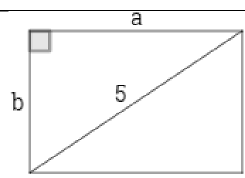
المادة: رياضيات – لغة إنكليزية الشهادة: المتوسطة نموذج رقم: 2 / 2019 المدّة: ساعتان	الهيئة الأكاديميّة المشتركة قسم: الرياضيات	المركز التربوي للبحوث والإنماء

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (3 points)

In the following table, only **one answer** to each question is correct. Write the number of each question then choose, **with justification**, its corresponding answer.

Nº	Questions	Answers		
		A	B	C
1)	If $a = 3\sqrt{3} + 2\sqrt{7}$ then $\frac{1}{a} =$	$2\sqrt{7} - 3\sqrt{3}$	$3\sqrt{3} - 2\sqrt{7}$	$-3\sqrt{3} - 2\sqrt{7}$
2)	x and y are two real numbers such that $x > y > 0$. If $B = \frac{\sqrt{x+y}}{\sqrt{x}} \times \frac{\sqrt{x^2 - xy}}{\sqrt{x^2 - y^2}}$ then the simplest form of B is:	$y\sqrt{x}$	$x\sqrt{y}$	1
3)	The two positive numbers a and b represent the length and width of the rectangle whose length of the diagonal is equal to 5.  If the area of the rectangle is 12, then $(a+b)^2 =$	5	25	49
4)	In a class, there are 15 boys and 10 girls. 40% of boys and 20% of girls participate in an activity. The percentage of students participating in this activity is:	60%	50%	32%

II- (3 points)

Given $A(x) = \frac{x^2}{9} - \frac{2x}{3} + 1 - (3 - x)^2$.

1) Develop $\left(\frac{x}{3} - 1\right)^2$ and show that $A(x) = \frac{-8(x-3)^2}{9}$

2) Let $F(x) = \frac{A(x)}{\frac{x^2}{9} - 1}$

   a. For what values of x, the expression F(x) is not defined?

   b. Simplify F(x).

III- (3 points)

Jad and Mazen bought telephones type A and type B.

The table below shows the total amount in LL paid by each of them.

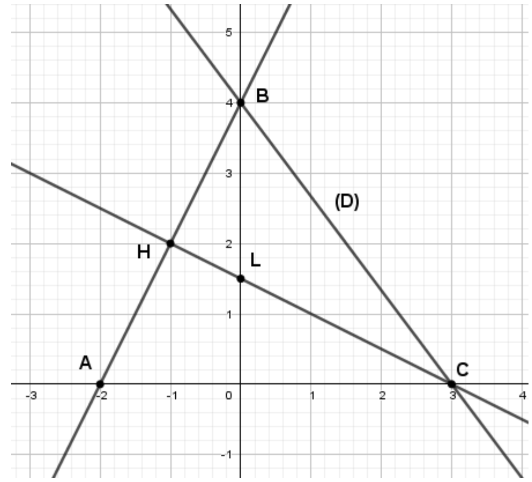
	Number of telephones type A	Number of telephones type B	Total amount paid in LL
Jad	3	2	3 000 000
Mazen	2	3	3 250 000

1) Verify that the price of a telephone type A is 500 000 LL and that type B is 750 000 LL.

2) During the month of sales, the price of type A is reduced by 20%, and type B is reduced by 30%.
   Lynne bought 7 telephones and paid 3 300 000 LL.
   Calculate the number of each type of telephone bought by Lynne.

IV- (5.5 points)

In an orthonormal system of axes $x'Ox$ and $y'Oy$, consider the points $A(-2\,;0), B(0\,;4)$ and the line (D) with the equation $y = -\frac{4}{3}x + 4$.



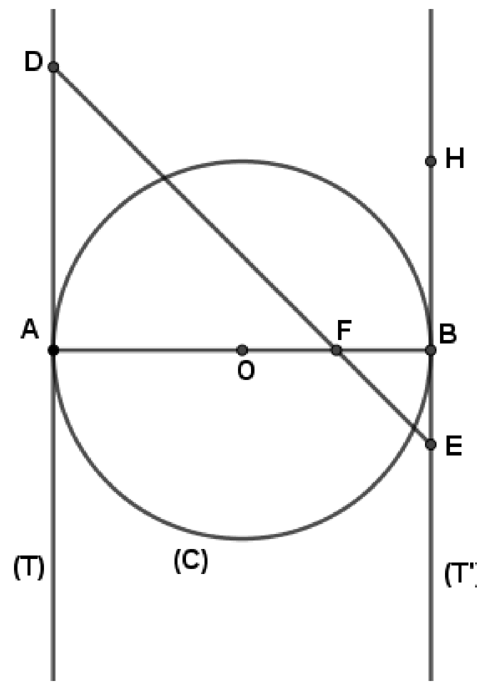
The line (D) intersects the $x'Ox$ at a point C.

