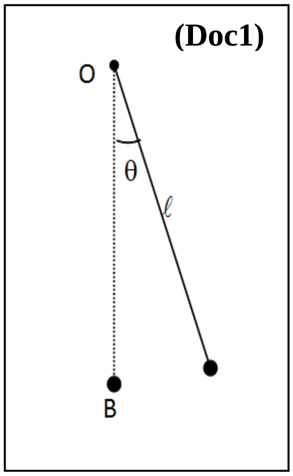
المادة: الفيزياء – لغة إنكليزية الشهادة: الثانوية العامّة الفرع: العلوم العامّة نموذج رقم: 1 / 2019 المدّة: ثلاث ساعات	الهيئة الأكاديميّة المشتركة قسم: العلوم	المركز التربوي للبحوث والإنماء

This test includes four mandatory exercises. The use of non-programmable calculators is allowed.

Exercise 1 (8 points) Conservation of mechanical energy

A simple pendulum (S) consists of an inextensible string, of negligible mass and of length $\ell = 1.0$ m, carrying, at one of its ends, a particle (M_1) of mass m = 0.10 kg, the other end being fixed, at O, to a fixed support (Doc 1). Let B be the position of (M_1) at equilibrium. Neglect any loss in energy and take the horizontal plane passing through B as the reference level for the gravitational potential energy.

Take: $g = 10\ m/s^2 = \pi^2\ m/s^2$; $\cos\theta \approx 1 - \frac{\theta^2}{2}$ for small angles θ, θ being in radian.



(Doc1)

1) The pendulum as an oscillator

We give (S), from its equilibrium position, the angular elongation $\theta_m = 10° = 0.175$ rad, then we release it without speed at the instant $t_0 = 0$.

At an instant t, the angular elongation of (S) is θ.

1-1) Calculate the mechanical energy of the system [(S), Earth] at the instant $t_0 = 0$.

1-2) Determine the absolute value of the angular velocity of (M_1) at B.

1-3) Derive the differential equation that describes the oscillations of (S).

1-4) Deduce the value T_0 of the proper period of the oscillations of (S).

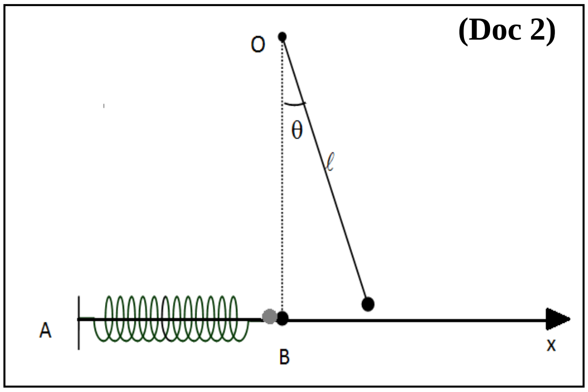
1-5) The solution of the differential equation is of the form: $\theta = \theta_m\cos(\omega_0 t + \varphi)$, (θ in rad and t in s). Determine the value of φ and write the expression of θ.

1-6) Determine the position and the linear speed of (M_1) at the instant $t = T_0/4$.

2) Collision

The pendulum (S) is held in equilibrium over a horizontal support AB (Doc 2). Neglect any loss in energy.

Another particle (M_2), of mass m' = m = 0.10 kg, attached to a spring of un-jointed turns, is placed against (M_1), the spring, of stiffness k, having its natural length. We shift the pendulum (S) by an angle of 10° and then we release it without speed at the instant $t_0 = 0$. When the pendulum passes through the equilibrium position B, (M_1) enters in a perfectly elastic collision with (M_2), all velocities being carried by the horizontal axis (Bx) (Doc 2).



(Doc 2)

2-1) Indicate the instant t_1 at which the first collision between (M_1) and (M_2) takes place.

2-2) Determine then the algebraic values V_1' and V_2' of the velocities $\overrightarrow{V_1'}$ and $\overrightarrow{V_2'}$ of (M_1) and (M_2), just after the collision.

