ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (1.5 point)

Answer true or false and explain.

1) Let z be a complex number, if $z = re^{i\theta}$ with $r > 0$ and $0 < \theta < \frac{\pi}{2}$, and $z' = \frac{-\bar{z}^2}{isin\theta cos\theta}$, then $\frac{\pi}{2} + \theta$ is an argument of z'.
2) If a complex number $z \neq -i$ is so that $\left|\frac{2z-4i}{\bar{z}-i}\right| = 2$, then z is real.
3) The solution set of the inequality $ln(e^{-2x} - 2e^{-x} + 1) \leq 0$ is $[-ln2\,;\, +\infty[$.
4) If g is the function defined as $g(x) = ln\, x$ and f is another function so that $D_f =]-\infty, 1[$. Then the domain of definition of the function fog is $]0, e[$.
5) If $I_n = \int_0^1 x^n e^x dx$, then $I_{n+1} = e - (n+1)\, I_n$.

II- (2.5 point)

In the space referred to a direct orthonormal system $\left(O; \vec{i}, \vec{j}, \vec{k}\right)$, consider the points *E(1,–1, 2)*,

F(0,–6,0), *L (3,–1,1)* and the line (d) with equations $\begin{cases} x = 2m+1 \\ y = -1 \\ z = -m+2 \end{cases}$ where m is a real parameter.

Let (P) be the plane determined by O and (d) and let (Δ) be the line through F and parallel to (d).

1) a- Verify that E and L are on (d).

   b-Show that $x + 5y + 2z = 0$ is an equation of (P).

2) Write the parametric equations for the line (Δ).
3) Denote by (Q) the plane determined by (d) and (Δ).

   a-Calculate $\overrightarrow{FE} \wedge \overrightarrow{FL}$, then deduce a normal vector to (Q).

   b-Verify that (P) and (Q) are perpendicular.

   c- Prove that F is the orthogonal projection of E on (Δ).

4) Consider in the plane (Q) the parabola (C) with focus F and directrix (d).

   Determine the coordinates of A and B, the meeting points of (C) and (Δ).

5) Show that the volume of tetrahedron $AOEL$ is equal to the volume of tetrahedron $FOEL$, then calculate this volume.

III- (2 point)

Consider two urns:

Urn U contains 10 cards: 3 marked with letter A; 5 marked with letter B; and 2 marked with letter C.

Urn V contains 6 balls: 2 of them are red and 4 are green.

Part A

A player plays a game according to the following rules:

The player starts the game by drawing one card from U.

- If the card is marked A, then he draws two balls from V one after another with replacement
- If the card is marked B, then he draws two balls from V one after another without replacement
- If the card is marked C, then one red ball is added to urn V, and the player draws two balls simultaneously from V.

The player wins the game if the two balls drawn from V are red.

Consider the events:

A: The card drawn from U is marked A.

B: The card drawn from U is marked B.

C: The card drawn from U is marked C.

G: The player wins the game.

1) Calculate $P({}^{G}/_{A})$. Deduce that $P(G \cap A) = \frac{1}{30}$.
2) Prove that $P(G \cap C) = \frac{1}{35}$.
3) Prove that $P(G) = \frac{2}{21}$.
4) Assume that the player lost the game, find the probability that a card marked A or B was selected from U.

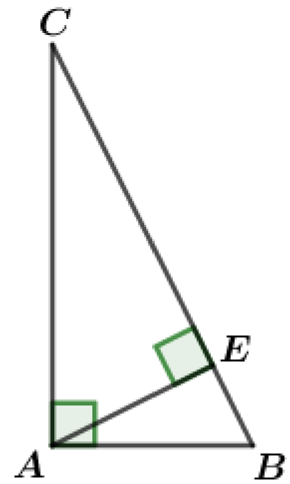
Part B.

In this part, consider only urn V, and assume that n red balls ($n \geq 1$) are added to it, then two balls are drawn simultaneously from urn V.

