ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the point A(1 ; 1 ; 1)

and the two lines (d_1) and (d_2) defined as $(d_1):\begin{cases} x=t+1 \\ y=t+1 \; ; t\in\mathbb{R} \\ z=t+1 \end{cases}$ and $(d_2):\begin{cases} x=-k+3 \\ y=k-1 \; ; k\in\mathbb{R} \\ z=k-1 \end{cases}$.

1) a. Show that the two lines (d_1) and (d_2) are not parallel and intersect at the point A.

   b. Show that $y-z=0$ is an equation of the plane (P) determined by (d_1) and (d_2).

2) Let $B(1\,;0\,;0)$ be a point on a bisector (δ) of an angle formed by (d_1) and (d_2) in the plane (P).

   a. Determine the coordinates of E the orthogonal projection of B on (d_1).

   b. Write a system of parametric equations of the line (Δ) perpendicular to (P) at A.

   c. Denote by F the orthogonal projection of B on (d_2) and M is a point on (Δ) with $y_M \neq 0$.

   Determine the coordinates of M so that the volume of the tetrahedron MABF is equal to $\frac{2}{9}$ units of volume.

II- (4 points)

An urn U contains **six** balls: **four** red balls and **two** blue balls.
A bag S contains **five** bills: **one** 50 000 LL bill, **two** 20 000 LL bills and **two** 10 000 LL bills.

Part A

One ball is randomly drawn from U

- If this ball is red, then **two** bills are drawn successively without replacement at random from S.
- If this ball is blue, then **three** bills are drawn simultaneously at random from S.

Consider the following events:

R: " the drawn ball is red ″.
A: ″ the sum of the values of the bills drawn is 70 000 LL ″.

1) Calculate the probabilities $P(R)$, $P\left(A/R\right)$ then verify that $P(A\cap R)=\frac{2}{15}$.

2) Calculate $P(A\cap\overline{R})$. Deduce P(A).

Part B

In this part, **two** bills are drawn successively with replacement at random from the bag S.
Designate by X, the random variable that is equal to the sum of the values of the two drawn bills.

1) Determine the six possible values of X.

2) Show that P(X = 70 000) = $\frac{4}{25}$.

3) Calculate P(X< 70 000).

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B, M and M' with respective affixes 1, 2, z and z' so that $z' = \dfrac{z-2}{2-\bar{z}}$ with $z \neq 2$.

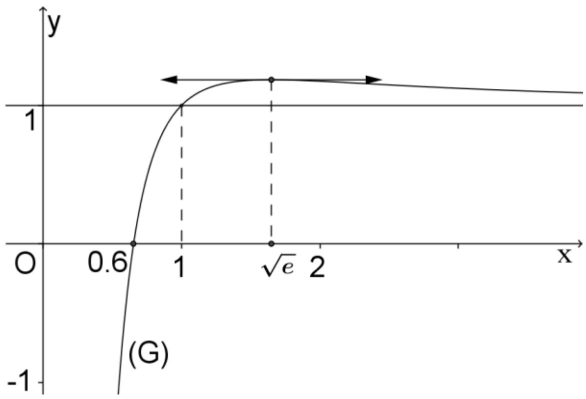
1) In this part, let $z = 1-i$.
   a. Write z' in algebraic form and exponential form.
   b. Prove in this case that the quadrilateral ABMM' is a parallelogram.
2) Let $z = x+iy$ where x and y are two real numbers.
   Determine the complex number z so that the points M and M' are confounded.
3) a. Show that $|z'| = 1$ for all $z \neq 2$. Deduce that M' moves on a circle whose center and radius are to be determined.
   b. Show that $|z'-1| \leq 2$ for all $z \neq 2$.

IV- (8 points)

Let f be the function defined over $]0;+\infty[$ as $f(x) = x - \dfrac{1+\ln x}{x}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. (1 graphical unit = 2 cm).

Let (d) be the line with equation $y = x$.

1) a. Study, according to the values of x, the relative position of (C) and (d).
   b. Determine $\lim\limits_{x\to+\infty} f(x)$ and show that the line (d) is an asymptote to (C).
2) Determine $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)$ then deduce an asymptote to (C).
3) In the adjacent figure, we have:
   - (G) is the representative curve of the function **f ', the derivative of f**.
   - (G) admits a maximum for $x = \sqrt{e}$.
   - (G) intersects the x-axis at a point of abscissa 0.6.



   a. Set up the table of variations of f.
   b. Show that the equation f(x) = 0 admits **exactly** two roots so that one of them is equal to1.
   c. Denote by α the second root of the equation $f(x) = 0$. Verify that $0.4 < \alpha < 0.5$.
   d. Show that (C) admits a point of inflection whose coordinates are to be determined.
   e. Determine the coordinates of the point A on (C) where the tangent (T) at A is parallel to (d).
4) Draw (d), (T) and (C).
5) a. Calculate, in cm^2, the area $A(\alpha)$ of the region bounded by (C), (d) and the two lines with equations $x = \alpha$ and $x = 1$.
   b. Prove that $A(\alpha) = (2-2\alpha^4)\ cm^2$.