3) Consecutive elastic collisions

3-1) Express the value of the maximum compression of the spring in terms of m, k and V_2'.

3-2) Deduce the proper period T'_0 of the horizontal elastic pendulum knowing that the two particles collide again at B at the instant $t_2 = \frac{3}{4}T_0$.

3-3) Deduce the instant t_3 at which the third collision will take place.

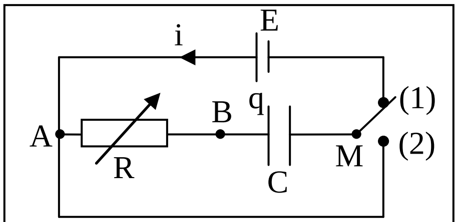
3-4) What can we conclude?

Exercise 2 (7 points) Air Humidity Sensor

In meteorology, we can measure the relative humidity rate of air, expressed in (%RH), by means of a capacitive sensor constituted by a capacitor whose capacity can vary with humidity.

1) Theoretical study

We set up the circuit represented in (Doc 3). The circuit is formed of an ideal generator of constant voltage E, a resistor of adjustable resistance R, a capacitive sensor represented by a capacitor of variable capacitance C and a switch K.



(Doc 3)

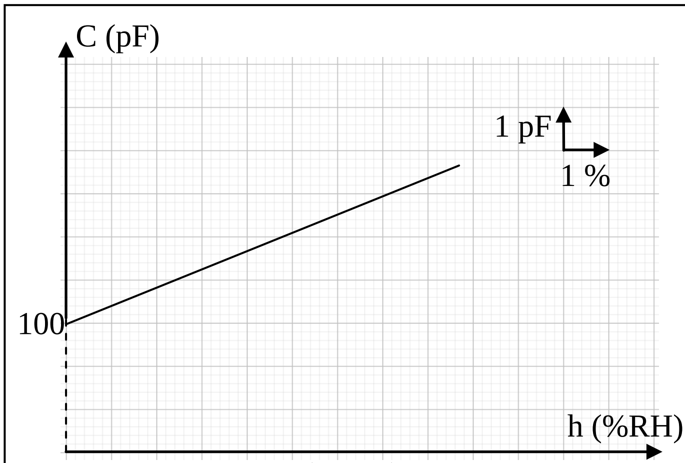
The capacitor being initially uncharged, the switch K is placed in position (1) at the instant $t_0 = 0$. At an instant t, the voltage across the capacitor is $u_C = u_{BM}$ and the circuit carries a current i. A suitable device records the variation of the voltage u_C as a function of time.

1-1) Derive the differential equation that governs the variation of the voltage u_C as a function of time.

1-2) The solution of this differential equation is given by: $u_C = A + B\, e^{-t/\tau}$. Determine the expressions of the constants A, B and τ in terms of E, R and C.

2) Measure of the humidity rate

The capacitance C of the sensor in the circuit varies with the relative humidity rate h of the air according to the graph of (Doc 4).



(Doc 4)

2-1) Determine the expression of C as a function of h.

2-2) In a first measurement, we find $h = h_1 = 75$ (%RH).

2-2-1) Calculate the value C_1 of C.

2-2-2) The document (Doc 5) shows the variation of u_C as a function of time t. The straight line (OT) represents the tangent to the curve $u_C(t)$ at the instant $t_0 = 0$. Determine the value R_1 of R.

2-3) In a second measurement, we find $h = h_2 = 50$ (%RH), C_2 being the capacitance of the sensor. We adjust the value of R so that the time constant τ of the circuit keeps the same value as that in the first measurement. R_2 is the value of R.

