المادة: رياضيات – لغة إنكليزية الشهادة: الثانوية العامة الفرع: الاقتصاد والاجتماع نموذج رقم: ١/ ٢٠١٩ المدّة: ساعتان	الهيئة الأكاديميّة المشتركة قسم: الرياضيات	المركز التربوي للبحوث والإنماء

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- [4 points]

A factory produces a certain article. The table below shows the supply y of this product, expressed in thousands of articles, in terms of the unit price x in thousands LL.

The sale price x_i in thousands LL.	2	3	5	7	10
Supply y_i in thousands of articles	5	7	8	9	11

The pattern remains valid for a price 20 000 LL

1) Determine an equation of the regression line $(D_{y/x})$.
2) Estimate the supplied number of articles for a unit price of 15 000 LL.
3) The demand is modeled by the line $(D_{z/x})$ with equation $z = -0.567x + 8.266$.
   The two lines $(D_{y/x})$ and $(D_{z/x})$ intersect at the $I(3.158\,;\,6.474)$.
   Give an economical interpretation of the coordinates of this point.
4) Calculate, in LL, the revenue for a unit price of 7000 LL.
5) Let $e(x)$ be the elasticity of demand in terms of the unit price x.
   a) Show that $e(x) = \frac{567x}{-567x+8266}$.
   b) Calculate $e(4)$. Give an economical interpretation of the value thus obtained.

II- [4 points]

Part A

Nisrine deposits 4 000 000 LL in a bank at an annual interest rate of 0.5% compounded yearly.
After compounding the interest, Nisrine adds yearly 200 000 LL to her account.
Denote by Vn the amount, in millions LL, that Nisrine has in her account after n years (n is a natural number).

Thus $V0 = 4$.

1) Justify, for all natural numbers n, that $V_{n+1} = 1.005V_n + 0.2$.
2) Denote by (Wn) the sequence defined, for all natural numbers n, as $W_n = V_n + \alpha$.
   a) Calculate α if (W_n) is a geometric sequence with common ratio 1.005.
   b) Take $\alpha = 40$. Express W_n in terms of n and deduce that $V_n = 44(1.005)^n - 40$.
3) a) Calculate the amount in Nisrine's account after ten years.
   b) Calculate the amount of interest gained by Nisrine over ten years.
4) Prove that the sequence (V_n) is increasing.

Part B

At the same date when Nisrine deposited the 4 000 000 LL, Nadia invested 8 000 000 LL in the same bank following the rule: $U_{n+1} = 1.005U_n$, where U_n is the amount, in millions LL, in Nadia's account after n years (n is a natural number). Thus $U_0 = 8$.

1) Prove that (U_n) is a geometric sequence and that $U_n = 8(1.005)^n$.
2) We admit that (U_n) is also an increasing sequence.
   After how many years would the amount in Nisrine's account exceed the amount of money in Nadia's account for the first time?

III- [4 points]
An urn U contains 4 white balls, 5 red balls and 3 green balls.
Part A
We select randomly and successively 3 balls from U without replacement.
1) Calculate the probability of selecting three balls of the same color.
2) Prove that the probability of selecting at least one green ball and at least one red ball among the three selected balls is $\frac{21}{44}$.

Part B
A player pays an amount of 10 000 LL to participate in a game.
This game runs as follows:
The player selects at random and simultaneously two balls from the urn U.
- If at most one of the two selected balls is green, the player receives 5 000 LL and the game ends.
- If the two selected balls are both green, they are kept outside the urn U and the player receives 8 000 LL. After that, the player selects randomly and simultaneously two balls from the remaining 10 balls in U.
  - If these two selected balls are of the same color the player receives12 000 LL and the game ends;
  - Otherwise, the player receives 2 000 LL and the game ends.

Let X be the random variable equal to the algebraic gain of the player

(The algebraic gain could be zero, positive or negative).

1) Justify that the values of X are: -5 000 ; 0 and 10 000.
2) Prove that $P(X = 0) = \frac{29}{990}$.
3) Determine the probability distribution of X.
4) Do you expect this player to win? Justify.

IV- [8 points]
Let f be the function defined on $[0;+\infty[$ as $f(x) = (-2x-1)e^{-x} + 2$ and denote by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.
Part A
1) Determine $\lim\limits_{x \to +\infty} f(x)$. Deduce an equation of an asymptote to (C).
2) Prove that $f'(x) = (2x-1)e^{-x}$ and set up the table of variations of f.
3) Draw (C) and its asymptote.
4) The line (d) with equation $y = \frac{x}{2}$ intersects the curve (C) at one point only with abscissa α.