Consider the event D: The two balls have distinct colors.

1) Show that $P(D) = \frac{8(n+2)}{n^2+11n+30}$.
2) Can you find n so that the probability of selecting two balls of the same color is equal to that of selecting two balls with different colors? Justify.

IV- (4 point)



In the next figure,

- Triangle *ABC* is right at *A*
- *AB* =2, *AC* = 4
- [*AE*] is an altitude in the triangle *ABC*
- Let *S* be the similitude that maps *A* onto *B* and *E* onto *C*

Part A.

1) Determine an angle of *S* and show that its scale factor is $\frac{5}{2}$.
2) Let *F = S(B)* and *L = S(C).*
   a-Construct F.
   b-Show that *L* is the meeting point of (*CF*) and (*AB*).
3) a-Construct *(d) = S(AF)*, then determine *S*(*d*).
   b-Deduce that the center *I* of *S* is the meeting point of (*d*) and (*AF*).
4) Let h be the dilation that maps *F* onto *A* and *B* onto *C*.
   a-Determine the center *J* of *h*, then verify that the scale factor of *h* is $-\frac{4}{5}$.
   b-Construct *G = h(L).*
5) a-Determine the nature of *Soh*.
   b-Show that *C* is the center of *Soh*.
   c-Deduce that *E* is the center of *hoS*.

Part B

The complex plane is referred to the system $(A;\vec{u},\vec{v})$ with $\vec{u}=\frac{1}{2}\overrightarrow{AB}$ and $\vec{v}=\frac{1}{4}\overrightarrow{AC}$.

1) a-Write the complex form of *hoS*.
   b-Deduce z_E.
2) Determine *hoS(C)*, then find z_G.
3) Determine the nature of the quadrilateral *LAGC*.

V- (4 point)

In the complex plane $(O;\vec{u},\vec{v})$, consider the points M, M', I and B so that
$z_M = z,\ z_{M'} = z',\ z_I = 1$ and $z_B = -1$.
The two complex numbers z and z' are so that $z' = z^2 - 2z$.

1) a) Verify that: $(z'+1) = (z-1)^2$.
   b) If *M* moves on a circle *(C)* with center I and radius IB, prove that *M'* moves on a circle *(C′)* with center and radius to be determined.
2) Let $z = x + iy$ and $z' = x' + iy'$, where x, y, x' and y' are real numbers.
   a-Show that $x' = x^2 - y^2 - 2x$ and $y' = 2y(x-1)$.
   b-Write a relation between *x* and *y* so that *z'* is pure imaginary.
3) a- if *z'* is pure imaginary, prove that *M* moves on a rectangular hyperbola (*H*) with center *I*.
   b-Determine the vertices and the asymptotes of (*H*).
   c-Draw (*H*).
4) Consider the two points *L* and *G* so that: $z_L = i\sqrt{3}$ and $z_G = 1+\frac{\sqrt{6}}{2}+i\frac{\sqrt{2}}{2}$.
   a- Show that z_L and z_G satisfy the relation $z_L + 1 = (z_G - 1)^2$.
   b- Deduce that *G* is on (*H*).
   c- Prove that the tangent at *G* to (*H*) is parallel to (*BL*).

5) Let (E) be the ellipse with vertices O, $A(2,0)$ and $K(1,3)$.
   a-Write an equation of (E).
   b-Prove that (E) is tangent to (BL) at J so that $x_J = \frac{1}{2}$.
6) The lines (BL) and (IK) intersect at P.
   Calculate the area of the region inside the triangle (BIP) and outside (E).

VI- (6 point)

Part A.

Let g be the function defined over $]0,+\infty[$ as $g(x) = x + \ln x - 1$.

1) a- Determine $\lim\limits_{x\to 0^+} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$.
   b- Calculate $g'(x)$ and set up the table of variations of g.
2) Calculate $g(1)$, then discuss according to x the sign of $g(x)$.