أسس تصحيح مسابقة الرياضيات

Q.I	Answer key	4 pts
1.a	$\overrightarrow{u_1}(1;1;1)$ and $\overrightarrow{u_2}(-1;1;1)$ are not collinear. $A\in(d_1)$ for $t=0$ and $A\in(d_2)$ for $k=2$.	**1**
1.b	$(d_1)\subset(P): t+1-(t+1)=0$ for all t and $(d_2)\subset(P): k-1-(k-1)=0$ for all k. **OR** Let M(x ; y ; z) be a point in (P) and $\overrightarrow{AM}.\left(\overrightarrow{u_1}\wedge\overrightarrow{u_2}\right)=0$	**0,75**
2.a	$\overrightarrow{BE}(t\,;\ t+1\,;\ t+1)$ and $\overrightarrow{BE}.\overrightarrow{u_1}=0$ so $t=-\frac{2}{3}$ hence $E\left(\frac{1}{3}\,;\ \frac{1}{3}\,;\ \frac{1}{3}\right)$	**1**
2.b	$\overrightarrow{n_P}(0\,;\ 1\,;\ -1)$ is a direction vector of (Δ) so (Δ): $\begin{cases}x=1\\y=m+1\\z=-m+1\end{cases}$; $m\in\mathbb{R}$	**0,5**
2.c	Area of the triangle AEB = Area of the triangle AFB since [AB) is a bisector of the angle $E\hat{A}F$ so $V_{MABE}=V_{MABF}$, $M\in(\Delta)$ so M(1 ; m+1 ; –m+1) $V_{MABE}=\frac{1}{6}\left\|\left(\overrightarrow{AM},\overrightarrow{AB},\overrightarrow{AE}\right)\right\|=\frac{2}{9}$ so $\|m\|=1$ hence m = 1 or m = –1 $M_1(1\,;\ 2\,;\ 0)$ or $M_2(1\,;\ 0\,;\ 2)$ so $M_1(1\,;\ 2\,;\ 0)$ accepted . **OR** $V_{MABE}=\frac{A_{AEB}\times AM}{3}$	**0,75**

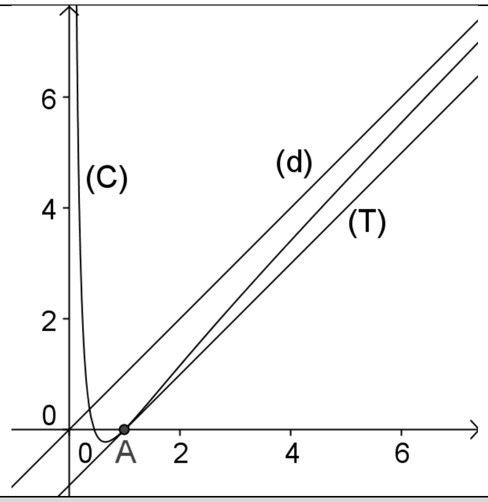
Q.II	Answer key	4 pts
A.1	$P(R)=\frac{C_4^1}{C_6^1}=\frac{2}{3}$ $P(A/R)=P(50000 \text{ and } 20000)$ $=P\left(\left(1^{st}\ 50000 \text{ then } 2^{nd}\ 20000\right) \text{ or } \left(1^{st}\ 20000 \text{ then } 2^{nd}\ 50000\right)\right)$ $=P(1^{st}\ 50000 \text{ then } 2^{nd}\ 20000)+P(1^{st}\ 20000 \text{ then } 2^{nd}\ 50000)$ $=\frac{C_1^1}{C_5^1}\times\frac{C_2^1}{C_4^1}+\frac{C_2^1}{C_5^1}\times\frac{C_1^1}{C_4^1}=\frac{1}{5}$ $P(A\cap R)=P(R)\times P(A/R)=\frac{2}{3}\times\frac{1}{5}=\frac{2}{15}$	**1,5**
A.2	$P(A\cap\overline{R})=P(\overline{R})\times P(A/\overline{R})=\frac{1}{3}\times\frac{C_1^1C_2^1}{C_5^3}=\frac{1}{15}$ $P(A)=P(A\cap R)+P(A\cap\overline{R})=\frac{2}{15}+\frac{1}{15}=\frac{1}{5}$	**1**
B.1	$50000+50000=100000$; $50000+20000=70000$; $50000+10000=60000$ $20000+20000=40000$; $10000+10000=20000$; $20000+10000=30000$ $X\in\{20000\ ;\ 30000\ ;\ 40000\ ;\ 60000\ ;\ 70000\ ;\ 100000\}$	**0,5**
B.2	$P(X=70000)=P(50000 \text{ and } 20000)$ $=P((1^{st}\ 50000 \text{ then } 2^{nd}\ 20000) \text{ or } (1^{st}\ 20000 \text{ then } 2^{nd}\ 50000))$ $=P(1^{st}\ 50000 \text{ then } 2^{nd}\ 20000)+P(1^{st}\ 20000 \text{ then } 2^{nd}\ 50000)$ $=\frac{C_1^1}{C_5^1}\times\frac{C_2^1}{C_5^1}+\frac{C_2^1}{C_5^1}\times\frac{C_1^1}{C_5^1}=\frac{4}{25}$	**0,5**
B.3	$P(X<70000)=1-P(70000)-P(100000)=1-\frac{4}{25}-\frac{1}{5}\times\frac{1}{5}=\frac{4}{5}$	**0,5**

