وزارة التربية والتعليم العالي
المديريّــة العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة الثانوية العامة
فرع: علوم الحياة

دورة الـعام ٢٠١٩ العاديّة
الثلاثاء ١٨ حـزيران ٢٠١٩
مكيفة

عدد المسائل: اربع

مسابقة في مادة الرياضيات
المدة: ساعتان

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

مسابقة في مادة الرياضيات

المدة: ساعتان

(باللغة الإنكليزية)

الاسم: ....

الرقم: ....

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, given :

- The points **A(4 , 1 , 4), B(1 , 0 , 1), E(3 , –1 , 1)**
- The plane **(P)** of equation **x + 2y + 3z – 4 = 0**.

1) - **Show that (AE)** is perpendicular to plane **(P).**

- **Verify that** the point **E** is on plane **(P).**

2) a- Determine an equation of the plane (Q) determined by A, B and E.

b- Verify that the two planes **(P)** and **(Q)** are perpendicular.

3) Let **(d)** be the line of intersection of **(P)** and **(Q).**

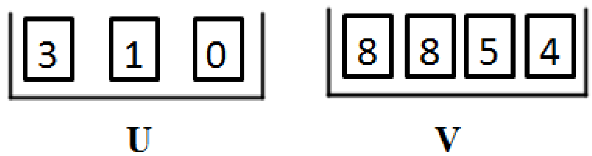
Show that a system of parametric equations of (d) is $\begin{cases} x=-2t+1 \\ y=t \\ z=1 \end{cases}$ $(t \in \mathbb{R})$.

4) Consider, in plane (P), the circle (C) with center E and radius $\sqrt{5}$.
Show that the line (d) intersects the circle (C) in two points whose coordinates are to be **determined.**

II- (4 points)

U and **V** are two urns such that:

- **U** contains three cards holding the numbers **3, 1**, and **0**
- **V** contains four cards holding the numbers **8, 8, 5**, and **4**



One card is selected randomly from urn **U**:

- If the selected card from **U** holds the number **0**, then two cards are selected randomly and simultaneously from urn **V**;
- If the selected card from **U** does not hold the number **0**, then three cards are selected randomly and simultaneously from urn **V**.

Consider the following events:

A: "The selected card from urn **U** holds the number **0**"

S: "The sum of the numbers held on the selected cards from urn V is even"

1) a- Verify that $P(S/A)=\frac{1}{2}$

b- Calculate $P(S\cap A)$.

c- Verify that $P\left(S\cap\overline{A}\right)=\frac{1}{6}$

d- calculate $P(S)$.

2) The sum of the numbers held on the selected cards from urn **V** is even.

Calculate the probability that the selected card from urn **U** does not hold the number **0**.

3) Let **X** be the random variable equal to the <u>product of numbers held by the cards</u> selected from the two urns **U** and **V**.

a- Calculate $P(X = 0)$.

b- Deduce $P(X \le 160)$.

III- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points **M** and **M'** with respective affixes z and **z'** such that **z' = (1 + i)** $\mathbf{\bar{z}}$.

1) In this part, let **z =** $e^{i\frac{\pi}{3}}$.

a- Write z' in exponential form.

b- Verify that $(z')^6$ is pure imaginary.

2) a- Show that $|z'| = \sqrt{2}\,|z|$.

b- Deduce that, when **M** moves on the circle with center **O** and radius $\sqrt{2}$, **M'** moves on a circle whose center and radius are to be determined.

3) Let **z = x + iy** and **z' = x' + iy'**, where **x , y , x'** and **y'** are real numbers.

a- Verify that x' = x + y et que **y' = x - y**.

b- For all **z ≠ 0**, denote by **N** the point with affix $\mathbf{\bar{z}}$. **Prove that** the triangle ONM' is right isosceles with principal vertex **N**.

IV- (8 points)

Consider the function f **defined** over $]0;+\infty[$ as $\mathbf{f(x) = 2x(1 - \ln x)}$. Denote by (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) **Determine** $\lim\limits_{\substack{x\to 0\\x>0}} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.

2) **a-** Let **A** be the point of intersection of (C) and the x-axis.

Determine the coordinates of **A**.

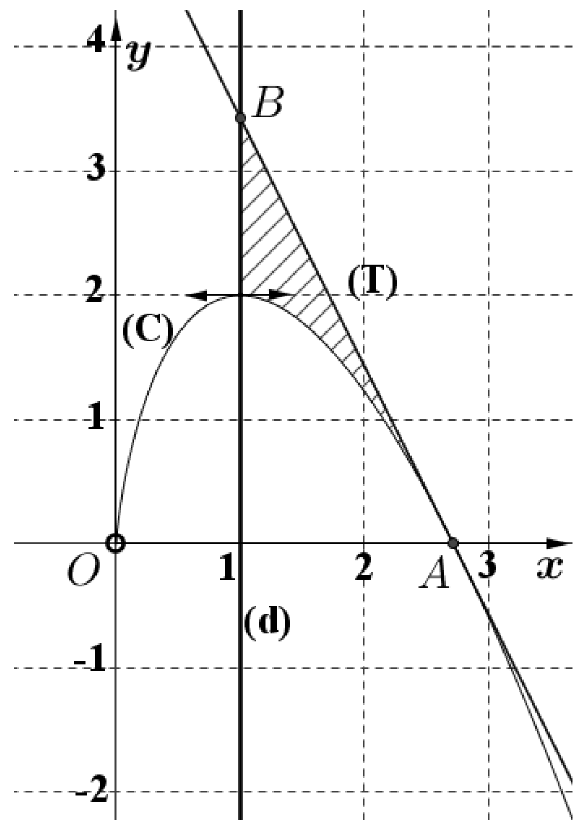
b- Show that $f'(x) = -2\ln x$

c- Set up the table of variations of f.

d- Determine an equation of the tangent **(T)** to (C) at **A**.

In the adjacent figure:

- **(C)** is the representative curve of **f.**
- **(T)** is the tangent to **(C)** at **A**.
- **(d)** is the line of equation x = 1
- **B(1, 2e -2)** is the point of intersection of **(d)** and **(T).**



3) **a- Show that** the function f has on $]1;+\infty[$ an inverse function **g**.

b- Determine the domain of definition of g

c- Set up the table of variation of **g**.

Denote by (C') its representative curve in the same system.

d- Copy (C), then **draw** (C') in the same system.

4) **Denote** by **(d)** the line with equation $x=1$ and by $B(1,2e-2)$ the point of intersection of **(d)** and **(T).**

a- Using integration by parts, verify that $\int x\ln(x)\,dx=\frac{x^2}{2}\ln x-\frac{x^2}{4}+c$

b- Deduce that $\int_1^e f(x)dx=\frac{e^2-3}{2}$.

c- Calculate the area of the shaded region bounded by the curve (C), the tangent **(T)** and the line **(d)**.