المادة: رياضيات- لغة انكليزية الشهادة: الثانوية العامة الفرع: علوم الحياة نموذج رقم: 2/ 2019 المدّة: ساعتان	الهيئة الأكاديميّة المشتركة قسم: الرياضيات	المركز التربوي للبحوث والإنماء

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, with justification, the answer corresponding to it.

In these exercise Z and Z' are two complex numbers

No	Question	a	b	c
1	If Z is real, then an argument for $Z(iZ+\bar{Z})$ is:	$\frac{\pi}{4}$	0	$\frac{\pi}{2}$
2	If $M(Z)$, $M'(Z')$, $A(-1)$ and $B(2i)$ are 4 points on the complex plane such that $Z'=\frac{Z+1}{2i-Z}$, then $AM'\times BM=$	$\sqrt{AB}$	$2AB$	AB
3	If $M(Z)$, $M'(Z')$ and $A(2i)$ are 3 points on the complex plane such that $Z'=\overline{(Z-2i)^2}$ and Z' is real negative, then $M(Z)$ moves on:	$y'Oy$	the line: $y=2$	$x'Ox$
4	If Z and Z' are two nonzero complex numbers such that $Z'=\frac{2i-3}{\bar{Z}}$, then $Z\times\overline{Z'}=$	$2i+3$	$-2i-3$	$-2i+3$

II- (4 points)

In the space referred to an orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the point $A(1,2,0)$ and the two lines (d) and (d') with parametric equations:

$(d)\begin{cases}x=t+1\\y=2\\z=-2t\end{cases}$ and $(d')\begin{cases}x=-m+4\\y=3\\z=2m-1\end{cases}$ where m and t are two real parameters.

1- a) Show that A is on (d) but not on (d').

b) Verify that (d) and (d') are parallel.

2- Let (P) be the plane determined by (d) and (d').

Prove that $2x-5y+z+8=0$ is an equation of (P).

3- Consider in the plane (P) the circle (C) which is tangent to (d) at A.

a) The center I of circle (C) is on (d'), calculate the coordinates of I.

b) Show that the radius of (C) is equal to $\sqrt{6}$ units of length.

4- (C) intersects (d') at E and F, and (Δ) is the line passing through A and perpendicular to (P).

a) Verify that, for any point L on (d) the area of triangle LEF is a constant to be calculated.

b) Write the parametric equations for (Δ).

c) Determine a point B on (Δ) so that the volume of tetrahedron $BLEF$ is equal to $2\sqrt{30}$ units of volume.

III- (4 points)
An urn contains 3 white balls and 7 black balls.
A game runs as follows:
The player chooses randomly one ball from this urn.
If this ball is black, then the game ends.
If the ball is white, then the player does not replace it back in the urn and he chooses a second ball.
This process will continue until a black ball is chosen and the game ends.
The player wins 10 000 LL for each white ball and nothing for the black ball.

Part A
1- Calculate the probability that the player wins exactly 10 000 LL.
2- Calculate the probability that the player wins at least 10 000 LL.
3- Knowing that the first chosen ball is white, what is the probability that he wins 30 000 LL?

Part B
Let X be the random variable that is equal to the amount gained by the player.
1- Find the four possible values of X.
2- Prove that P(X= 30 000) = $\frac{1}{120}$.
3- Find the probability distribution and the expected value of X.
4- Suppose that there are 10 players during each day of April. Calculate the expected total amount gained by all the players.

IV- (8 points)
Part A
Consider the function g defined over $\mathbb{R}$ as $g(x) = 1 + (1 - x)e^x$.
1- a) Determine $\lim\limits_{x\to-\infty} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.
b) Calculate $g'(x)$ and set up the table of variations of g.

2- a) Prove that the equation $g(x) = 0$ has only one root α, then verify that $1.27 < \alpha < 1.28$.
b) Discuss according to the values of x, the sign of $g(x)$.

Part B
Consider the function f defined over $\mathbb{R}$ as: $f(x) = (2 - x)e^x + x - 2$, and denote by (C) its representative curve in an orthonormal system (O, $\vec{i}, \vec{j}$).

1- a) Determine $\lim\limits_{x\to+\infty} f(x)$ and calculate $f(2.5)$.
b) Determine $\lim\limits_{x\to-\infty} f(x)$ and prove that the straight line (d) with equation $y = x - 2$ is an asymptote to (C).
c) Study the relative positions of (C) & (d).

2- a) Verify that $f'(x) = g(x)$, then set up the table of variations of f.
b) Show that $f(\alpha) = \frac{(\alpha-2)^2}{\alpha-1}$.

3- Show that the origin is an inflection point for (C).

