المادة: رياضيات – لغة إنكليزية الشهادة: المتوسطة نموذج رقم: ١ / ٢٠١٩ المدّة: ساعتان	الهيئة الأكاديميّة المشتركة قسم: الرياضيات	المركز التربوي للبحوث والإنماء

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (2 points)

All the steps of calculation must be shown.

Consider three distinct points A, P and N such that:

$$AN = 3 - \frac{1}{5} \times \frac{10}{3} - \frac{1}{3} \quad ; \quad NP = \frac{4}{5-\sqrt{5}} - \frac{\sqrt{5}}{5} \quad \text{and} \quad AP = \frac{2\times 10^8 + 10^7}{7\times 10^4 \times 10^3}$$

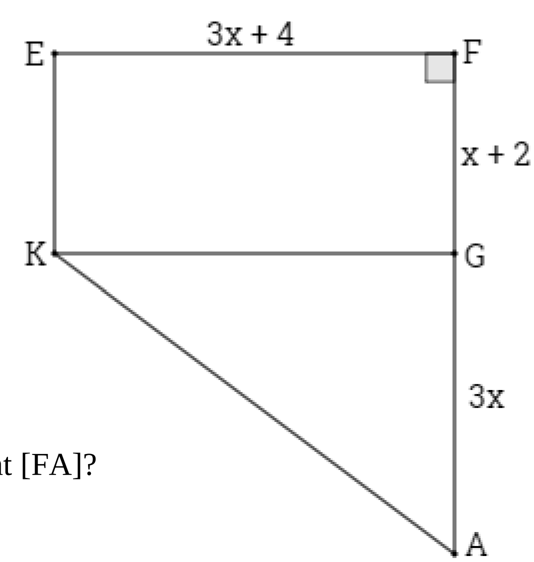
1) Show that AN, NP, and AP are natural numbers.
2) Verify that the three points A, N, and P are collinear.

II- (3 points)

In a school there are two sections for grade 9, section A and section B.

1) In grade 9 section A, 40% of the students are boys.
   Let x be the number of girls and y be the number of boys.
   a. Show that $2x = 3y$.
   b. Knowing that $x = y + 5$. Write a statement that describes this relation between x and y.
   c. Use parts **a.** and **b.** to calculate the number of girls and the number of boys in section A.
2) In grade 9 section B, $\frac{4}{9}$ of students are girls while the number of boys in section B is equal to 10.
   Calculate the number of girls in section B.

III- (4 points)



In the adjacent figure,

- EFGK is a rectangle
- $EF = 3x + 4$ and $FG = x + 2$ with x is a real positive number
- The points F, G and A are collinear such that $AG = 3x$.

Denote by S_1 the area of the rectangle EFGK and by S_2 the area of the triangle KFA.

1) **a.** Calculate S_1 and S_2 in terms of x.
   b. Show that $S_2 - S_1 = (3x+4)(x-1)$.
   c. Calculate x knowing that $S_2 = S_1$.
   In this case what does the line (KG) represent for the segment [FA]?
2) **a.** Calculate KA^2 in terms of x.
   b. Verify that $3x^2 + 4x - 4 = (3x-2)(x+2)$.
   c. Determine x knowing that $KA = 2\sqrt{10}$.

IV- (5 points)

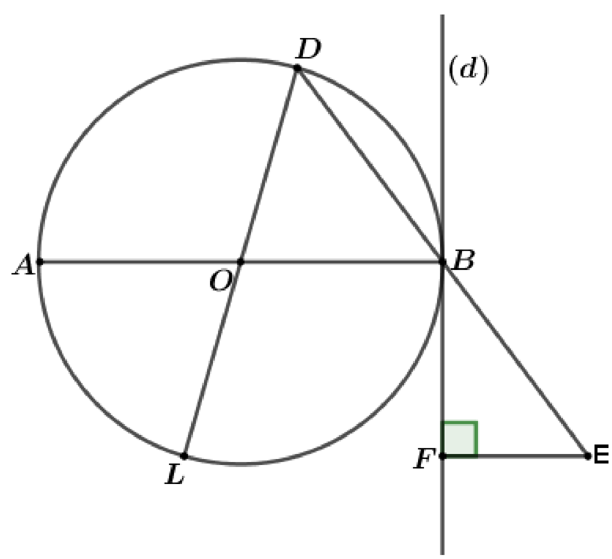
In an orthonormal system of axes $x'Ox$ and $y'Oy$, consider the line (d) with equation $y = x + 5$ and the point $A(-3\ ;\ 2)$.