   Verify that 3.5 < α < 3.6.
5) Let F be the function defined on $[0;+\infty[$ as $F(x) = (2x+3)e^{-x} + 2x$.
   a) Show that F is a primitive of f.
   b) Deduce the area of the region limited by (C), the x-axis and the lines of equations x = 0 and x = 1.

Part B

In what follows, suppose that $\alpha = 3.55$

A factory produces souvenirs. The total cost of production of souvenirs C_T, expressed in millions LL, is modeled as $C_T(x) = f(x)$ **only** for all x in [0.5 ; 4], where x, expressed in thousands, represents the number of souvenirs produced. $(0.5 \leq x \leq 4)$.

1) Calculate, in LL, the total cost of production of 3 000 souvenirs. In this case, deduce the average cost of production of one souvenir.
2) The adjacent table is the table of variations of the profit function P of this factory, in millions LL, over [0.5 ; 4].

x	0.5	0.844	4
$P'(x)$		+	
$P(x)$	-1	0	2.164

   a) Study if this factory can achieve a profit equal to 3 000 000 LL.
   b) Determine the minimum number of souvenirs to be sold so that the factory realizes a profit.
   c) The profit function P is defined over [0.5 ; 4] as
   $$P(x) = x - 2 + (2x+1)e^{-x}.$$
   Prove that the revenue function R, in millions LL, is modeled as $R(x) = x$ knowing that the whole production is sold.
   d) Calculate the number of souvenirs that should be sold so that the revenue is double the total cost.

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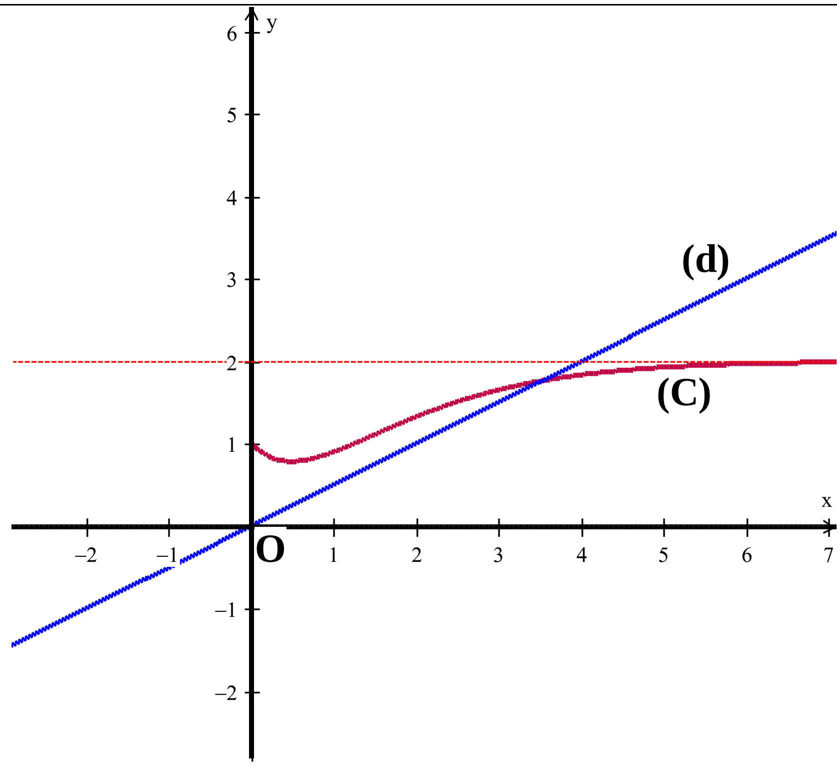
أسس التصحيح

QI	Answers	Mark
1	$(D_{y/x}): y = 0.679x + 4.330$	0.5
2	Supply = 0.679(15) + 4.330 = 14, 515 thousand articles, then 14515 articles	1.5
3	The equilibrium price is 3158 LL and the equilibrium quantity is 6474 articles.	0.75 0.75
4	Revenue = (7)(-0.567(7) + 8.266)(1000) = 30079 LL	1.5
5a	$e(x) = -x.\frac{z'}{z} = \frac{567x}{-567x + 8266}$	1
5b	e(4) = 0.378. For a price of 4000 LL, if the price increases by 1% then the demand decreases by 0.378%.	0.25 0.75