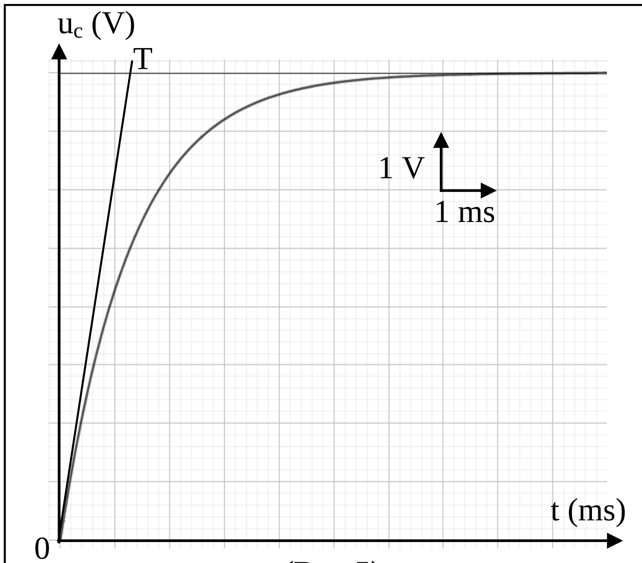
2-3-1) Determine R_2.

2-3-2) Draw the expression of the ratio C_2/C_1 in terms of R_1 and R_2.

2-3-3) The relation between C and h being linear, so by fixing the value of τ (that of (Doc 5)), the humidity rate will be a function of a single variable R; therefore, by adjusting R, we deduce h.

2-3-3-1) Show that: $h = \frac{1.3\times10^{9} - 100\,R}{0.4R}$
(R in Ω and h in (%RH)).

2-3-3-2) Deduce the value of h for $R = 10^{7}$ Ω.



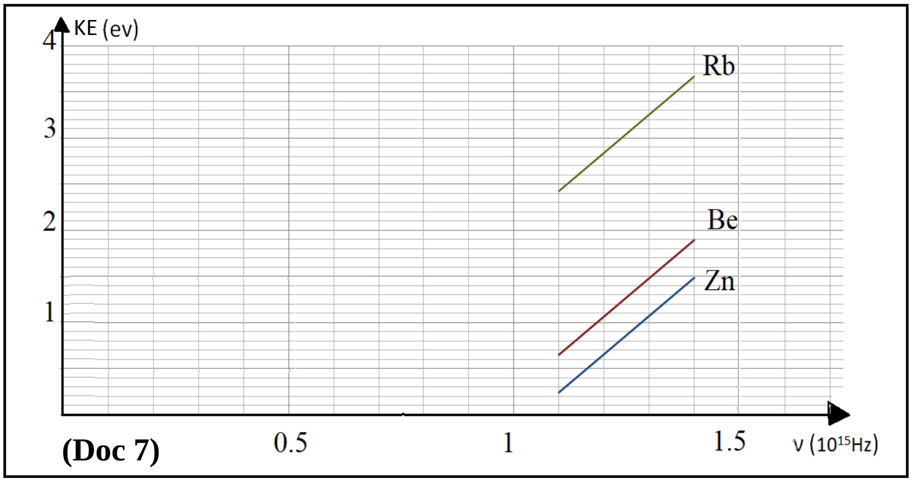
(Doc 5)

Exercise 3 (6½ points) Photoelectric effect

An experimenter uses a mono-energetic electromagnetic radiation source of adjustable frequency ν to illuminate, respectively, three metal plates, one in Zinc (Zn), another one in Beryllium (Be) and a third one in Rubidium (Rb).
The experimenter varies the frequency ν of the incident radiation and records, for each value of ν, the value of the maximum kinetic energy of an electron emitted by each of the three plates in the table (Doc 6).
He obtains the graph giving KE = f(ν) for each of the three plates, these graphs being represented in (Doc 7).
Take: 1 eV = 1.60×10^{-19} J; c = 3×10^{8} m/s.