1) **a.** Verify that A is a point on the line (d).
   b. Let B be the intersection of (d) with $y'Oy$. Calculate the coordinates of B.
   c. Plot the points A and B. Draw the line (d).
2) Let (d') be the line through B and perpendicular to (d).
   a. Write the equation of (d').
   b. Verify that the point $E(5\ ;0)$ is the intersection point of (d') with $x'ox$.
   c. Draw (d').
3) Let (C) be the circle circumscribed about triangle ABE.
   a. Calculate the coordinates of point I, the center of (C), and verify that its radius is equal to $\sqrt{17}$.
   b. Show that the point $F(0\ ;\ -3)$ is on the circle (C).
   c. Show that the triangle AFE is right isosceles.
4) Let L be the translate of E by the translation with vector $\overrightarrow{FI}$. Determine the coordinates of L.
5) Let G be the fourth vertex of the parallelogram IELG. Show that G is on the circle (C).

V-(6 points)

In the adjacent figure,

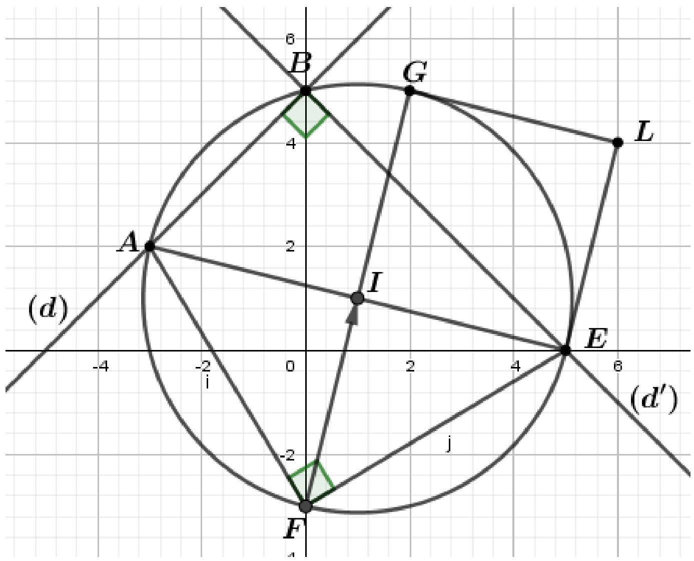
- (C) is a circle with diameter [AB] such that AB = 10
- D is a point on (C) such that DB = 6
- [DL] is a diameter of (C)
- (*d*) is the tangent to (C) at B
- E is the symmetric of D with respect to B
- F is the orthogonal projection of E on (*d*).

