ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $(O, \vec{i}, \vec{j}, \vec{k})$, consider the plane (P) with equation: $x + y + z - 1 = 0$, and the straight lines (d) and (d') with equations:

$$(d)\begin{cases} x = t - 1 \\ y = t + 1 \\ z = -2t + 1 \end{cases} \quad \text{and} \quad (d')\begin{cases} x = -1 \\ y = m + 1 \\ z = 2m - 2 \end{cases}$$ where m and t are two real parameters.

1- a) Verify that (d) is contained in (P).
   b) Calculate the coordinates of I, the meeting point of (d') and (P).
   c) Show that (d) and (d') are noncoplanar (skew).

2- Let (Q) be the plane containing (d') and perpendicular to (P), and denote by (Δ), the intersection line of (P) and (Q).
   a) Show that $x - 2y + z + 5 = 0$ is an equation of (Q).
   b) Write a system of parametric equations for (Δ).
   c) Show that (d) and (Δ) intersect at $E(0, 2, -1)$.

3- Let F be the point on (d) so that $\overrightarrow{IE}.\overrightarrow{IF} = \frac{1}{2}$.
   a) Calculate the coordinates of F.
   b) Show that IEF is a semi equilateral triangle.

II- (4 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points $M(Z)$, $M'(Z')$, $I(1 + 2i)$, and $E(5)$. The complex numbers Z and Z' are so that: $Z' = 2iZ + 5$.

1- a) If Z is pure imaginary, prove that Z' is real.
   b) If $Z' = 5i\sqrt{3}$, write Z in exponential form.

2- a) Prove that $Z_{\overrightarrow{IM'}} = 2iZ_{\overrightarrow{IM}}$.
   b) Express IM' in terms of IM and show that $(\overrightarrow{IM}, \overrightarrow{IM'}) = \frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$.
   c) Deduce that if M moves on the line (Δ) with equation $(x = 1)$, then M' moves on a line whose equation is to be determined.

3- Let $Z = x + iy$ and $Z' = x' + iy'$ where $x, y, x', and\ y'$ are real numbers.
   a) Express x' and y' in terms of y and x.
   b) If $x + 2y = 5$, prove that (MM') is parallel to $(y'y)$. Then use the result $(\overrightarrow{IM}, \overrightarrow{IM'}) = \frac{\pi}{2} + 2k\pi$ to construct M' when $x + 2y = 5$.
   c) If M' moves on the circle (C') with center E and radius 2, prove that M moves on the circle (C) with center O and radius 1.

III- (4 points)

Below are the results of a survey conducted on a sample of 500 persons:

- 70% of the persons are women
- 300 of the women are on diet
- 80% of the participants in the survey are on diet.

A) A person is chosen randomly from the above sample. Consider the following events:

W: "the chosen person is a woman"

D: "the chosen person is on diet"

1- Prove that $P(D/W) = \frac{6}{7}\cdot$

2- a) Calculate $P(D \cap W)$, and deduce $P(D \cap \overline{W})$.

b) Prove that $P\left(D/\overline{W}\right) = \frac{2}{3}\cdot$

3- Knowing that the chosen person is not on diet, prove that the probability that this person is a man is 0.5.

B) In this part, two persons are chosen randomly and simultaneously from the group of persons that **are not on diet**. Let *X* be the random variable that is equal to the number of men chosen.

1- Prove that $P(X=2) = \frac{49}{198}\cdot$

2- Determine the probability distribution of *X*.

3- If X designates the number of women chosen, would the probability distribution change? Justify.

IV- (8 points)

Let *f* be the function defined over $\mathbb{R}$ as $f(x) = 2 - \frac{4e^x}{1+e^x}\cdot$ Denote by (C) its representative curve in an orthonormal system $(O;\ \vec{i},\vec{j})$.

1- a) Determine $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$, then deduce that (C) has two asymptotes.

b) Prove that *f* is an odd function and interpret graphically the result thus obtained.

2- a) Calculate $f'(x)$ and set up the table of variations of *f*.

b) Write an equation of (T), the tangent to (C) at *O*.

c) Draw (T) and (C).

3- a) Prove that *f* has an inverse function g.

b) Determine the domain of definition of g, then express $g(x)$ as function of *x*.

c) Prove that the graph (C') of g is tangent at *O* to (C). Then draw (C') in the same system as that of (C).

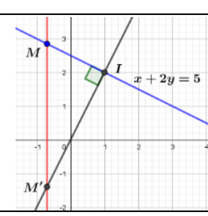
4- Let (D) be the region bounded by (C'), $(y'y)$ and the line with equation $y = a$ where $a > 0$.

a) Calculate in terms of *a* the area of (D).