ν (10^{15} Hz)	KE (eV)		
	Zn	Be	Rb
1.10	0.241	0.651	2.421
1.15	0.448	0.858	2.628
1.20	0.655	1.065	2.835
1.25	0.862	1.272	3.042
1.30	1.069	1.479	3.249
1.35	1.276	1.686	3.456
1.40	1.483	1.893	3.663

(Doc 6)



(Doc 7)

1) We notice that the photoelectric effect does not occur for some visible and infrared incident radiation regardless of the intensity of radiation and the duration of exposure. Why does this result defy the wave theory of light?
2) Indicate the aspect of light that the photoelectric effect phenomenon shows in evidence.
3) Interpret, based on Einstein's hypothesis relative to the photoelectric effect, the fact that the three graphs are parallel line segments.
4) Calculate, referring to (Doc 6), the value of Planck's constant.
5) Determine, referring to the graphs in (Doc 7), the threshold frequency of each of the metal plates.
6) Deduce the value of the extraction energy corresponding to each of the metal plates.
7) The experimenter illuminates each plate with an incident radiation of wavelength, in vacuum, 333 nm.
7-1) Specify, for each plate, whether there is an emission of electrons or not.
7-2) Calculate, in case we have an emission of electrons, the maximum kinetic energy of an emitted electron.

Exercise 4 (6 points) Chlorine Dating

Chlorine possesses several isotopes of which only three exist in the natural state, the $^{35}_{17}Cl$, the $^{37}_{17}Cl$ and the $^{36}_{17}Cl$; the first two are stable whereas the chlorine 36 is radioactive with half-life T = 3.08×10^{5} years.
In surface waters (seas, lakes), the chlorine 36 is constantly renewed and, as a result, the chlorine content 36, which is generally large, remains constant over time. This finding gives us a reference.
In the deep ice, several meters below the surface, the renewal is no longer happening and the proportion of chlorine 36 decreases with time.
The ice also contains carbon dioxide bubbles, these dioxides being formed by carbon atoms which are the isotopes $^{12}_{6}C$ (stable) and $^{14}_{6}C$ (radioactive). Once trapped, the carbon dioxides are not renewed, but geologists know that the amount of carbon 14 would be too low to be used in dating, its half-life T' = 5730 years being too short.

1) Radioactive chlorine $^{36}_{17}Cl$
1-1) Give:
1-1-1) The composition of the chlorine nucleus 36;
1-1-2) The definition of the word "isotope";

1-1-3) The definition of radioactivity.

1-2) The chlorine nucleus 36 undergoes a β^- disintegration and transforms into a stable $^{36}_{18}Ar$ nucleus with the emission of a γ radiation.

1-2-1) Write the equation of the disintegration of a chlorine nucleus 36 knowing that the β^- disintegration is always accompanied with the emission of an antiparticle.

1-2-2) Due to what is the emission of the γ radiation?

2) Geological dating of deep ice sheets by chlorine

We intend to determine the age t_1 of a sample of ice of mass m taken from a depth of several meters in the Antarctic and for which there is 60% of chlorine nuclei 36 compared with a recent sample of the same mass. Let N_0 and N be the numbers of chlorine nuclei 36 present in the sample, respectively at the instants $t_0 = 0$ and t, λ being the radioactive constant of chlorine 36.

2-1) Give the value of the ratio $\frac{N(t_1)}{N_0}$ for the studied piece of ice.

2-2) Show that the age of the sample is expressed by: $t_1 = -\frac{1}{\lambda}\ln\left(\frac{N(t_1)}{N_0}\right)$.

2-3) Calculate t_1.

2-4) Calculate the ratio $\frac{N(t_1)}{N_0}$ for carbon 14 and justify the cause for which geologists choose chlorine dating 36.

