


المادة: رياضيات – لغة إنكليزية الشهادة: الثانوية العامة الفرع: الآداب والانسانيات نموذج رقم: 1 / 2019 المدة: ساعة واحدة	الهيئة الأكاديمية المشتركة قسم: الرياضيات	
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ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (10 points)

Let f be the function defined over \mathbb{R} as $f(x) = \frac{4}{x^2 + 2x + 2}$.

Denote by (C) its representative curve in an orthonormal system (O, \vec{i}, \vec{j}) .

1- a) Determine $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.

Deduce that (C) has an asymptote.

b) For all x in \mathbb{R} , prove that (C) is above its asymptote.

c) Determine the coordinates of A and B , the meeting points of (C) and the line with equation $y = 2$; ($x_A < 0$).

2- a) Show that $f'(x) = \frac{-8(x+1)}{(x^2 + 2x + 2)^2}$, then set up the table of variations of f .

b) Calculate $f(-3)$ and $f(1)$, then draw (C) .

3- S is the vertex of (C) .

a) Prove that (SA) is tangent to (C) at A .

b) Solve graphically $f(x) < 2$.

c) Write an equation of (T) , the tangent at B to (C) .

Verify that (T) is passing through S .

4- Let g be the function defined as $g(x) = ax + \frac{b}{x-1}$. (C') is the representative curve of g in the same system as that of (C) .

Calculate a and b so that (C') is tangent at B to (C) .

5- In what follows, let $a = -4$ and $b = -2$.

a) Determine the domain of definition of g .

b) Determine the asymptotes for (C') .

II- (5 points)

1- Solve the system $\begin{cases} x - y = 4 \\ 2x - 3y = 0 \end{cases}$

2- A bag contains red and blue pens.

The blue pens are four more than the red pens, and the red pens represent $\frac{2}{5}$ the total of pens.

a) Show that this text is modeled by the system given in part 1.

b) Determine the number of blue pens and that of red pens.

3- The red pen worths twice the blue pen and these two pens worth together 5 250 LL.
Find the price of each pen.

III- (5 points)

The following table represents the students' distribution in three sections of grade 10.

	Section A	Section C	Section D	Total
Boys	15		12	
Girls		17		
Total	25	30		80

1- Complete the table given above.

2- One student is randomly selected from these three sections.

Consider the following events:

A: The student selected is from the section A.

C: The student selected is from the section C.

D: The student selected is from the section D.

B: The student selected is a boy.

G: The student selected is a girl.

a) Show that $P(D) = \frac{5}{16}$.

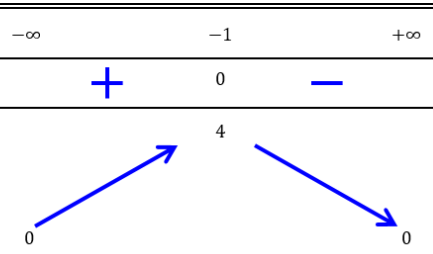
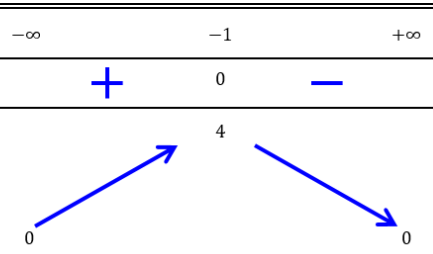
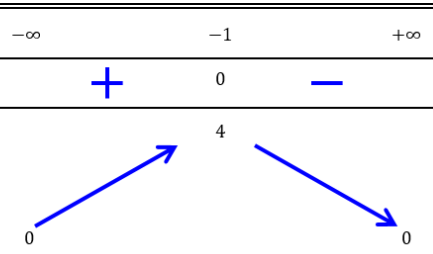
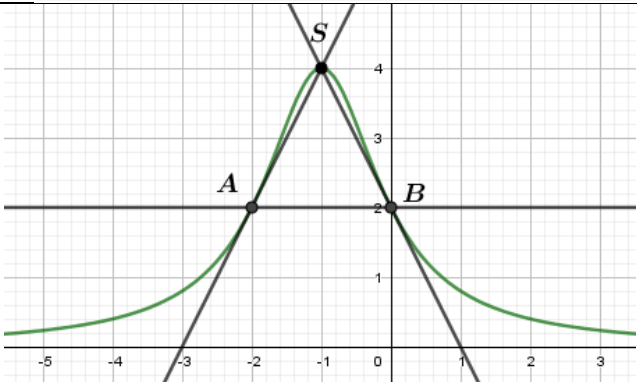
b) Calculate the following probabilities:

$$P(D \cap G), P(D \cup G), P(C/B), P(G/A).$$

c) Knowing that the student selected is from section A or D, calculate the probability that he is a boy.

2- Two students from grade 10 are randomly selected one after another to meet the director so that the first student selected does not come back to his section.