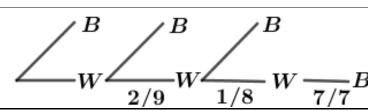
4- Draw (C) and (d).

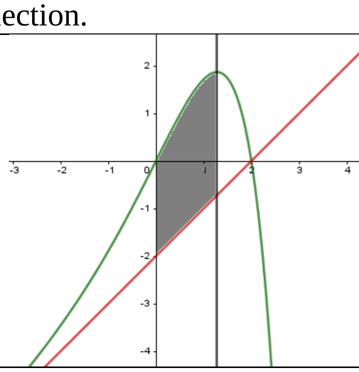
5- Denote by A the area bounded by (C), (d), (yy') and the line with equation $x = \alpha$.
Show that $A = \frac{6-4\alpha}{\alpha-1}$ units of area.

أسس التصحيح

Q I	Answers	pts
1	a is correct since Z is real then $\bar{Z} = Z$ then $Z\,(i\,Z\,+\,\bar{Z}\,)\,=\,Z\,(i\,Z\,+\,Z\,) = \,Z^2\,(1+i)$ and $\arg(Z^2(1+i)) = 0 + \frac{\pi}{4} + 2k\pi$.	1
2	c is correct since $AM' \times BM = \left\lvert\frac{Z+1}{2i-Z} + 1\right\rvert \times \lvert Z - 2i\rvert = \frac{\lvert 1+2i\rvert}{\lvert (Z-2i)\rvert} \times \lvert Z-2i\rvert = \lvert 1+2i\rvert = AB$.	1
3	a is correct since Z is –ve real then $(Z-2i)^2 = -ve\ real$ so $(Z-2i)$ is pure imaginary, Z is pure imaginary.	1
4	b is correct since $Z\ \times \overline{Z'} = \ \overline{2\imath - 3} = -2i - 3$.	1

Q II	Answers	pts
1-a	For $t = 0, A$ belongs to (d); $y_A = 2 \neq y_{(d')} = 3$ then A does not belong to (d').	0.5
1-b	$\vec{V}_{(d)} = -\vec{V}_{(d')}$ & A belongs to (d) and does not belong to (d') then (d) is parallel to (d').	0.5
2	Substitute their parametric equations in (P): $2(\,t+1) - 10 - 2t + 8 = 0,\ (d) \subset (P)$ and $2(-m+4) - 15 + 2m - 1 + 8 = 0, (d') \subset (P)$. **Or** $\overrightarrow{AM}.\left(\vec{V}_d \times \overrightarrow{AG}\right) = 0$, with G a certain point on (d').	0.5
3-a	$I(-m+4, 3, 2m-1)$; $\overrightarrow{AI}.\vec{V}_{(d)} = 0 \Rightarrow -m + 3 - 4m + 2 = 0 \Rightarrow I(3,3,1)$.	0.5
3-b	The radius of (C) is $AI = \sqrt{6}$ units of length.	0.25
4-a	The height issued from L to (EF) is constant since $(d)$and $(d')$are parallel and $[EF]$ is diameter. Thus, the area is constant and it is equal to $\frac{2\sqrt{6}\sqrt{6}}{2} = 6$ units of area.	0.75
4-b	(Δ): $\overrightarrow{AM} = \alpha\vec{N}_{(P)}$ (Δ):$\begin{cases} x = 2\alpha + 1 \\ y = -5\alpha + 2 \\ z = \alpha \end{cases}$	0.25
4-c	$B(2\alpha+1, -5\alpha+2, \alpha)$; $2\sqrt{30} = \frac{Area\Delta LEF \times BA}{3} = 6$ then $\sqrt{30} = \sqrt{30\alpha^2} \Rightarrow \alpha = \pm 1$ then $B(3,-3,1)$ or $B(-1,7,-1)$.	0.75

<table>
<tr><th>QIII</th><th>Answers</th><th>pts</th></tr>
<tr><td>A-1</td><td>$P(10\,000)$ = P(WB)= $\frac{3}{10} \times \frac{7}{9} = \frac{7}{30}$.</td><td>0.5</td></tr>
<tr><td>A-2</td><td>$P(A)$ =1- $P(0) = 1 - \frac{7}{10} = \frac{3}{10}$ where A is the event that the player wins at least 10 000LL.</td><td>0.5</td></tr>
<tr><td>A-3</td><td>$P(^{30\,000}/_{W}) = \frac{2}{9} \times \frac{1}{8} \times \frac{7}{7} = \frac{2}{72}$ where W is the event that the first ball is white.