أسس التصحيح

Exercise 1 (8 points) Conservation of mechanical energy

Question	Answer	Mark
1-1	$ME_0 = KE_0 + PE_0 = 0 + mgh_0 = mg\ell(1 - \cos\theta_0) = 0.015$ J ($KE_0 = 0$ since $V_0 = 0$)	½
1-2	The mechanical energy of the system [(S), Earth] is conserved (no loss in energy), thus: $ME_B = ME_0$; $KE_B + PE_B = ME_0$, but $PE_B = 0$ hence: $\frac{1}{2} mV_B^2 = ME_0$ Knowing that $V_B = \ell\theta'_B$, we get: $\frac{1}{2} m\ell^2\theta'^2_B = ME_0$; $\lvert\theta'_B\rvert = 0.548$ rad/s	¾
1-3	$ME = KE + PE = \frac{1}{2} m\ell^2\theta'^2 + mg\ell(1-\cos\theta)$; with θ small, hence $ME = \frac{1}{2} m\ell^2\theta'^2 + mg\ell\theta^2/2$ ME = constant; $\frac{dE_m}{dt} = 0$; $m\ell^2\theta'\theta'' + mg\ell\theta'\theta = 0$ with θ' is not always null Thus: $\theta'' + \frac{g}{\ell}\theta = 0$ (θ in rad and t in s)	¾
1-4	This equation is of the form: $\theta'' + \omega_0^2\theta = 0$ (S) performs then a simple harmonic motion of proper angular frequency ω_0 where $\omega_0^2 = \frac{g}{\ell}$; So: $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{\ell}{g}} = 2$ s	¾
1-5	$\theta = \theta_m\cos(\omega_0 t + \varphi)$; $\theta' = -\omega_0\theta_m\sin(\omega_0 t + \varphi)$; $\theta'' = -\omega_0^2\theta_m\cos(\omega_0 t + \varphi)$ At $t_0 = 0$; $\theta'_0 = -\omega_0\theta_m \sin(\varphi) = 0$ But $\omega_0\theta_m \neq 0$ so $\sin\varphi = 0$; $\varphi = 0$ or $\varphi = \pi$ rad But: $\theta_0 = 0.175$ rad > 0 so $\theta_0 = \theta_m\cos(\varphi) > 0$ with $\theta_m > 0$ We get: $\cos\varphi > 0$ So: $\varphi = 0$ $\theta_m = \theta_0 = 0.175$ rad and $\omega_0 = \sqrt{\frac{g}{\ell}} = \pi$ rad/s The solution is then: $\theta = 0.175\cos(\pi t)$ (θ in rad and t in s)	¾
1-6	At $t = T_0/4$, (M_1) will be at B because $\theta(T_0/4) = 0$ $\lvert\theta'_B\rvert = 0.548$ rad/s ; We get: $\lvert V_B\rvert = \ell\lvert\theta'_B\rvert = 0.548$ m/s	¾
2-1	The first collision takes place at $t_1 = T_0/4 = 0.5$ s	¼
2-2	The collision being elastic, the linear momentum and the kinetic energy of the system $[(M_1), (M_2)]$ are conserved. The system $[(M_1), (M_2)]$ is mechanically isolated (during the collision) because we neglect all the external forces with respect to those due to the collision. Hence $\sum \vec{F}_{ex} = \vec{0}$; $\frac{d\vec{P}}{dt} = \vec{0}$; $\vec{P}_{before} = \vec{P}_{after}$; $m\vec{V}_1 = m\vec{V}'_1 + m\vec{V}'_2$; $\vec{V}_1 = \vec{V}'_1 + \vec{V}'_2$; The three vectors carried by (Bx) are collinear. Algebraically: $V_1 = V'_1 + V'_2$ and $V_1 = -\lvert V_B\rvert$ (1) (the motion of (M_1) is in the negative direction) The collision is elastic: $KE_{before} = KE_{after}$; $\frac{1}{2} mV_1^2 = \frac{1}{2} mV'^2_1 + 1/2mV'^2_2$ $V_1^2 = V'^2_1 + V'^2_2$ (2) the resolution of system (1) and (2) gives $V'_1 = 0$ and $V'_2 = -0.548$ m/s	1½

