المركز التربوي للبحوث والإنماء	**الهيئة الأكاديميّة المشتركة** **قسم: الرياضيات**	**المادة: رياضيات – لغة إنكليزية** **الشهادة: الثانوية العامة** **الفرع: الاقتصاد والاجتماع** **نموذج رقم: 2/ 2019** **المدّة: ساعتان**

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

The table below shows the year end price of 1kg antique coins for six consecutive years.

Year	2008	2009	2010	2011	2012	2013
Rank of the year (x_i)	1	2	3	4	5	6
Price (y_i in hundred thousand LL)	5	7.5	8.5	10	11	12

1) Sketch in a rectangular system of axes the scatter plot, as well as the center of gravity $G(\overline{x}, \overline{y})$ of the given data.
2) Determine an equation of the regression line $(D_{y/x})$ of y in terms of x, and draw this line.
3) *Suppose that the above pattern remains valid until the end of the year 2026.*
   a- Estimate the price of 1 kg antique coins at the end of the year 2018.
   b- Calculate the percentage error of estimation knowing that the actual price of 1 kg of antique coins at the end of the year 2018 was 1 900 000 LL.
4) An exporter sold 50 kg of antique coins at the end of the year 2018 for a price of 1 900 000LL per kg. The exporter decides to buy a new office that costs 150 million LL. He pays all the amount of the selling coins as a down-payment and borrows the remaining amount from a bank as a loan for a period of 7 years. The bank charges an annual interest rate of 6.5% compounded monthly to be paid back equally at the end of each month. Calculate the value of the monthly payment.

II- (4 points)

A graduate wishes to work abroad. He receives two offers on his annual salary from two companies A and B.

Company A offers:

An annual salary of 54 000 000 LL with 6 % annual increase, at the beginning of every year, added to his salary of the previous year.

Company B offers:

An annual salary of 60 000 000 LL with an annual increase of 2 300 000 LL, at the beginning of every year, added to his salary of the previous year.

1) Denote by u_n the annual salary of the graduate at the beginning of the nth year if he chooses to work in company A. Let u_1 = 54 000 000.
   a- Calculate u_2.
   b- Prove that (u_n) is a geometric sequence, then calculate u_n in terms of n.

2) Denote by v_n the annual salary of the graduate at the beginning of the nth year if he chooses to work in company B. Let v_1 = 60 000 000.
   a- Calculate v_2 .
   b- Calculate v_n in terms of n.

3) This graduate plans to work for 10 years abroad.
   a- Calculate the sum S_A of the amount of money he would receive if he works in company A during 10 years.
   b- Which among these two offers is more advantageous for the graduate.

III- (4 points)

In an Interschool Competition, each question is written on a separate card, and all these cards are placed in the same bag. 60% of these cards carry math questions while others carry general knowledge questions.
A student, who is a contestant in this competition, has to choose randomly **one card** from the bag and answer the question that is written on it.
Consider the following events:

M: « The student chooses a **math** question ».
G: « The student chooses a **general** knowledge question ».
C: « The student's answer is **correct**».

1) Calculate the probabilities p(M) and p(G).
2) We are given the following information, which may be used to answer the remaining questions:
   - The probability that the student answers **c**orrectly knowing that it is a **m**ath question is $\frac{2}{3}$.
   - The probability that the student answers **c**orrectly knowing that it is a **g**eneral knowledge question is $\frac{3}{4}$.

   a- Show that the probability $p(M \cap C) = \frac{2}{5}$.
   b- Calculate the probability $p(C)$.
3) The competition rules are as follows: Each contestant draws at first **one** question:
   - If the contestant answers this question correctly, he/she scores **10 points** and does not draw a second question.
   - But if the contestant does not answer the first question correctly, then this question is **put back** in the bag, after which he/she draws randomly one more question from the bag.
     If his/her answer is correct he/she scores **6 points**;
     otherwise if his/her answer is not correct then she scores 0 **points.**

   Let X be the random variable that is equal to the number of points that may be scored by contestant.
   a- Determine the three possible values of X.
   b- Determine the probability distribution for X.

IV- (8 points)

Part A

Consider the function f defined over $]0;+\infty[$ as $f(x) = 1 + \frac{(\ln x)^2}{x}$ and denote by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.

1) Determine $\lim\limits_{x \to 0^+} f(x)$. Deduce an equation of an asymptote to the curve (C).
2) Prove that $\lim\limits_{x \to +\infty} f(x) = 1$. Deduce an equation of another asymptote to the curve (C).
3) Show that $f'(x) = \frac{(\ln x)(2 - \ln x)}{x^2}$, then set up the table of variations of f.
4) Draw (C).