Q.III	Answer key	4 pts
1.a	$z' = -i = e^{-i\frac{\pi}{2}}$	0,75
1.b	$Z_{\overrightarrow{AB}} = Z_{\overrightarrow{MM'}} = 1$; moreover, A,B ,M and M ' are not collinear.	0,75
2	$z = \frac{z-2}{2-\bar{z}}$ so $z - z\bar{z} + 2 = 0$ hence $x - x^2 - y^2 + 2 + iy = 0$ so $x - x^2 + 2 = 0$ and $y = 0$ hence $z_1 = 2$ (rejected)or $z_2 = -1$	1
3.a	$\lvert z'\rvert = \frac{\lvert z-2\rvert}{\lvert 2-\bar{z}\rvert} = \frac{\lvert z-2\rvert}{\lvert \bar{z}-2\rvert} = \frac{\lvert z-2\rvert}{\lvert z-2\rvert} = 1$ then OM ' = 1. Hence M' moves on a circle (C) with center O and radius 1.	1
3.b	$\lvert z' - 1\rvert \leq \lvert z'\rvert + \lvert -1\rvert \leq 1+1 \leq 2$ **OR** geometric method ……	0,5

Q.IV	Answer key	8 pts
1.a	$f(x) - x = -\left(\frac{1+\ln x}{x}\right)$ $f(x) - x > 0$ so $-\left(\frac{1+\ln x}{x}\right) > 0$ so $1+\ln x < 0$, $\ln x < -1$, $x < \frac{1}{e}$ then (C) is above (d) For $x > \frac{1}{e}$ then (C) is below (d) and for $x = \frac{1}{e}$ then (C) and (d) intersect	1
1.b	$\lim\limits_{x\to+\infty} f(x) = +\infty$ and $\lim\limits_{x\to+\infty} [f(x) - y] = \lim\limits_{x\to+\infty}\left[\frac{-1}{x} - \frac{\ln x}{x}\right] = 0$ then (d) is an asymptote to (C) at + ∞	1
2	$\lim\limits_{\substack{x\to 0\\ x>0}} f(x) = 0 + \infty = +\infty$, so $x = 0$ is V.A	1
3.a	Variation table (below)	0,75
3.b	• On $]0;0.6[$: f is continuous and strictly decreasing from $+\infty$ to $-0.2 < 0$ then the equation f(x) = 0 has a unique solution. • On $]0.6;+\infty[$: f is continuous and strictly increasing from $-0.2 < 0$ to $+\infty$ then the equation f(x) = 0 has a unique solution. f(1) = 0, then 1 is one of the roots. Or we have exactly two solutions 1 and another	0,5
3.c	f(0.4) × f(0.5) = (0,19) × (–0,11) < 0, then 0.4< α < 0.5.	0,25
3.d	$f''(x)$ vanishes and changes sign for $x = \sqrt{e}$ So (C) admits an inflection point ($\sqrt{e}$; f($\sqrt{e}$)).	0,5
3.e	$f'(x_A) = 1$ so $x_A = 1$ then A(1 ;0)	0,75

Variation table (3.a):

x	0		0.6		$+\infty$
f '(x)	‖	–	0	+	
f(x)	‖ $+\infty$	↘	–0.2	↗	$+\infty$

4		1	5.a	$A(\alpha) = \int_{\alpha}^{1} \frac{1+\ln x}{x}dx = [\frac{(1+\ln x)^2}{2}]_{\alpha}^{1}$ $= [\frac{1}{2} - \frac{(1+\ln\alpha)^2}{2}]$ u^2 $= 2 - 2(1+\ln\alpha)^2$ cm^2	0,75
			5.b	$f(\alpha) = 0$ $\alpha - \left(\frac{1+\ln\alpha}{\alpha}\right) = 0$ $1 + \ln\alpha = \alpha^2$ $A(\alpha) = 2 - 2(\alpha^2)^2 = 2 - 2\alpha^4$ cm^2	0,5