1) **a.** Calculate the coordinates of the point C.
   b. Verify, by calculation, that B is a point on (D).
2) Let H be the orthogonal projection of C on (AB).
   a. Show that the triangle ABC is isosceles with vertex C.
   b. Verify that the coordinates of point H are $(-1\,;2)$.
3) (CH) intersects $y'Oy$ at a point L.
   a. Write an equation of the line (CH).
   b. Calculate the coordinates of point L.
4)
   a. Show that the two triangles OLC and CBH are similar and write their ratio of similarity.
   b. Deduce the length of the segment [CL].
5) Calculate $\tan O\hat{C}L$, deduce the measure rounded to the nearest degree, of the angle $A\hat{B}C$.

V- (5.5 points)

In the adjacent figure, we have:

- (C) is the circle of center O and radius 4
- [AB] is a diameter of (C)
- (T) is the tangent to (C) at A
- D is a point of (T) such that AD = 6
- (T’) is the tangent to (C) at B
- E is a point of (T’) such that BE = 2
- [DE] intersects [AB] in F.



1) Draw the figure.
2) Show that $\frac{FB}{FA} = \frac{1}{3}$.
3) Verify that FB = 2.
4) Show that $A\hat{F}D = 45^\circ$.
5) Let H be a point on the line (T') such that OBH is a right isosceles triangle.
   The two segments [OH] and [DF] intersect at a point I.
   Show that $O\hat{I}F = 90°$.
6) **a.** Show that the four points O, I, B and E belong to the same circle (C’) and determine a diameter of (C’).
   b. Calculate the radius of (C’).
7) Let M be the symmetric of B with respect to H.
   a. Verify that $OM = 4\sqrt{5}$.
   b. Show that the line (OM) is tangent to the circle (C’).

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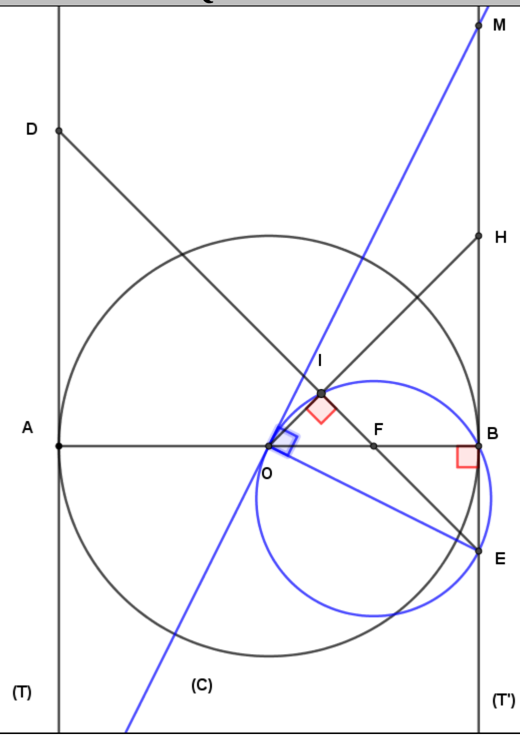
	Question I	Note
1	$\frac{1}{(3\sqrt{3}+2\sqrt{7})} = 2\sqrt{7} - 3\sqrt{3}$ the answer is (A).	0.75
2	$\frac{\sqrt{x+y}}{\sqrt{x}} \times \frac{\sqrt{x(x-y)}}{\sqrt{(x-y)(x+y)}} = \frac{\sqrt{x+y}}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x+y}} = 1$ the answer is (C).	0.75
3	$(a+b)^2 = a^2 + 2ab + b^2 = 5^2 + 2(12) = 25 + 24 = 49$ the answer is (C).	0.75
4	$\frac{8}{25} = 32\%$ the answer is (C).	0.75
	Question II	
1	$\left(\frac{x}{3} - 1\right)^2 = \frac{x^2}{9} - \frac{2}{3}x + 1$ $A(x) = \left(\frac{x}{3} - 1\right)^2 - (3-x)^2 = \frac{1}{9}(x-3)^2 - (x-3)^2 = -\frac{8}{9}(x-3)^2$.	0.5 1
2.a	F(x) is not defined if $x^2 - 9 = 0$ then $x = 3$ or $x = -3$.	0.5
2.b	$F(x) = \frac{-8(x-3)}{x+3}$.	1
	Question III	
1	$3(500\,000) + 2(750\,000) = 3\,000\,000$ and $2(500\,000) + 3(750\,000) = 3\,250\,000$	1
2	Reduction of 20% on the price of the phone type A then the new price will be 400 000 LL. Reduction of 30% on the price of the phone type B then the new price will be 525 000 LL. Let m be the number of phones type A and n the number of phones type B. By solving the system: $\begin{cases} 400000m + 525000n = 3300000 \\ m + n = 7 \end{cases}$ We get m = 3 “type A” and n = 4 “type B”	2
	Question IV	Note
1.a	$y_C = 0$, then, $0 = -\frac{4}{3}x + 4$ so C(3 ; 0).	0.25
1.b	Since $y_B = -\frac{4}{3}x_B + 4$ then B is a point of (D).	0.25
2.a	CA = CB = 5, so ABC is an isosceles triangle with vertex C.	0.5
2.b	ABC is an isosceles triangle with vertex C, so [CH] is perpendicular bisector and H is the midpoint of [AB], then $x_H = \frac{x_A + x_B}{2} = -1$ and $y_H = \frac{y_A + y_B}{2} = 2$, so H(–1 ; 2).	1
3.a	The equation of the line (CH) is: $y = -\frac{1}{2}x + \frac{3}{2}$.	0.5
3.b	The line (CH) intersects the axis *y'Oy* in L then $x_L = 0$ and $y_L = -\frac{1}{2}x_L + \frac{3}{2} = \frac{3}{2}$ then $L(0 ; \frac{3}{2})$.	0.5
4.a	The two triangles OLC et CBH are similar: $O\hat{C}L = H\hat{C}B$: [CH] is the bisector of the angle $A\hat{C}B$ in the isosceles triangle ABC. $C\hat{O}L = C\hat{H}B = 90°$. The ratio of similarity is: $\begin{array}{c\|} OLC \\ HBC \end{array} \frac{OL}{HB} = \frac{OC}{HC} = \frac{CL}{BC}$	1
4.b	According to the ratio of similarity: $CL = \frac{OC \times BC}{HC} = \frac{3 \times 5}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{2}$	0.5