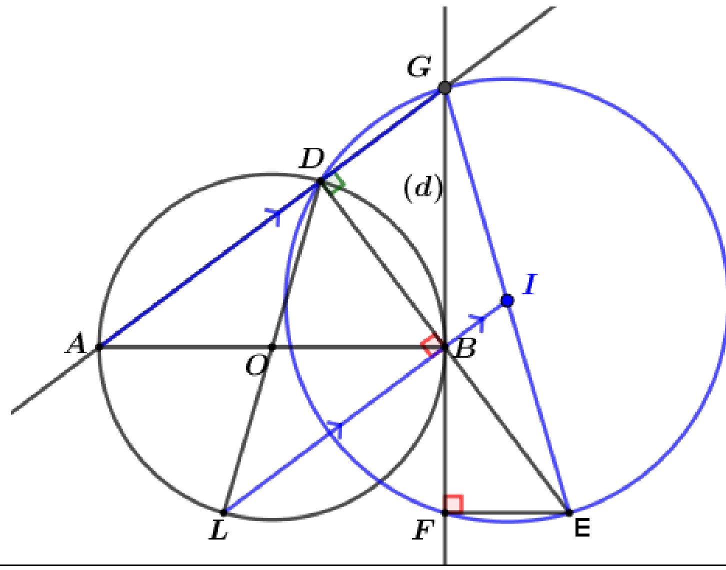


1) Draw the figure.
2) Calculate AD.
3) **a.** Show that the two triangles ABD and BEF are similar and write the ratio of similarity.
   b. Calculate FE and verify that FB = 4.8.
4) Let G be the point of intersection of (*d*) and (AD).
   Show that the points D, G, F and E are on the same circle (C′) and determine a diameter of (C′).
5) Let I be the center of the circle (C′).
   a. Show that the two lines (IB) and (DG) are parallel.
   b. Show the points L, B and I are collinear.
6) **a.** Calculate $\tan \widehat{BAD}$, and deduce that BG =7.5.
   b. Calculate the radius of the circle (C′).

أسس التصحيح

	Question I	pts
1	AN=$3-\frac{1}{5}\times\frac{10}{3}-\frac{1}{3}=3-\frac{2}{3}-\frac{1}{3}=2$; NP=$\frac{4}{5-\sqrt{5}}-\frac{\sqrt{5}}{5}=\frac{4}{5-\sqrt{5}}\times\frac{5+\sqrt{5}}{5+\sqrt{5}}-\frac{\sqrt{5}}{5}=\frac{20+4\sqrt{5}}{20}-\frac{\sqrt{5}}{5}=1$; AP= $\frac{2\times10^8+10^7}{7\times10^4\times10^3}=\frac{2\times10^8+10^7}{7\times10^7}=\frac{10^7(2\times10+1)}{7\times10^7}=\frac{21}{7}=3$;	1.5
2	AP=AN+NP , then the points A, N and P are collinear.	0.5
	Question II	
1.a	$\frac{y}{40}=\frac{x}{60}$ then $2x=3y$ **OR** $\frac{40}{100}(x+y)=y$, $\frac{40}{100}x-\frac{60}{100}y=0$, so $2x=3y$.	0.75
1.b	The number of girls exceeds the number of boys by 5.	0.75
1.c	The two equations: $2x=3y$ and $x=y+5$ are written as the system: $\begin{cases}2x-3y=0\\ x-y=5\end{cases}$, the number of girls is 15 and the number of boys is 10.	0.75
2	The number of boys in section B is 10, let n be the number of girls. So $\frac{4}{9}(n+10)=n$. Then, in section B: the number of girls is 8.	0.75
	Question III	
1.a	$S_1=L\times w=(3x+4)(x+2)$ $S_2=\frac{h\times b}{2}=\frac{(3x+4)(3x+x+2)}{2}=\frac{(3x+4)(4x+2)}{2}=(3x+4)(2x+1)$	1
1.b	$S_2-S_1=(3x+4)(2x+1)-(3x+4)(x+2)=(3x+4)(x-1)$	0.5
1.c	$S_2=S_1$, then $S_2-S_1=0$, $(3x+4)(x-1)=0$, $x=-\frac{4}{3}$ (rejected) or $x=1$ (accepted). If x = 1, FG = GA = 3, then G is the midpoint of [FA], and (KG) is perpendicular to [FA] at G, then (KG) is the perpendicular bisector of [FA].	1
2.a	By Pythagoras theorem in the right triangle KGA we have: $KA^2=KG^2+GA^2=(3x+4)^2+9x^2=18x^2+24x+16$.	0.5
2.b	$(3x-2)(x+2)=3x^2+6x-2x-4=3x^2+4x-4$ verified.	0.5
2.c	$KA^2=40$, $18x^2+24x+16=40$, $18x^2+24x-24=0$, $6(3x^2+4x-4)=0$. Since $3x^2+4x-4=(3x-2)(x+2)$ (from (**2.b**)), then $x=\frac{2}{3}$ (accepted) or $x=-2$ (rej).	0.5