Part B

An enterprise produces a certain type of articles. The total cost function, expressed in millions LL, is modeled as $C(x) = 1 + \frac{(\ln x)^2}{x}$ for all $x \in [1; e^2]$, where x is the number, in thousands LL, of articles produced.

1) Calculate, in LL, the total cost of 2000 articles.
2) Suppose that the whole production is sold.

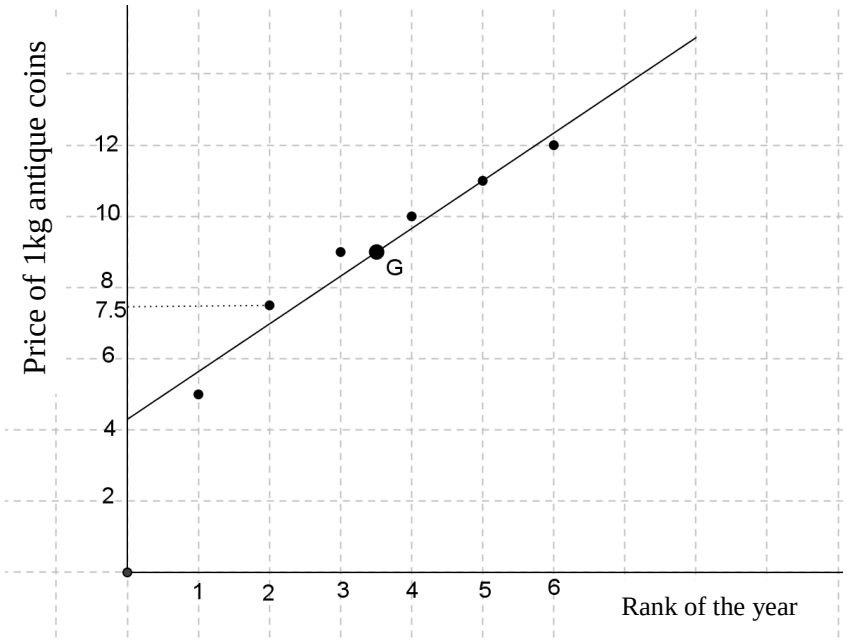
   The profit function, expressed in millions LL, is modeled as $P(x) = 2x - 1 - \frac{(\ln x)^2}{x}$, and its table of variations is shown below:

x	1		e^2
$P'(x)$	$\|$	$+$	$\|$
	$\|$		$-$
$P(x)$	$\|$		$\|$
	$-$		$-$

   a- Complete the given table.

   b- Study if this enterprise can achieve a profit equal to 2 500 000 LL.

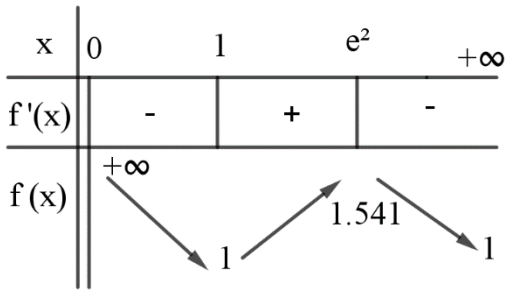
   c- Prove that the selling price of one article is 2 000 LL.