3-1	The mechanical energy of the system [(M_2), spring, Earth] is conserved (no loss in energy), between the initial instant just after the collision and the final instant corresponding to the maximum compression. $ME(x_0 = 0) = ME(X_m)$; $KE_0 + PEe_0 = KE_m + PEe_m$; $\frac{1}{2} mV'^2_2 = \frac{1}{2} kX_m^2$ (because at maximum compression, the speed is null). $\lvert X_m \rvert = \lvert V'_2 \rvert \sqrt{\frac{m}{k}}$	¾
3-2	The duration between the first two shocks is ½ T'_0 (duration of a half-oscillation of the elastic pendulum) ; As a result : $t_2 - t_1 = \frac{3}{4} T_0 - \frac{1}{4} T_0 = = \frac{1}{2} T_0 = \frac{1}{2} T'_0$; $T'_0 = T_0 = 2$ s	½
3-3	The time between the second and the third collision is: ½ T_0 Hence: $t_3 = t_2 + \frac{1}{2} T_0 = \frac{3}{4} T_0 + \frac{1}{2} T_0 = 5/4\ T_0$	½
3-4	Collisions are repeated regularly (periodically) for each $T_0/2$.	¼

Exercise 2 (7 points) Air Humidity Sensor

Question	Answer	Mark
1-1	$u_{AM} = u_{AB} + u_{BM}$; $E = Ri + u_C$ with $i = \frac{dq}{dt} = C\frac{du_c}{dt}$; $E = RC\frac{du_c}{dt} + u_C$	**1**
1-2	$u_C = A + Be^{-\frac{t}{\tau}}$ $\frac{du_C}{dt} = -\frac{B}{\tau} e^{-\frac{t}{\tau}}$; Therefore $E = -RC\frac{B}{\tau} e^{-\frac{t}{\tau}} + A + Be^{-\frac{t}{\tau}} \quad \forall t$ $B e^{-\frac{t}{\tau}}\left(-\frac{RC}{\tau} + 1\right) + A - E = 0\ \forall t$ By identification: $-\frac{RC}{\tau} + 1 = 0$ et $A - E = 0$ $\tau = RC$ and $A = E$ and at the instant $t_0 = 0$; $u_{C0} = 0 = A + B$ So, B = -A = -E	**1**
2-1	C is a linear function of h: C = ah + b ; For h = 0 ; C = b = 100 pF (according to the graph) $a = \frac{\Delta C}{\Delta h} = \frac{2}{5} = 0.4\ pF/(\%RH)$ Therefore $C = 0.4h + 100$ (h in (%RH) and C in pF)	**1**
2-2-1	$C_1 = 0.4 \times 75 + 100 = 130$ pF	¼
2-2-2	According to the graph, the tangent (OT) meets the asymptote at a point of abscissa τ; So τ = 1.3 ms ; $\tau = R_1C_1$; $R_1 = \tau/C_1 = 10^7\ \Omega$	**1**
2-3-1	$C_2 = 0.4h_2 + 100 = 0.4 \times 50 + 100 = 120$ pF $R_2 = \frac{\tau}{C_2} = \frac{1.3\times10^{-3}}{120\times10^{-12}} = 1.08 \times 10^7\Omega$	¾
2-3-2	$\frac{C_2}{C_1} = \frac{\frac{\tau}{R_2}}{\frac{\tau}{R_1}} = \frac{R_1}{R_2}$	½
2-3-3-1	$C = (0.4h + 100) \times 10^{-12}$ (h in (%RH) and C in F) or $C = \frac{\tau}{R} = \frac{1.3 \times 10^{-3}}{R} = (0.4h + 100) \times 10^{-12}$ $h = \frac{1.3\times10^9 - 100R}{0.4R}$ (h in (%RH) and R in Ω)	**1**
2-3-3-2	For $R = 10^7\ \Omega$; $h = \frac{1.3\times10^9 - 100\times10^7}{0.4\times10^7} = 75$ (%RH)	½

