وزارة التربية والتعليم العالي
المديريّة العامّة للتربية
دائرة الامتحانات الرسميّة

امتحانات الشهادة الثانوية العامّة
فرع الاجتماع والاقتصاد

دورة العام ٢٠١٩ العاديّة
الاثنين ٢٤ حزيران ٢٠١٩
مكيّفة

عدد المسائل: أربع	**مسابقة في مادة الرياضيات** **المدة: ساعتان**	

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

مسابقة في مادة الرياضيات

المدة: ساعتان

باللغة الإنكليزيّة

الاسم : ....

الرقم : ....

I- (4 points)

The table below represents the number of passengers **, in millions,** in an airport, for each year from **year 2014** to **year 2018.**

Year	**2014**	**2015**	**2016**	**2017**	**2018**
Rank of the year: x_i	4	5	6	7	8
Number of passengers (in millions): y_i	15	18	22	24	25

1) **Represent** the scatter plot of points (x_i, y_i) in a rectangular system.

2) **- Calculate** the coordinates of the center of gravity $G(\bar{x}, \bar{y})$.

   - Plot G in the preceding system.

3) **- Calculate** the correlation coefficient r.

   - Give an interpretation of the value thus obtained.

4) **- Determine** an equation of the regression line $(D_{y/x})$, of y in terms x.

   - Draw $(D_{y/x})$ in the preceding system.

5) Assume that the above model remains valid till the year 2023.

   a- **Estimate** the number of passengers in 2021.

   b- **Estimate** the percentage increase in the number of passengers from the year 2018 till the year 2022.

II- (4 points)

A cafeteria sells dessert and coffee only.

A customer can buy **one dessert**, **one cup of coffee**, **both** or **none**.

In this cafeteria:

- **70 %** of the customers **buy dessert**, among which **40 % buy coffee**,
- **Among** the customers who **do not buy dessert, 35 % do not buy coffee.**

One customer of this cafeteria is randomly selected and **interviewed**.

Consider the following events:

D: "The interviewed customer **buys a dessert**",

C: "The interviewed customer **buys a cup of coffee**".

1) a- **Calculate** the probabilities $P(C \cap D)$ and $P(C \cap \overline{D})$.

   b- **Deduce** that $P(C) = 0.475$.

2) **A customer does not buy a cup of coffee**.

   Calculate the probability that this customer does not buy a dessert.

3) In this cafeteria:

   - the price of a dessert is 7 000 LL.
   - the price of a cup of coffee is 3 000 LL.

   Denote by **X the random variable equal to the sum paid by a customer**.

   a- **Verify that** the four possible values of X are :

   0 ; 3 000 ; 7 000 ; 10 000 .

   b- **Justify that** $P(X = 0) = 0.105$.

   c- **Determine** the probability distribution of X.

   d- During a certain day, 500 **customers entered** the cafeteria.

   Estimate the total revenue during that day.

III- (4 points)

A company produces a certain type of objects.

At the end of January 2018, the monthly total cost was **9 million** LL.

At the end of each month, and during the following months,

The monthly total cost receives:

- **an increase of 21%**
- **a fixed sum of 840 000 LL**

For all natural numbers n > 0, denote by:

- **C_1 the monthly total cost,** in millions LL**, at the end of January 2018**
- **C_n the monthly total cost,** in millions LL**, at the end of the nth month**.

Thus $C_1 = 9$ and $C_{n+1} = 1.21C_n + 0.84$.

1) **- Calculate** C_3.

   - What is the monthly total cost at the end of March 2018?

2) Consider the **sequence (V_n)** defined by $V_n = C_n + 4$.

   a- **Show that** (V_n) is a geometric sequence of common ratio r = 1.21.

   - Determine the first term V_1 of this sequence.

   b- **Show that** $C_n = 13 \times (1.21)^{n-1} - 4$.

   c- **Calculate** $C_{n+1} - C_n$ in terms of n**.**

   - Deduce that the sequence (C_n) is strictly increasing.

   d- Calculate n so that $C_n > 300$.

3) The monthly revenue in millions LL, at the end of the nth month, for this company is modeled as $R_n = 100 \times (1.07)^{n-1}$ for all natural numbers n > 0.

   - **Express the profit** P_n in terms of n.
   - Does the company achieve **profit** at the end of June 2019?

IV- (8 points)

Consider the function f defined over $[0, +\infty[$ as $f(x) = \frac{2}{2+e^{x-1}}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

Part A

1)a- **Calculate** f(0) and f(2) to the nearest 10^{-3}.

2) **Determine** $\lim_{x \to +\infty} f(x)$.

- **Deduce** an asymptote to (C).

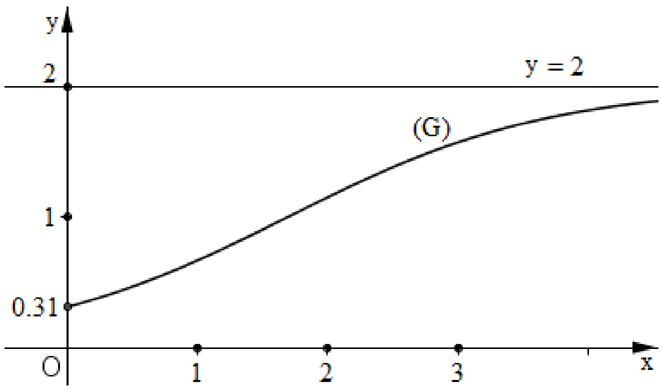
3) a- **Calculate** f '(x).

- **Show that** f is strictly decreasing.

b- **Set up** the table of variations of f.

4)Consider the function g defined over $[0, +\infty[$ as $g(x) = \frac{2e^{x-1}}{2+e^{x-1}}$.

The representative curve (G) of the function g and its asymptote are given in the figure below.



a- **Solve** the equation f(x) = g(x).

b- **Copy** (G).

- **Draw** (C) in the same system.

Part B

A factory produces a certain type of objects.

The demand function is modeled as:

$$f(x) = \frac{2}{2 + e^{x-1}}$$ and expressed in thousands of objects.

The supply function is modeled as:

$$g(x) = \frac{2e^{x-1}}{2 + e^{x-1}}$$ and expressed in thousands of objects.

x is the unit price expressed in millions LL with $x \in]0\,,5]$.

1) **Calculate** the demanded number of objects for a unit price of 2 million LL.
2) **Calculate** the unit price for a supply of 1000 objects.
3) **Determine** the equilibrium price.
4) Denote by E(x) the elasticity of the demand with respect to the unit price x.

   a- **Show that** $E(x) = \dfrac{xe^{x-1}}{2 + e^{x-1}}$.

   b- **Calculate** E(2).

   c- If the unit price of 2 million LL increases by 1 %, then **calculate** the number of demanded objects.