Part B.

Let f be the function defined over $]0,+\infty[$ as $f(x) = \left(\frac{1}{x} - 1\right) \ln x$.

Denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to +\infty} f(x)$. Deduce an asymptote to (C).
   b- Determine $\lim\limits_{x\to +\infty} \frac{f(x)}{x}$. Interpret graphically the result obtained.
2) a-Show that $f'(x) = \frac{-g(x)}{x^2}$, then set up the table of variations of f.
   b-Draw (C).
3) a- For $x \in]0, 1]$, prove that f has an inverse function h.
   b-Determine the domain of h, then draw the graph (C') of h in the same system as that of (C).
4) Let A be the area of the region bounded by (C'), $y'y$ and the line (d) with equation $y = \alpha$ where $0 < \alpha < 1$. Determine α so that $A = (\alpha - \alpha \ln\alpha)$ units of area.
5) Let p be the function defined as $p(x) = \ln(\alpha - h(x))$ with $\alpha = e^{-\sqrt{2}}$.
   a-Prove that the domain of definition p is $]-\infty; \sqrt{2}(1 - e^{\sqrt{2}})[$.
   b-Determine the limits of p at the boundaries of its domain.

Part C.

Let (U_n) be the sequence defined as $U_n = e^{f(n)}$ with $n \in \mathbb{N}$ and $n \geq 1$.

1) a-Show that (U_n) is strictly decreasing
   b- Show that (U_n) has a lower bound.
   c- Deduce that (U_n) is convergent, then find its limit.
2) Show that $U_n = n^{\frac{1-n}{n}}$.

أسس تصحيح

QI	Answers	pts
1-	Arg $(z')=\pi-2\theta-\frac{\pi}{2}=\frac{\pi}{2}-2\theta\ [2\pi]$ (F).	0.5
2-	$\lvert z-2i\rvert=\lvert\bar{z}-i\rvert$ then $x^2+(y-2)^2=x^2+(y+1)^2$ therefore $y=\frac{1}{2}$ (F).	0.5
3-	$e^{-2x}-2e^{-2x}+1>0$ then $(e^{-x}-1)^2>0$ therefore $x\neq 0$ and $e^{-2x}-2e^{-x}+1\leq 1$ then $e^{-x}(e^{-x}-2)\leq 0$ thus $x\geq -\ln 2$. D = $[-\ln 2;\ 0[\cup]0;+\infty[$. (F) .	1
4-	$x\in D_g$ then $x>0$ and $\ln x<1$; $x<e$. $D_{fog}=]0,e[$ (T) .	0.5
5-	$I_{n+1}=e-(n+1)I_n$ (Integration by parts) (T).	0.5

