصحيح مسابقة الرياضيات

Q.I	Answer key	4 pts
1	$x_E + 2(y_E) + 3(z_E) - 4 = 0$, $3 - 2 + 3 - 4 = 0$ then $E \in (P)$. $\overrightarrow{EA}(1,2,3) = \overrightarrow{n_P}$ then E is the orthogonal projection of point A on plane (P). **OR:** $(AE): \begin{cases} x = n+4 \\ y = 2n+1 \\ z = 3n+4 \end{cases}$; $E(n+4 ; 2n+1; 3n+4)$; $x_E + 2(y_E) + 3(z_E) - 4 = 0$ then $n = -1$ so, $E(3; -1; 1)$	1
2.a	Let $M(x, y, z) \in (Q)$ $\overrightarrow{AM}.(\overrightarrow{AB} \wedge \overrightarrow{AE}) = 0$ $\begin{vmatrix} x-4 & y-1 & z-4 \\ -3 & -1 & -3 \\ -1 & -2 & -3 \end{vmatrix} = 0$ Then (Q): $3x + 6y - 5z + 2 = 0$.	1
2.b	$\overrightarrow{n_Q}.\overrightarrow{n_P} = 3 + 12 - 15 = 0$. Then the two planes (P) and (Q) are perpendicular.	0.5
3	For every $M(-2t + 1; t ; 1) \in (d)$ $x_M + 2(y_M) + 3(z_M) - 4 = 0$ then $M \in (P)$ $3x_M + 6y_M - 5z_M + 2 = 0$ then $M \in$ **(Q)**	0.5
4	$M(-2t + 1; t ; 1)$ $\overrightarrow{EM}(-2t - 2; t + 1; 0)$ $EM = \sqrt{5}$; $(-2t - 2)^2 + (t + 1)^2 = 5$ then $t = 0$ or $t = -2$ Therefore B(1 ; 0 ; 1) (5 ; –2 ; 1).	1
Q.II	**Answer key**	**4 pts**
1.a	$P(S/A) = \frac{C_3^2}{C_4^2} = \frac{1}{2}$, $P(S \cap A) = P(S/A) \times P(A) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$	1
1.b	$P(S \cap \overline{A}) = P(S/\overline{A}) \times P(\overline{A}) = \frac{C_3^3}{C_4^3} \times \frac{2}{3} = \frac{1}{6}$ $P(S) = P(S \cap A) + P(S \cap \overline{A}) = \frac{1}{3}$	1
2	$P(\overline{A}/S) = \frac{P(S \cap \overline{A})}{P(S)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$	1
3	$P(X = 0) = \frac{1}{3} \times 1 = \frac{1}{3}$ $P(X \leq 160) = P(X = 0) + P(X = 160)$ $= \frac{1}{3} + \frac{1}{3} \times \frac{C_2^1 C_1^1 C_1^1}{C_4^3}$ $= \frac{5}{12}$	1
Q.III	**Answer key**	**4 pts**
1.a	$z' = \sqrt{2}e^{i\frac{\pi}{4}}e^{i\frac{-\pi}{3}} = \sqrt{2}e^{i\frac{-\pi}{12}}$	0.5
1.b	$(z')^6 = (\sqrt{2}e^{i\frac{-\pi}{12}})^6 = 8e^{i\frac{-\pi}{2}} = -8i$ is pure imaginary Or: Ou $\arg(z'^6) = 6\arg(z')\ [2\pi] = 6 \times (\frac{-\pi}{12})\ [2\pi] = -\frac{\pi}{2}\ [2\pi]$, donc (z'^6) est imaginaire pur .	0.5

2.a	$\lvert z'\rvert = \lvert 1+i\rvert\lvert\bar{z}\rvert$; $\lvert z'\rvert=\sqrt{2}\lvert z\rvert$	**0.5**
2.b	$OM=\sqrt{2}$; $\lvert z'\rvert=\sqrt{2}\lvert z\rvert$; $OM'=\sqrt{2}OM=2$ Then M' moves on a circle of center O and radius 2.	**1**
3.a	x' +iy' = (1 + i)(x – iy) = x + y + i(x – y) then x' = x + y and y' = x – y.	**0.5**
3.b	$N(\bar{z})$ then N(x; –y) ; M' (z') then M' $(x+y; x-y)$ $\overrightarrow{ON}(x; -y)$; $\overrightarrow{NM'}(y; x)$ ON = N M' = $\sqrt{x^2+y^2}$ and $\overrightarrow{ON}.\overrightarrow{NM'}$ = xy – yx = 0. Then ONM' is right isosceles of vertex N. **OR:** $\frac{z'}{\bar{z}}=1+i=\sqrt{2}e^{i\frac{\pi}{4}}$, then $OM'=\sqrt{2}ON$ $\left(\overrightarrow{ON};\overrightarrow{OM'}\right)=\frac{\pi}{4}[2\pi]$ Then ONM' is right isosceles of vertex N. **OR** $\frac{z'-\bar{z}}{\bar{z}}=i$ **……** then ONM' is right isosceles of vertex N.	**1**
Q.IV	**Answer key**	**8 pts**
1	$\lim\limits_{\substack{x\to0\\x>0}} f(x)=\lim\limits_{\substack{x\to0\\x>0}} 2x-2x\ln x=0$ and $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty} 2x(1-\ln x)=-\infty$	**1**
2.a	2x(1 – lnx) = 0 ; x = 0 rej 1 – lnx = 0 ; lnx = 1 then x = e hence A(e; 0)	**0.5**
2.b	f '(x) = 2(1 – lnx) + (2x)(-1/x) = –2lnx $\begin{array}{c\|ccccc} x & 0 & & 1 & & +\infty \\ \hline f'(x) & & + & 0 & - & \\ \hline f(x) & 0 & \nearrow & 2 & \searrow & -\infty \end{array}$	**1**
2.c	f '(e) = –2 (T): y = –2x + 2e	**0.5**
3.a	f is continuous and strictly decreasing over $]1;+\infty[$ then f admits an inverse function g. $D_g=\ ]-\infty;2[$	**0.5**
3.b	$\begin{array}{c\|ccc} x & -\infty & & 2 \\ \hline g'(x) & & - & \\ \hline g(x) & +\infty & \searrow & 1 \end{array}$	**1**
3.c		**1.5**
4.a	$\int x\ln(x)dx=\frac{x^2}{2}\ln x-\int\frac{x}{2}dx=\frac{x^2}{2}\ln x-\frac{x^2}{4}+C$	**1**
4.b	$\int_1^e f(x)dx=\int_1^e 2xdx-2\int_1^e x\ln xdx=x^2-2\left(\frac{x^2}{2}\ln x-\frac{x^2}{4}\right)=\left[\frac{3x^2}{2}-x^2\ln x\right]_1^e=\frac{e^2-3}{2}$	**0.5**
4.c	Area = $\frac{(e-1)(2e-2)}{2}-\int_1^e f(x)dx=e^2-2e+1-\frac{e^2-3}{2}=\frac{e^2-4e+5}{2}=0.758u^2$	**0.5**