</td><td>0.75</td></tr>
<tr><td>B-1</td><td>$\Omega_X = \{\,0; 10\,000; 20\,000; 30\,000\}$.</td><td>0.25</td></tr>
<tr><td>B-2</td><td>$P(30\,000)$ = P(WWWB)= $\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} \times \frac{7}{7} = \frac{1}{120}$.</td><td>0.25</td></tr>
<tr><td>3</td><td>
<table>
<tr><td>X_i</td><td>0</td><td>10 000</td><td>20 000</td><td>30 000</td><td>Total</td></tr>
<tr><td>P_i</td><td>$\frac{84}{120}$</td><td>$\frac{28}{120}$</td><td>$\frac{7}{120}$</td><td>$\frac{1}{120}$</td><td>1</td></tr>
<tr><td>P_iX_i</td><td>0</td><td>$\frac{280\,000}{120}$</td><td>$\frac{140\,000}{120}$</td><td>$\frac{30\,000}{120}$</td><td>E(X)= 3750</td></tr>
</table>
</td><td>1.25</td></tr>
<tr><td>4</td><td>Total amount = $10 \times 30 \times 3\,750 = 1\,125\,000$ LL.</td><td>0.5</td></tr>
</table>

QIV	Answers	pts
A-1-a	$\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to-\infty} 1+\frac{(1-x)}{e^{-x}} \overset{L'H}{=} \lim\limits_{x\to-\infty} 1+\frac{-1}{-e^{-x}} = 1$; $\lim\limits_{x\to+\infty} g(x) = -\infty$	0.75
A-1-b	g'(x) = $-xe^{x}$ With $e^{x}>0$ $\begin{array}{cccccc} x & -\infty & & 0 & & +\infty \\ g' & & + & & - & \\ g(x) & 1 & \nearrow & 2 & \searrow & -\infty \end{array}$	0.5
A-2-a	Over]-∞, 0] $g(x)>0$ *then* $g(x)=0$ has no solution in this interval. In $[0,+\infty[$ g is continuous and strictly decreasing, and changes its sign from positive to negative thus $g(x)=0$ has one solution α in this interval. Consequently, $g(x)=0$ has only one root α over ℝ. $g(1.27)\times g(1.28) = 0.04\times(-0.01) < 0 \Longrightarrow 1.27<\alpha<1.28$.	0.75
A-2-b	By referring to the table of variations, over]-∞, α[$g(x)>0$, over]$\alpha,+\infty$[$g(x)<0$	0.5
B-1-a	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} x[\left(\frac{2}{x}-1\right)e^{x}+1-\frac{2}{x}] = -\infty$ and $f(2.5) = -5.59$.	0.5
B-1-b	$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \frac{(2-x)}{e^{-x}} + x - 2 \overset{L'H}{=} \lim\limits_{x\to-\infty} \frac{1}{e^{-x}} + x - 2 = -\infty$ $\lim\limits_{x\to-\infty} [f(x)-(x-2)] = \lim\limits_{x\to-\infty} \frac{2-x}{e^{-x}} = \lim\limits_{x\to-\infty} \frac{+1}{e^{-x}} = 0 \Rightarrow (d)$: $y = x - 2$ is an asymptote.	0.75
B-1-c	$y_{(C)} - y_{(d)} = (2-x)e^{x}$ With $e^{x}>0$ $\begin{array}{cccccc} x & -\infty & & 2 & & +\infty \\ y_c - y_d & & + & 0 & - & \end{array}$ Relative position: (C) above (d) | (C) below (d) (C) intersects (d)at A(2,0)	0.5
B-2-a	$f'(x) = -e^{x} + (2-x)e^{x} + 1 = g(x)$ $\begin{array}{cccccc} x & -\infty & & \alpha & & +\infty \\ f' & & + & & - & \\ f(x) & -\infty & \nearrow & f(\alpha) & \searrow & -\infty \end{array}$	0.5
B-2-b	*f(α)*= $(2-\alpha)e^{\alpha}+\alpha-2$ but *g(α)*= 0 then $(1-\alpha)e^{\alpha} = -1$ so $e^{\alpha} = -\frac{1}{(1-\alpha)}$ *f(α)*= $e^{\alpha}-1+\alpha-2 = \frac{(\alpha-2)^2}{\alpha-1}$.	0.5
B-3	$f''(x) = g'(x) = -xe^{x}$, then f '' (0) = 0, and changes its sign at x = 0 from positive to negative & f(0) = 0, thus O is a point of inflection.	0.5
B-4		1.25
B-5	$A = \int_o^{\alpha}(2-x)e^{x}dx = (2-x)e^{x}+e^{x}]_o^{\alpha}$ "By parts" so $A = (3-\alpha)e^{\alpha} - 3 = \frac{6-4\alpha}{\alpha-1}$ units of area.	1