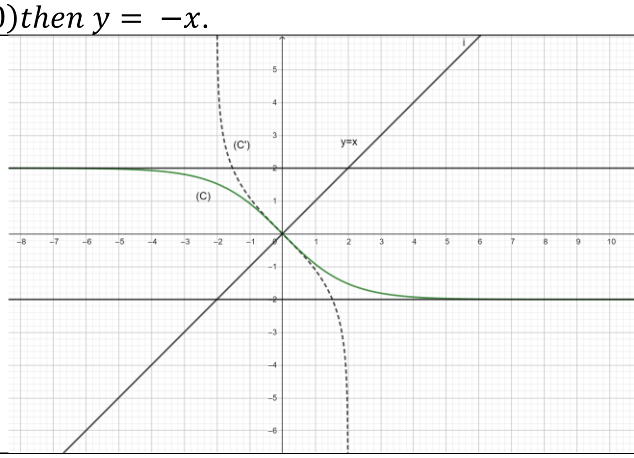
b) Calculate *a* so that this area is equal to *4Ln2* unit of area.

أسس التصحيح

Q I	Answers	Pts
1-a	Substitute parametric equations of *(d)* in *(P)*. *t–1+t+1–2t+1–1=0* then *(d)*⊂ *(P)*	0.25
1-b	Substitute parametric equations of *(d')* in *(P)*. *–1+ m + 1 + 2m–2–1= 0* then *m=1*, and *(d')* intersects *(P)* at *I(–1,2,0)*.	0.5
1-c	$\vec{V}_d \neq \alpha \vec{V}_{d'}$ skew or intersecting, but *I* is in *(P)* and belongs to*(d')* but does not belong to *(d)* then *(d)* & *(d')* are skew. **Or** solve system of 3 equations with two unknowns “no solution”	0.5
2-a	Substitute parametric equations of *(d')* in *(Q)*. *–1–2m –2 + 2m–2+5= 0* then *(d')* ⊂ *(Q)* $\vec{N}_{(Q)}.\vec{N}_P = 0$ then they are perpendicular. **Or** $\overrightarrow{IM}.\left(\vec{V}_{d'} \times \vec{n}_{(p)}\right) = 0$	0.5
2-b	$\vec{V}_{(\Delta)} = \vec{N}_{(P)} \times \vec{N}_{(Q)} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 3\vec{i} - 3\vec{k}$ and $I \in (\Delta)$ then $\overrightarrow{IM} = \alpha \vec{V}_{(\Delta)}$. $(\Delta): \begin{cases} x = 3\alpha - 1 \\ y = 2 \\ z = -3\alpha \end{cases}$	0.5
2-c	Substitute coordinates of *E* in *(d)*. then *t=1* ⇒ *E* ∈ *(d)*. and for $\alpha = \frac{1}{3}$ $E \in (\Delta)$.	0.5
3-a	$F \in (d) \Rightarrow F(t-1, t+1, -2t+1), \overrightarrow{IF}(t, t-1, -2t+1), \overrightarrow{IE}(1, 0, -1)$ $\overrightarrow{IF}.\overrightarrow{IE} = \frac{1}{2} \Rightarrow t = \frac{1}{2} \Rightarrow F(\frac{-1}{2}, \frac{3}{2}, 0)$,	0.5
3-b	$\overrightarrow{IF}\left(\frac{1}{2}, \frac{-1}{2}, 0\right)$ and $\overrightarrow{IF}.\overrightarrow{IE} = \frac{1}{2}$ then $IE.IF\cos(\overrightarrow{IF}, \overrightarrow{IE}) = \frac{1}{2}$ so $\cos(\overrightarrow{IF}, \overrightarrow{IE}) = \frac{1}{2}$ then $\widehat{EIF} = \frac{\pi}{3}$ *and* $\overrightarrow{FE}\left(\frac{1}{2}, \frac{1}{2}, -1\right)$, $\overrightarrow{FI}.\overrightarrow{FE} = 0$ then $\widehat{IFE} = \frac{\pi}{2}$ Thus *IEF* is a semi equilateral triangle right at F.	0.75

Q II	Short Answers	Pts
1-a	If Z is pure imaginary, then $Z = yi$ where y is a nonzero real number. $Z' = 2i(yi) + 5 = 5 - 2y$ which is real.	0.5
1-b	$Z' = 5i\sqrt{3}$ then $Z = \frac{5i\sqrt{3}-5}{2i} = \frac{5\sqrt{3}}{2} + \frac{5}{2}i = 5e^{\frac{\pi}{6}i}$	0.5
2-a	$Z_{\overrightarrow{IM'}} = Z' - Z_I = 2iZ + 4 - 2i = 2i(Z - 1 - 2i) = 2iZ_{\overrightarrow{IM}}$	0.25
2-b	$\frac{Z_{\overrightarrow{IM'}}}{Z_{\overrightarrow{IM}}} = 2i$ then: $IM' = 2IM$ and $(\overrightarrow{IM}, \overrightarrow{IM'}) = \frac{\pi}{2} + 2k\pi$	0.5
2-c	M moves on a line (Δ) passing through I, and $\left(\overrightarrow{IM}, \overrightarrow{IM'}\right) = \frac{\pi}{2} + 2k\pi$ then M'moves on a straight line (L) passing through I and ⊥ (Δ) with an equation (L) : y=2.	0.5
3-a	$x' + iy' = 2i(x + iy) + 5 = 5 - 2y + 2xi$. $x' = 5 - 2y$ and $y' = 2x$.	0.5
3-b	If $x + 2y = 5$, then $x = 5 - 2y$ and $x' = x$ so $(MM') \nearrow\nearrow (y'y)$. Since M is the point on the line (d) with equation $x + 2y = 5$ and passing through I. Then M' is the point of intersection between the perpendicular to (d) drawn through I and line through M and parallel to $(y'y)$. 	0.75
3-c	$EM' = 2 \Longrightarrow (x' - 5)^2 + y'^2 = 4 \Longrightarrow x^2 + y^2 = 1 \Longrightarrow M$ moves on the circle *(C)* of center *O* & radius 1.	0.5