QII	Answers	Mark
A1	$V_{n+1} = (1+0.5/100)V_n + 200000/1000000 = 1.005V_n + 0.2$	0.5
A2a	$W_{n+1} = 1.005W_n$; so $\alpha = 40$	1
A2b	$W_n = 44(1.005)^n$; and $V_n = 44(1.005)^n - 40$	0.25 0.25
A3a	$V_{10} = 6.250165$. So: 6250165 LL	1
A3b	I = 6 250 165 – (4 000 000+200 000x10)= 250 165 LL	0.5
A4	$V_{n+1} - V_n = 44(1.005)^{n+1} - 44(1.005)^n = 44(1.005)^n(1.005-1)$ $= 0.22(1,005)^n > 0$. Therefore (V_n) is an increasing sequence.	1.5
B1	The common ratio is 1.005 ; $U_n = U_0 \times q^n = 8(1.005)^n$	0.25 0.25
B2	$V_n > U_n$ gives $n > 21.12$ so $n = 22$. Thus after 22 years.	1.5

QIII	Answers	Mark
A1	$P(WWW \text{ or } RRR \text{ or } GGG) = \dfrac{A_4^3 + A_5^3 + A_3^3}{A_{12}^3} = \dfrac{3}{44}$	1
A2	$(2G1R \text{ or } 1G2R \text{ or } 1G1R1W)$ where $2G1R$ can be written $\dfrac{3!}{2!} = 3$ ways: GGR, GRG, RGG and similarly for *2R1G*. where *RGW* can be written in $3!$ ways $P = \dfrac{A_3^2 \times A_5^1 \times 3 + A_3^1 \times A_5^2 \times 3 + A_3^1 \times A_5^1 \times A_4^1 \times 3!}{A_{12}^3} = \dfrac{21}{44}$	1.5
B1	5000 – 10000 = -5000; the first 2 selected balls are not green and the game ends. 8000 + 2000 – 10000 = 0; the first 2 selected balls are green and the second 2 selected balls are of different color. 8000 + 12000 – 10000 = 10000. The first 2 selected balls are green and the second 2 selected balls are of the same color.	1
B2	$P(X=0) = \dfrac{C_3^2}{C_{12}^2} \times \left(1 - \dfrac{C_4^2 + C_5^2}{C_{10}^2}\right) = \dfrac{29}{990}$. The first 2 balls are both green $\dfrac{C_3^2}{C_{12}^2}$ <u>and</u> the second 2 balls from 10 are of different color (RG, RW, WG) $\dfrac{C_5^1 C_1^1 + C_5^1 C_4^1 + C_4^1 C_1^1}{C_{10}^2} = \left(1 - \dfrac{C_4^2 + C_5^2}{C_{10}^2}\right)$ "opposite of 2 balls same color"	1
B3	$P(X = -5000) = 1 - \dfrac{C_3^2}{C_{12}^2} = \dfrac{21}{22}$ $P(X = 10000) = \dfrac{C_3^2}{C_{12}^2} \times \dfrac{C_4^2 + C_5^2}{C_{10}^2} = \dfrac{8}{495}$ and $P(X=0) = \dfrac{29}{990}$	2
B4	EX = 0 + (-5000)(21/22) + (10000)(8/495) = -4611.11 < 0 The player is therefore expected to lose.	0.5

QIV	Answers	Mark
A1	$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} (-2xe^{-x} - e^{-x} + 2) = 2$; y = 2 HA	1
A2	$f'(x) = (-2 + 2x + 1)e^{-x} = (2x - 1)e^{-x}$ $\begin{array}{c\|ccccc} x & 0 & & 1/2 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & 1 & \searrow & 0.786 & \nearrow & +\infty \end{array}$	2

A3		1.5
A4	f(3.5) = 1.758 > 3.5/2 f(3.6) = 1.775 < 3.5/2 Then $3.5 < \alpha < 3.6$	1
A5a	$F'(x) = (2-2x-3)e^{-x} + 2 = (-2x-1)e^{-x} + 2 = f(x)$	1
A5b	Area $= \int_0^1 f(x)dx = [F(x)]_0^1 = 5e^{-1} - 1 = 0.839\ u^2$	1.5
B1	$C_T(3)$ = 1.651490 LL ; therefore 1 651 490 LL Average cost of production of a souvenir = 1651490/3000 ≈ 550.5 LL	2
B2a	According to the table of variations of the profit function: 2164000 < 3000000 therefore NO.	0.5
B2b	P(0.844) = 0 and P strictly increasing. For the production of 845 souvenirs the factory realizes profit.	1.5
B2c	R(x) = P(x) + C(x) = x	0.5
B2d	R(x) = 2C(x) gives f(x) = x/2, then x = α, therefore 3 550 articles.	1.5