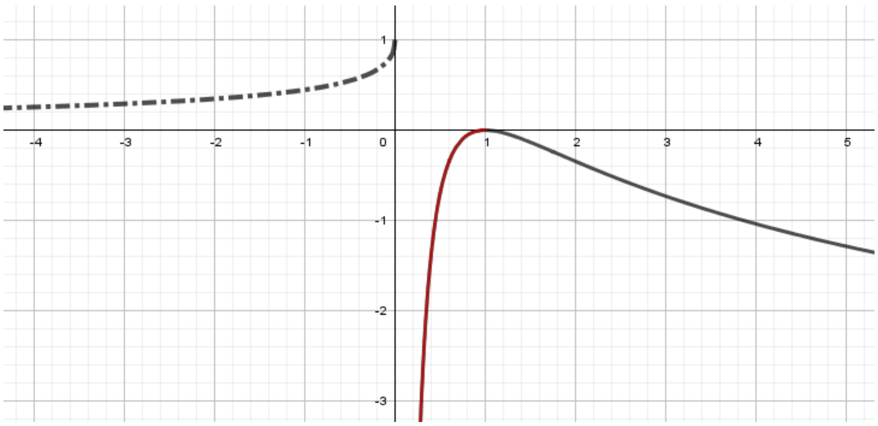
QIII	Answers	pts
1-a)	For $m=0$, $E(1,-1,2)$ is on (d) and for $m=1$, $L(3,-1,1)$ is on (d) .	0.5
b)	$\overrightarrow{OM}.(\overrightarrow{OE}\wedge\overrightarrow{V_d})=0$; $x+5y+2z=0$	0.5
2-	(Δ) : $x=2k$, $y=-6$, $z=-k$ where k is real.	0.5
3-a-	$(Q)=(LEF)$ and $\overrightarrow{n_Q}=\overrightarrow{FE}\wedge\overrightarrow{FL}=-5\vec{i}+5\vec{j}-10\vec{k}$	0.5
b-	$\overrightarrow{n_P}.\overrightarrow{n_Q}=0$ then (P) is perpendicular to (Q).	0.25
c-	F is on (Δ) and $\overrightarrow{FE}.\overrightarrow{V_\Delta}=0$	0.75
4)	A and B are on (C), then $AF=BF=FE=\sqrt{30}$ Therefore $5k^2=30; k^2=6$ and $k=\pm\sqrt{6}$ $A(2\sqrt{6},-6,-\sqrt{6})$ and $B(-2\sqrt{6},-6,\sqrt{6})$	1
5)	(Δ) is parallel to (P) then for any point M on (Δ) the volume of $MOEL$ is the same. Then the volume $(AOEL)$ = volume $(FOEL)$ = $\frac{1}{6}\begin{vmatrix}0 & -6 & 0\\1 & -1 & 2\\3 & -1 & 1\end{vmatrix}=5\ units\ of\ volume$	1

QIII	Answers	pts
	Part A .	
1-	$P(G/A)=\frac{2}{6}\times\frac{2}{6}=\frac{1}{9}$. $P(G\cap A)=P(A)\times P(G/A)=\frac{3}{10}\times\frac{1}{9}=\frac{1}{30}$.	0.5
2-	$P(G\cap C)=P(C)\times P(G/C)=\frac{2}{10}\times\frac{C_3^2}{C_7^2}=\frac{1}{35}$.	0.5
3-	$P(G\cap B)=P(B)\times P(G/B)=\frac{5}{10}\times\frac{2}{6}\times\frac{1}{5}=\frac{1}{30}$ $P(G)=\frac{1}{30}+\frac{1}{35}+\frac{1}{30}=\frac{1}{15}+\frac{1}{35}=\frac{7+3}{105}=\frac{10}{105}=\frac{2}{21}$.	1
4-	$P((A\cup B)/\overline{G})=\frac{P[(A\cup B)\cap\overline{G}]}{P(\overline{G})}=\frac{P(A\cap\overline{G})+P(B\cap\overline{G})}{P(\overline{G})}=\frac{\frac{3}{10}\times\frac{8}{9}+\frac{5}{10}\times\frac{14}{15}}{\frac{19}{21}}=\frac{\frac{11}{15}}{\frac{19}{21}}=\frac{77}{95}$.	1
	Part B .	
1-	$P(D)=P(\text{two different colors })=\frac{4(n+2)}{C_{n+6}^2}=\frac{8(n+2)}{n^2+11n+30}$	0.5
2-	$P(\overline{D})=1-P(D)$ then $\frac{16(n+2)}{n^2+11n+30}=1$ no solutions.	0.5