Q III	Answers	Pts
A-1	The number of women is $0.7 \times 500 = 350$. The number of women that are on diet is 300, thus $P(D/W) = \frac{300}{350} = \frac{6}{7}$	0.5
A-2a	$P(D \cap W) = P(W) \times P(D/W) = 0.7 \times \frac{6}{7} = 0.6$ $P(D) = P(W \cap D) + P(\overline{W} \cap D)$; $0.8 = 0.6 + P(\overline{W} \cap D)$ so $P(\overline{W} \cap D) = 0.2$	0.75
A-2b	$P\left(D/_{\overline{W}}\right) = \frac{P(\overline{W} \cap D)}{P(\overline{W})} = \frac{0.2}{0.3} = \frac{2}{3}$	0.5
A-3	$P(D/\overline{W}) = \frac{2}{3}$ and $P(\overline{D}/\overline{W}) = \frac{1}{3}$ then $P(\overline{W}/\overline{D}) = \frac{P(\overline{W} \cap \overline{D})}{P(\overline{D})} = \frac{0.3 \times \frac{1}{3}}{0.2} = \frac{1}{2}$	0.75
B-1	$P(X=2) = \frac{C_{50}^2}{C_{100}^2} = \frac{49}{198}$	0.5
B-2	$P(X=0) = \frac{C_{50}^2}{C_{100}^2} = \frac{49}{198}$; $P(X=1) = \frac{C_{50}^1 \times C_{50}^1}{C_{100}^2} = \frac{50}{99}$	0.75
B-3	The probability distribution of X would not change since the number of women who are not on diet is the same as that of men.	0.25

Q IV	Answers	Pts
1-a	$\lim_{x \to -\infty} f(x) = 2$ *then* $y = 2$ *H.A.* $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} 2 - \frac{4}{1+e^{-x}} = \lim_{x \to +\infty} 2 - 4 = -2$ *then* $y = -2$ *H.A.*	1
1-b	The domain is centered at 0 and $f(-x) = 2 - \frac{4e^{-x}}{1+e^{-x}} = 2 - \frac{4}{1+e^{x}} = \frac{2e^x - 2}{1+e^x}$ $-f(x) = -2 + \frac{4e^x}{1+e^x} = \frac{2e^x-2}{1+e^x} = f(-x)$; then *(O)* is the center of symmetry of (C).	0.75
2-a	$f'(x) = \frac{-4[e^x(e^x+1) - e^x e^x]}{(1+e^x)^2} = \frac{-4e^x}{(1+e^x)^2} < 0$ x: $-\infty$ … $+\infty$ f': $-$ $f(x)$: $+2$ ↘ -2	0.75
2-b	$y - f(0) = f'(0)(x - 0)$ *then* $y = -x$.	0.25
2-c		1
3-a	f is continuous and strictly decreasing over $\mathbb{R}$ then it admits an inverse g.	0.25
3-b	$D_g = R_f =]-2, 2[$; $y = 2 - \frac{4e^x}{1+e^x} \Longrightarrow y + ye^x = 2 + 2e^x - 4e^x$ *so* $e^x(y+2) = 2 - y$ then $e^x = \frac{2-y}{2+y} \Longrightarrow x = Ln(\frac{2-y}{2+y}) \Longrightarrow g(x) = Ln(\frac{2-x}{2+x})$	1.25
3-c	The tangent to *(C)* at *O* is symmetric to itself with respect to *y=x*, thus it is tangent to (C'), which means that (C') is tangent to (C) at O.(C') see figure.	0.75
4-a	$A = \int_o^a \left(-2 + \frac{4e^x}{1+e^x}\right) dx = 4Ln(1+e^x) - 2x]_o^a = 4Ln(1+e^a) - 4Ln2 - 2a$ unit of area.	1
4-b	$4Ln(1+e^a) - 4Ln2 - 2a = 4Ln2 \Longrightarrow 2Ln\left(\frac{1+e^a}{4}\right) = a$ then $e^{2a} - 14e^a + 1 = 0$ *then* $a = Ln(7 + 4\sqrt{3})$	1