Question IV		
1.a	The coordinates of point A satisfy the equation of (d) ($y_A = x_A + 5$) then A is on (d).	0.25
1.b	B is the intersection of (d) with the axis $y'Oy$ so $x_B = 0$ and $y_B = 5$. $B(0;5)$	0.25
1.c		0.5
2.a	(d') is perpendicular to (d), slope of (d) × slope of (d') $= -1$, the equation of the line (d') is: $y = -x + b$.but (d') passes through $B(0;5)$ so (d') : $y = -x + 5$.	0.5
2.b	Coordinates of point the E verify the equation of (d'). We have $y_E = -x_E + 5$ and $y_E = 0$, then E is also on the $x'Ox$ axis.	0.5
2.c	Figure	0.25
3.a	I is the midpoint of [AE], so $x_I = \frac{x_A + x_E}{2} = 1$ and $y_I = \frac{y_A + y_E}{2} = 1$, so $I(1;1)$. Radius of the circle (C): $r = \frac{AE}{2} = \frac{\sqrt{(x_A - x_E)^2 + (y_A - y_E)^2}}{2} = \sqrt{17}$.	0.75
3.b	$IF = \sqrt{17} = r$.	0.25
3.c	F is on the circle and [AE] is a diameter, $\widehat{AFE} = 90°$ and $AF = FE = \sqrt{34}$, so AFE is a is a right isosceles triangle at F.	0.75
4	$\overrightarrow{FI} = \overrightarrow{EL}$ so $x_L - x_E = x_I - x_F$ then $x_L = 6$ similarly $y_L = 4$.and $L(6;4)$.	0.5
٥	$\overrightarrow{IG} = \overrightarrow{EL} = \overrightarrow{FI}$ so $IG = IF = r$, then G is on the circle (C).	0.5

Question V		
1		0.5
2	D is on (C) opposite to the diameter [AB] then $\widehat{ADB} = 90^{\circ}$. By using Pythagoras theorem: $AB^2=AD^2+BD^2$, then $AD=8$.	0.5
3.a	The two triangles ABD and BEF are right triangles and since (AB) and (EF) are parallel, then $\widehat{DBA} = \widehat{BEF}$ (corresponding angles). Then ABD and BEF are similar by two equal angles. The ratio of similarity: $\frac{ABD}{BEF}\Big\vert \frac{AB}{BE} = \frac{AD}{BF} = \frac{BD}{EF}$;	1
3.b	Since E is symmetric of D with respect to B then $BE = DB = 6$. By ratio of similarity: $\frac{AB}{BE} = \frac{BD}{FE}$, then $FE = \frac{BD\times BE}{AB} = \frac{6\times 6}{10} = 3.6$. $\frac{AB}{BE} = \frac{AD}{BF}$ then $\frac{10}{6} = \frac{8}{4.8}$ **OR** In the right triangle BEF: $BE^2=FB^2+FE^2$, then $FB = 4.8$.	1
4	$\widehat{EDG} = 90°$ and $\widehat{GFE} = 90°$, then D, G, F and E are on the same circle (C') whose diameter is [GE].	0.5
5.a	I is the midpoint of [GE] and B is the midpoint of [DE], by the midpoint theorem then (IB) // (DG).	0.5
5.b	$\widehat{LBD} = 90°$(angle opposite to the diameter [LD]) and $\widehat{GDB} = 90°$, (LB) // (DG) and (IB) // (DG). Then L, B and I are collinear.	0.5
6.a	In the right triangle ABD: $\tan \widehat{BAD} = \frac{BD}{AD} = \frac{6}{8} = 0.75$. In the right triangle ABG: $\tan \widehat{BAD} = \frac{BG}{AB} = \frac{BG}{10}$. By comparison: $\frac{BG}{10} = 0.75$ then $BG = 7.5$.	0.75
6.b	By using Pythagoras theorem in the triangle GFE: $GE^2=GF^2+FE^2 = (12.3)^2 + (3.6)^2 = \frac{675}{4}$, then $GE = \frac{3\sqrt{73}}{2}$ and the radius of the circle (C'): $r'=\frac{GE}{2} = \frac{3\sqrt{73}}{4}$	0.75