QIV	Answers	pts
	Part A .	
1-	$(\overrightarrow{AE},\overrightarrow{BC})=\frac{\pi}{2}+2k\pi$; $K=\frac{BC}{AE}=\frac{EC+EB}{AE}=\tan\hat{B}+\tan\hat{C}=2+\frac{1}{2}=\frac{5}{2}$	0.75
2-a)	F = The intersection of the perpendicular at C to (BC) and the perpendicular at B to (AB).	0.5
b)	Since (CL) is ⊥ to (EC) and (BL) is ⊥ to (AC) then $L=(AB)\cap(CF)$.	0.5
3-a)	(d) = line through B and perpendicular to (AF) ; S(d) = (AF).	0.5
b)	S((d) ∩ (AF)) = S(d) ∩ S(AF) = (AF) ∩ (d) = I (center)	0.5
4-a)	J is on (BC)& (AF)therefore $J=(BC)\cap(AF)$; $\overrightarrow{AC}=k\,\overrightarrow{FB}$ but $FB=\frac{5}{2}AB=5$;k $=-\frac{4}{5}$;	1
b)	G = (L J) ∩ parallel through C to (AB).	0.5
5-a)	Soh = S' (? , $\frac{4}{5}\times\frac{5}{2}=2$, $-\pi+\frac{\pi}{2}=-\frac{\pi}{2}$)	0.5
b)	C is on (FL) and h(FL) = (AG) then h(C) = E , but S(E) = C therefore Soh(C) = C C center of Soh.	0.75
c)	$C\xrightarrow{h}E\xrightarrow{S}C$ then $E\xrightarrow{S}C\xrightarrow{h}E$ hence, E center of hoS.	0.5
	Part B .	
1-a)	hoS (E , 2 , $-\frac{\pi}{2}$) ; $z'=-2iz+b$. hoS(A) = C then 4i = 0 + b and $z'=-2iz+4i$.	0.5
b)	$z_E(1+2i)=4i$ then $z_E=\frac{8}{5}+\frac{4}{5}i$	0.5
2	hoS(C) = h(S(C) = h(L) = G. $z_G=-2iz_C+4i=-2i(4i)+4i=8+2i$	0.5
3-	$(AG)=h(FL)$ then (AG) is parallel to (CL) and since (CG) is parallel to (AB) thus $LAGC$ is a parallelogram.	0.5

QV	Answers	Pts
1-a)	$z' = z^2 - 2z = (z-1)^2 - 1$ then $z'+1 = (z-1)^2$.	0.5
b)	$IM = 2$, then $BM' = 4$ and M' moves on the circle with center B and radius 4.	0.5
2-a)	$z = x+iy$ and $z' = x'+iy'$ then $x' = x^2 - y^2 - 2x$ and $y' = 2y(x-1)$.	0.5
b)	z' pure imaginary, then $x^2 - y^2 - 2x = 0$ and $y(x-1) \neq 0$. ($y \neq 0$, $x \neq 1$.	0.5
3-a)	$(x-1)^2 - y^2 = 1$, equation of rectangular hyperbola with center $I(1,0)$.	0.75
b)	Vertices $O(0,0)$ and $A(2,0)$. Asymptotes: $y = x-1$ and $y = -x+1$.	1
4-a)	$z_L + 1 = 1 + i\sqrt{3}$. $(z_G - 1)^2 = (\frac{\sqrt{6}}{2} + \frac{i\sqrt{2}}{2})^2 = 1 + i\sqrt{3} = z_L + 1$.	0.5
b)	Since z_L pure imaginary, then G is on (H).	0.5
c)	$2x - 2yy' - 2 = 0$ then $y' = \frac{x-1}{y} = \frac{\frac{\sqrt{6}}{2}}{\frac{\sqrt{2}}{2}} = \sqrt{3} = slope(BL)$	0.5
5-a)	$\frac{(x-1)^2}{1} + \frac{y^2}{9} = 1$	0.75
b)	(BL): $y = \sqrt{3}(x+1)$. Replace in (E): $(x-1)^2 + \frac{1}{3}(x+1)^2 = 1$ then $4x^2 - 4x + 1 = 0$. $(2x-1)^2 = 0$ then (BL) tangent to (E) at $J(\frac{1}{2}, \frac{3\sqrt{3}}{2})$.	1
6-	$P(1, 2\sqrt{3})$. Area = Area of the triangle of BIP - $\frac{1}{4}$ area of (E). $= \frac{2\times 2\sqrt{3}}{2} - \frac{1}{4}(\pi \times 1 \times 3) = 2\sqrt{3} - \frac{3\pi}{4}$ *units of area*	1