Exercise 3 (6½ points) Photoelectric effect

Question	Answer	Mark
1	According to the wave theory, the wave gives energy in a continuous way which means that whatever is the frequency of the incident radiation, a continuous and prolonged lighting of the metal must produce a photoelectric emission which is not the case.	½
2	The corpuscular aspect.	¼
3	According to Einstein's hypothesis: $E_{photon} = h\nu = W_0 + KE(e^-)$ $KE = h\nu - W_0$; $KE(e^-)$ is a linear function of ν with a slope h for any metal. (The extraction energy W_0, which depends on the metal, is the ordinate at the origin).	**1**
4	$h = \frac{\Delta E_c}{\Delta \nu} = \frac{(1.483 - 0.241) \times 1{,}6 \times 10^{-19}}{1.4 \times 10^{15} - 1.1 \times 10^{15}} = 6.624 \times 10^{-34} J.s$	**1**
5	The threshold frequency corresponds to an extraction without kinetic energy; KE(e-) = 0 Extending each segment of the graph, the intersection with the ν axis corresponds to ν_0 For the Zinc: $\nu_0 = 1.05 \times 10^{15}$ Hz For the Beryllium: $\nu_0 = 0.95 \times 10^{15}$ Hz For the Rubidium: $\nu_0 = 0.50 \times 10^{15}$ Hz	**1¼**
6	$W_0 = h\nu_0$ For the Zinc: $W_0 = 6.96 \times 10^{-19}$ J For the Beryllium: $W_0 = 6.29 \times 10^{-19}$ J For the Rubidium: $W_0 = 3.31 \times 10^{-19}$ J	**1**
7-1	$\nu = c/\lambda = 0.9 \times 10^{15}$Hz; To have an extraction, the frequency ν of the incident radiation must verify the condition: $\nu > \nu_0$. So, there is no extraction for the Zn and Be plates, but there is one for the Rb plate.	**1**
7-2	For the Rb : $h\nu = W_0 + KE(e-)$; $KE_{(e^-)} = h\nu - W_0 = 2.66$ J	½

Exercise 4 (6 points) Chlorine Dating

Question	Answer	Mark
1-1-1	17 protons ; 36 – 17 = 19 neutrons	½
1-1-2	Isotopes are nuclei, of the same element, having the same charge number but of different mass numbers.	½
1-1-3	Radioactivity is a spontaneous transformation of an unstable nucleus into another more stable one.	½
1-2-1	${}^{36}_{17}Cl \longrightarrow {}^{36}_{18}Ar + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \gamma$	½
1-2-2	The daughter nucleus ${}^{36}_{18}Ar$ is obtained in an excited state and it may stay there for a very short duration. After that, it undergoes a downward transition thus emitting a γ ray.	½
2-1	$\frac{N(t_1)}{N_0} = \frac{60}{100} = 0.60$	½
2-2	The law of radioactive decay is written: $N = N_0 e^{-\lambda t}$ $\ln(N) = \ln(N_0 e^{-\lambda t}) = \ln(N_0) - \lambda t$ $\ln(N) - \ln(N_0) = -\lambda t$; $\ln\left(\frac{N}{N_0}\right) = -\lambda t$ $t = -\frac{1}{\lambda}\ln\left(\frac{N}{N_0}\right)$; $t_1 = -\frac{1}{\lambda}\ln\left(\frac{N(t_1)}{N_0}\right)$	**1**
2-3	The radioactive constant is written: $\lambda = \frac{\ln(2)}{T} = 2.25 \times 10^{-6} an^{-1}$ The age of the sample: $t_1 = -\frac{\ln(0,6)}{2.25 \times 10^{-6}} = 2.27 \times 10^5 ans$	**1**
2-4	For the carbon nuclei present at instant t_1, the ratio is written: $\frac{N}{N_0} = e^{-\frac{\ln(2)t}{T}} = e^{-\frac{0.693 \times 2.27 \times 10^5}{5730}} = 1.19 \times 10^{-12}$ This ratio is too small, so the number of the remaining carbon nuclei in the sample is too small.	**1**