5	$\tan O\hat{C}L = \frac{OL}{OC} = \frac{\frac{3}{2}}{3} = \frac{1}{2}$ then $O\hat{C}L \approx 27°$ ABC is an isosceles triangle with vertex C then: $2\,O\hat{C}L + 2\,A\hat{B}C = 180°$, So $A\hat{B}C = 63°$.	**1**
	Question V	Note
1		**0.5**
2	According to Thales theorem: $\frac{FB}{FA} = \frac{FE}{FD} = \frac{BE}{AD} = \frac{2}{6} = \frac{1}{3}$	**0.5**
3	FB + FA = 8 since FA = 3FB then, 4FB = 8 then FB = 2.	**0.5**
4	ADF is a right isosceles triangle with vertex A because AD = AF = 6. $D\hat{A}F = 90°$ (the line (T) is tangent to the circle (C) at A), then $A\hat{F}D = 45°$.	**0.5** **0.5**
5	OBH is right isosceles triangle then $H\hat{O}B = 45°$. ADF is right isosceles triangle (part **4**) then $A\hat{F}D = 45°$. In the triangle OFI : $O\hat{I}F = 180° - (O\hat{F}I + I\hat{O}F) = 180° - (A\hat{F}D + H\hat{O}B) = 90°$.	**0.5**
6.a	$O\hat{I}E = 90°$ and $O\hat{B}E = 90°$, so O, I, B and E belong to the same circle (C') of diameter [OE].	**1**
6.b	OBE is a right isosceles triangle with vertex B therefore by Pythagoras theorem: $OE^2 = OB^2 + BE^2 = 4^2 + 2^2 = 20$, $OE = 2\sqrt{5}$ so the radius is $\sqrt{5}$.	**0.5**
7.a	OMB is a right isosceles triangle B (the line (T') is tangent to the circle (C) at B), according to the Pythagoras theorem: $OM^2 = OB^2 + BM^2 = 4^2 + 8^2 = 16 + 64 = 80$, $OM = 4\sqrt{5}$.	**0.5**
7.b	$OM^2 + OE^2 = \left(4\sqrt{5}\right)^2 + \left(2\sqrt{5}\right)^2 = 80 + 20 = 100$ and $ME^2 = 10^2 = 100$ By the converse of Pythagoras theorem OME is a right triangle at O. So $E\hat{O}M = 90°$ with [OE] is a diameter of the circle (C'), so the line (OM) is tangent to (C') at O.	**0.5**