QVI	Answers	pts
	Part A .	
1)a-	$\lim_{x\to 0} g(x) = -\infty$ and $\lim_{x\to +\infty} g(x) = +\infty$	0.5
b-	$g'(x) = 1 + \frac{1}{x} > 0$. x : 0 … $+\infty$ $g'(x)$: + $g(x)$: ↗	1
2)	$g(1) = 0$ then $g(x) > 0$ for $x > 1$.	0.5
	Part B .	
	$f(x) = (\frac{1-x}{x}) \ln x$	
1)a-	$\lim_{x\to 0} f(x) = -\infty$ and $\lim_{x\to +\infty} f(x) = -\infty$ Then $y'y$ is an asymptote to (C)	0.75
b-	$\lim_{x\to +\infty} \frac{f(x)}{x} = 0$ then (C) admits a parabolic branch parallel to $(x'x)$.	0.75

2)a-	$f'(x) = \frac{-g(x)}{x^2}$. x: 0, 1, $+\infty$ $f'(x)$: (undefined at 0), $+$, 0, $-$ $f(x)$: $-\infty$ ↗ 0 ↘ $-\infty$	1.25
b-		1
3) a-	$x \in]0,1]$, f is continuous and strictly increasing then it has an inverse function h.	0.5
b-	$D_h = R_f =]-\infty, 0]$ see the figure	0.75
4)	A= Area bounded by (C), $x'x$, $x = \alpha$, $x = 1$ $A = \int_1^\alpha f(x)\,dx = \left[\frac{1}{2}ln^2x - xlnx + x\right]_1^\alpha = \frac{1}{2}ln^2\alpha - \alpha ln\alpha + \alpha - 1$ If $A = \alpha - \alpha ln\alpha$, then $\frac{1}{2}ln^2\alpha - 1 = 0$; $ln^2\alpha = 2$ $ln\alpha = \sqrt{2}$ (rej); $ln\alpha = -\sqrt{2}$ and $\alpha = e^{-\sqrt{2}}$.	1
5) a-	$f(\alpha) = \left(e^{\sqrt{2}} - 1\right)\left(-\sqrt{2}\right)$ $\alpha - h(x) > 0$, then $h(x) < \alpha$ then $x \in \left]-\infty, \sqrt{2}\left(1 - e^{\sqrt{2}}\right)\right[$	1
b-	If $x \to -\infty$, $h(x) \to 0$ and $P(x) \to -\sqrt{2}$ If $x \to \sqrt{2}\left(1 - e^{\sqrt{2}}\right)$, $P(x) \to -\infty$	0.5
	Part C .	
1) a-	For $n \geq 1$,$n < n + 1$ but f is strictly decreasing so $f(n) > f(n+1)$ since the exponential is strictly increasing function then $e^{f(n)} > e^{f(n+1)}$, then (U_n) is strictly decreasing. **Or:** for $x \geq 1$ let $S(x) = e^{f(x)}$ but $S'(x) = f'(x)e^{f(x)} < 0$ then S is strictly decreasing function and hence (U_n) is strictly decreasing.	1
b-	$U_n = e^{f(n)}$, then $U_n > 0$	0.5
c-	$(U_{n)}$ is strictly decreasing, having a lower bound then (U_n) is convergent. If $n \to +\infty$, $f(n) \to -\infty$ $\lim_{x \to +\infty} U_n = 0$	0.5
2)	$U_n = e^{\left(\frac{1-n}{n}\right)\ln n} = e^{lnn^{\frac{1-n}{n}}} = n^{\frac{1-n}{n}}$	0.5