Calculate the probability that one student is from section A and the second student is a boy not from section A.

<u>QI</u>	<u>Answers</u>	<u>pts</u>												
1-a)	$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$ then $(x'x)$ is an asymptote to (C) .	1												
b)	$f(x) > 0$, then (C) is above $(x'x)$.	0.5												
c)	$f(x) = 2$, then $x^2 + 2x = 0$ thus $x = 0$ or $x = -2$. Hence $A(-2, 2)$ and $B(0, 2)$.	1												
2-a)	$f'(x) = \frac{-8(x+1)}{(x^2 + 2x + 2)^2}$ <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">$-\infty$</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">$+\infty$</td> </tr> <tr> <td style="padding: 5px;">$f'(x)$</td> <td style="padding: 5px; text-align: center;">+</td> <td style="padding: 5px; text-align: center;">0</td> <td style="padding: 5px; text-align: center;">-</td> </tr> <tr> <td style="padding: 5px;">$f(x)$</td> <td colspan="3" style="padding: 5px; text-align: center;">  </td> </tr> </table>	x	$-\infty$	-1	$+\infty$	$f'(x)$	+	0	-	$f(x)$				1.5
x	$-\infty$	-1	$+\infty$											
$f'(x)$	+	0	-											
$f(x)$														
b)	$f(-3) = \frac{4}{5} = f(1)$. 	1.5												
3-a)	Slope $(SA) = 2$ and $f'(-2) = 2$. Then (SA) is tangent to (C) at A .	0.75												
b)	$f(x) < 2$, consider the part of (C) below (AB) . thus $x < -2$ or $x > 0$.	0.5												
c)	$f'(0) = -2$; $(T) : y = -2x + 2$ and S satisfies the equation of (T) .	0.75												
4-	$g(x) = ax + \frac{b}{x-1}$ but $B(0, 2)$ is on (C') then $b = -2$. $g'(x) = a - \frac{b}{(x-1)^2}$ then $g'(0) = a + 2 = -2$ then $a = -4$.	1												
5-a)	$D_g =]-\infty, 1[\cup]1, +\infty[$	0.5												
b)	The equations of asymptotes for (C') are $x = 1$ since $\lim_{x \rightarrow 1} g(x) = \pm \infty$ and $y = -4x$ because $\lim_{x \rightarrow +\infty} (g(x) - 4x) = \lim_{x \rightarrow +\infty} \frac{2}{x-1} = 0$	1												

QII	Answers	pts
1-	$\begin{cases} x - y = 4 \\ 2x - 3y = 0 \end{cases}$ then $x = 12$ and $y = 8$	1
2-a)	$x =$ number of the blue pens $y =$ number of red pens Then $x = y + 4$ thus $x - y = 4$ And $5y = 2x + 2y$ then $2x - 3y = 0$	1
b)	The number of the blue pens = 12 the number of the red pens = 8	1
3-	$a =$ the price of blue pen, then $2a =$ the price of red pen. $a + 2a = 5250$ then $3a = 5250$ then $a = 1750$ LL and $2a = 3500$ LL	2

QIII		pts																				
1-	<table border="1"> <thead> <tr> <th></th> <th>A</th> <th>C</th> <th>D</th> <th>total</th> </tr> </thead> <tbody> <tr> <th>Boys</th> <td>15</td> <td>13</td> <td>12</td> <td>40</td> </tr> <tr> <th>Girls</th> <td>10</td> <td>17</td> <td>13</td> <td>40</td> </tr> <tr> <th>Total</th> <td>25</td> <td>30</td> <td>25</td> <td>80</td> </tr> </tbody> </table>		A	C	D	total	Boys	15	13	12	40	Girls	10	17	13	40	Total	25	30	25	80	1
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a)	$P(D) = \frac{25}{80} = \frac{5}{16}$	0.5																				
b)	$P(D \cap G) = \frac{13}{80}$ $P(D \cup G) = P(D) + P(G) - P(D \cap G) = \frac{25}{80} + \frac{40}{80} - \frac{13}{80} = \frac{52}{80} = \frac{13}{20}$ $P(C/B) = \frac{\text{number of boys in C}}{\text{total number of boys}} = \frac{13}{40}$ $P(G/A) = \frac{\text{number of girls in A}}{\text{total number of students in A}} = \frac{2}{5}$ $P(B/A \cup D) = \frac{\text{number of boys in A or D}}{\text{number of students in A or D}} = \frac{27}{50}$	2																				
3-	$P(\text{Student from A and Boy from } \bar{A}) = P(A, \text{Boy } \bar{A}) + P(\text{Boy } \bar{A}, A) =$ $\frac{25}{80} \times \frac{25}{79} + \frac{25}{80} \times \frac{25}{79} = \frac{125}{632}$	1.5																				