

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

مسابقة في مادة الرياضيات

المدة: ساعتان

(بالغة الإنكليزية)

الاسم:

الرقم:

I- (4 points)

In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the point $A(1; 1; 1)$ and the two lines (d_1) and (d_2) defined as:

$$(d_1): \begin{cases} x = t + 1 \\ y = t + 1 \\ z = t + 1 \end{cases} ; t \in \mathbb{R} \quad \text{and} \quad (d_2): \begin{cases} x = -k + 3 \\ y = k - 1 \\ z = k - 1 \end{cases} ; k \in \mathbb{R}$$

- 1) a. Show that the two lines (d_1) and (d_2) are not parallel and intersect at the point A.
b. Show that $y - z = 0$ is an equation of the plane (P) determined by (d_1) and (d_2) .
- 2) Let $B(1; 0; 0)$ be a point on a bisector of an angle determined by (d_1) and (d_2) in the plane (P)
 - a. Determine the coordinates of E the orthogonal projection of B on (d_1) .
 - b. Write a system of parametric equations of the line (Δ) perpendicular to (P) at A.
 - c. Denote by F the orthogonal projection of B on (d_2) and M is a point on (Δ) with $y_M \neq 0$.

Determine the coordinates of M so that the volume of the tetrahedron MABF is $\frac{2}{9}$ units of volume.

II- (4 points)

An urn U contains **six** balls: **four** red balls and **two** blue balls.

A bag S contains **five** bills: **one** 50 000 LL bill, **two** 20 000 LL bills and **two** 10 000 LL bills.

- 1) **One** ball is randomly drawn from U
 - If this ball is red, then **two** bills are drawn successively without replacement from S.
 - If this ball is blue, then **three** bills are drawn randomly and simultaneously from S.

Consider the following events:

R: " the drawn ball is red".

A: "the sum of the values of the bills drawn is 70 000 LL".

a. Calculate $P(R)$ and $P\left(\frac{A}{R}\right)$ then verify that $P(A \cap R) = \frac{2}{15}$.

b. Calculate $P(A \cap \bar{R})$. Deduce $P(A)$.

2) In this part, **two** bills are drawn successively with replacement from S.

Designate by X, the random variable representing the sum of the values of the drawn bills.

a. Determine the six possible values of the random variable X.

b. Show that $P(X = 70\,000) = \frac{4}{25}$.

c. Calculate $P(X < 70\,000)$.

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A,

B, M and M' with respective affixes 1, 2, z and z', such that $z' = \frac{z-2}{2-\bar{z}}$ with $z \neq 2$.

1) In this part, let $z = 1 - i$.

a. Write z' in algebraic form and exponential form.

b. Prove that the quadrilateral ABMM' is a parallelogram.

2) Let $z = x + iy$ where x and y are real numbers.

Determine the complex number z so that the points M and M' are confounded.

3)

a. Show that $|z'| = 1$, then deduce that M' moves on a circle whose center and radius are to be determined.

b. Show that $|z' - 1| \leq 2$.

IV- (8 points)

Let f be the function defined over $]0; +\infty[$ by $f(x) = x - \frac{1 + \ln x}{x}$ and denote by (C) its

representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$. (1 unit = 2 cm).

Let (d) be the line with equation $y = x$.

1) a. Study, according to the values of x, the relative position of (C) and (d).

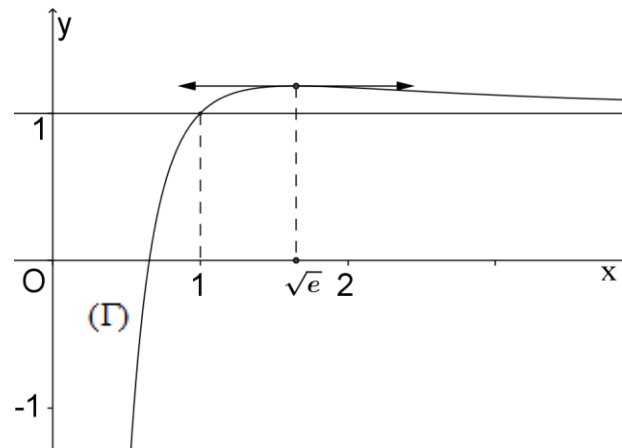
b. Calculate $\lim_{x \rightarrow +\infty} f(x)$ and show that the line (d) is an asymptote to (C).

2) Determine $\lim_{\substack{x \rightarrow 0 \\ x > 0}} f(x)$ then deduce an asymptote to

(C).

3) In the adjacent figure, we have:

- (Γ) is the representative curve of the function f' , **the derivative of f** .
- (Γ) admits a maximum at $x = \sqrt{e}$
- (Γ) intersects $(x'x)$ at a point of abscissa 0.6



a. Set up the table of variations of f .

b. Show that $f(x) = 0$ admits exactly two roots α and 1.

Verify that: $0.4 < \alpha < 0.5$.

c. Show that (C) admits a point of inflection whose coordinates are to be determined.

d. Determine the coordinates of the point A so that the tangent (T) at A to (C) is parallel to (d).

4) Draw (d), (T) and (C).

5) a. Calculate, in cm^2 , the area $A(\alpha)$ of the region bounded by (C), (d) and the two lines of equations

$x = \alpha$ and $x = 1$.

b. Verify that: $A(\alpha) = 2 - 2\alpha^4 \text{ cm}^2$.