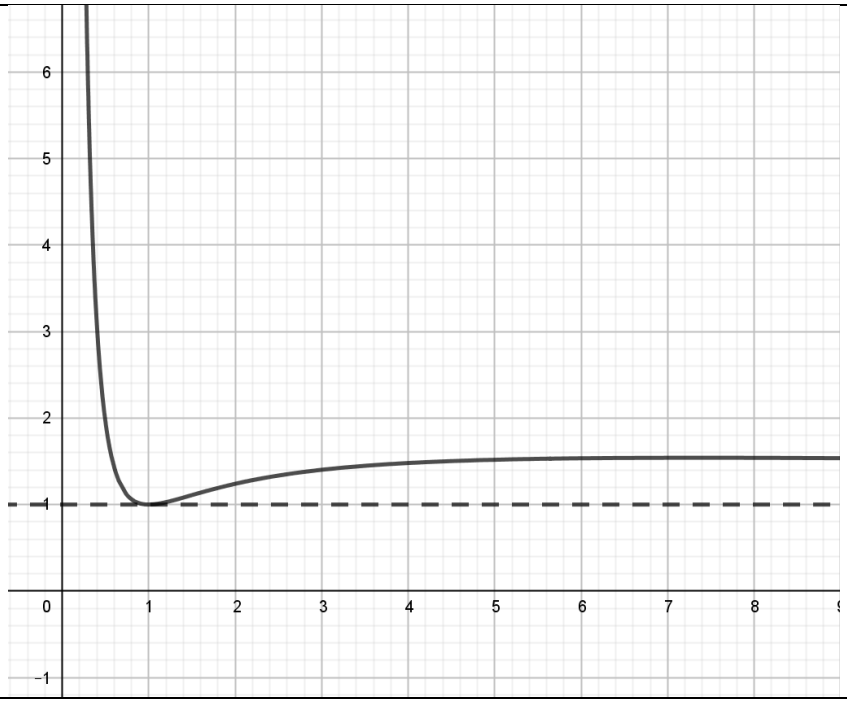
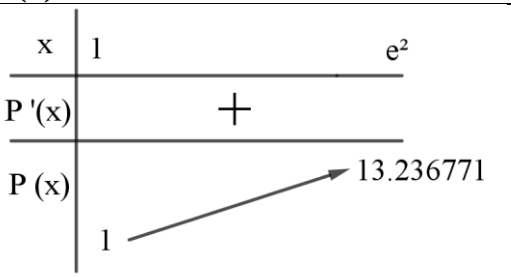
QI	Answers	Mark
1	 G(3.5 , 9)	1
2	$(D_{y/x})$: y = 1.34285x + 4.3	1
3a	y=1.34285(11) + 4.3= 19.07135; So predicted 1 907 135 LL	1
3b	Percentage error: $\frac{\lvert 1\,900000 - 1\,907\,135\rvert}{1\,900\,000} = \frac{7\,135}{1\,900\,000}$ = 0.003 then 0.3%	1
4	The price of 50 kgs would be 50(1 900 000) = 95 000 000 LL. He has to borrow 150 000 000 – 95 000 000 = 55 000 000; then using present annuity $55\,000\,000 = R\left(\frac{1-\left(1+\frac{0.065}{12}\right)^{-84}}{\frac{0.065}{12}}\right)$ So payment R = 816 719 LL $i = \frac{r}{k} = \frac{0.065}{12}$ $n = t \times k = 7 \times 12$	3

QII	Answers	Mark
1a	U_2 = 54 000 000 +0.06(54 000 000) = 57 240 000	1
1b	$U_{n+1} = 1.06U_n$ so (U_n) is a geometric sequence with common ratio 1.06 ; and $U_{n+1} = U_1\,(1.06)^{n-1} = 54\,000\,000(1.06)^{n-1}$	1.5
2a	$V_2 = 62\,300\,000$;	1/2
2b	(V_n) is an arithmetic sequence $V_n = V_1 + (n-1)(2\,300\,000)$=57 700 000 + 2 300 000n	1

3a	$S_A = 54000000\left(\frac{1-(1.06)^{10}}{1-1.06}\right) = 711762926.9LL$	1.5
3b	$S_B = \frac{10}{2}\left(2U_1 + 9(2300000)\right) = 703500000LL$ $S_A > S_B$ So Offer company A is more advantageous.	1.5

QIII	Answers	Mark
1	$P(M) = \frac{60}{100} = 0.6$ and $P(G) = \frac{40}{100} = 0.4$	1
2a	$P(M \cap C) = p(M) \times p(C/M) = 0.6 \times \frac{2}{3} = \frac{2}{5}$	1
2b	$p(C) = P(G \cap C) + P(M \cap C) = \frac{2}{5} + \frac{3}{10} = \frac{7}{10}$	2
3a	The possible values of X are :0, 6 and 10	1
3b	$P(X=0) = p(\overline{C}) \times p(\overline{C}) = \frac{9}{100}$ $P(X=6) = p(\overline{C}) \times p(C) = \frac{21}{100}$ and $P(X=10) = \frac{7}{10}$	2

QIV	Answers	Mark
A1	$\lim\limits_{x \to 0^+} f(x) = 1 + \frac{+\infty}{0^+} = +\infty$; x = 0 VA	1.5
A2	$\lim\limits_{x \to +\infty} f(x) = 1 + \lim\limits_{+\infty} \frac{(\ln x)^2}{x} = 1 + 0 = 1$ since $\lim\limits_{+\infty} \frac{(\ln x)^2}{x} = \lim\limits_{+\infty} \frac{2\ln x}{x} = 0$ So y =1 HA	1.5
A3	$f'(x) = \frac{(\ln x)(2 - \ln x)}{x^2}$ 	3

A4		2.5
B1	C(2) = 1.240 Thus 1 240 000 LL	1
B2a		1
B2b	Maximum profit is 13 236 771 LL so yes enterprise can achieve a profit 2 500 000	1.5
B2c	P(x) = R(x) –C(x) gives R(x) = 2x ; Therefore selling price is 2000 LL	2