ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

The table below shows the number of demanded televisions in terms of the sale price, in hundred thousands LL, of each television:

The sale price of a television in **hundred thousands LL** (x_i)	8	9	10	11	13	15
Number of demanded televisions (y_i)	25	22	20	16	10	7

Round all your answers to the nearest 10^{-1}.

1) a- Calculate the coordinates of the center of gravity $G(\bar{x}, \bar{y})$.
   b- Draw, in a rectangular system, the scatter plot of the points associated to the distribution $(x_i; y_i)$ and plot G.
   c- Determine an equation of the regression line $(D_{y/x})$ and draw it in the same system.
2) Calculate the percentage of decrease in the number of demanded televisions when the sale price of a television increases from 900 000 LL to 1 300 000 LL.
3) Suppose that the above pattern remains valid for a sale price less than or equal to 1 700 000 LL. Estimate the number of demanded televisions at a price of 1 590 000 LL.
4) a- Verify that the elasticity of demand in terms of the price x is $E(x) = \dfrac{-2.7x}{2.7x - 46.1}$.
   b- Calculate E(11) and give an economical interpretation of the obtained value.

II- (4 points)

A telecommunication enterprise conducted a survey about the clients who bought only one prepaid mobile line of type E or F. After buying the mobile line, a client either does not subscribe to the internet or subscribes to the internet by choosing only one of the two options A (500 mega bites) or B (1.5 gega bites). The enterprise declares that:

- 60% of the clients bought each a line of type E;
- Among the clients who bought each a line of type E:
  45% chose option A, 35% chose option B and 20% did not subscribe to the internet;
- Among the clients who bought each a line of type F, 55% chose option A;
- **18% of all the surveyed clients did not subscribe to the internet.**

A client is randomly interviewed. Consider the following events:

E: "The interviewed client bought a line of type E" ; A: "The interviewed client chose option A" ;
B: "The interviewed client chose option B" ; C: "The client did not subscribe to the internet".

1) What is the probability that the client bought a line of type F?
2) a- Calculate the probability $P(C \cap E)$ and deduce that $P(C \cap \bar{E}) = 0.06$.
   b- The client bought a line of type F. Calculate the probability that this client did not subscribe to the internet.
3) The monthly rate price of a line of type E is 30 000 LL and a line of type F is 40 000 LL.
   In addition, option A costs 10 000 LL and option B costs 20 000 LL per month.
   Denote by X the random variable equal to the sum paid monthly by a surveyed client.
   a- Complete the probability distribution table.
   b- Calculate E(X), the mathematical expectation of X.
   c- Estimate, in LL, the revenue when the enterprise sells 100 000 lines.

$X = x_i$	30 000	40 000	50 000	60 000
$P(X = x_i)$			0.43	0.12

III- (4 points)

In 2011, the number of students in a university was 3000.

Each academic year, 12% of the students leave this university for different reasons and 480 new students join in.

For all integers $n \geq 0$, denote by U_n the number of students in this university in the year (2011 + n).

So $U_0 = 3000$.

1) Verify that $U_1 = 3120$.
2) For all integers $n \geq 0$, justify that $U_{n+1} = 0.88U_n + 480$.
3) For all integers $n \geq 0$, consider the sequence (V_n) defined as $V_n = U_n - 4000$.

   a- Show that (V_n) is a geometric sequence whose common ratio and first term are to be determined.

   b- For all integers $n \geq 0$, show that $U_n = 4\,000 - 1\,000\,(0.88)^n$.

   c- Estimate the number of students in this university in 2017.
4) The profit of this university in 2017 was 3535000000 LL.

   In the aim of building a new laboratory, the university decided to invest 10% of the profit achieved in 2017 in a bank for 5 years at an annual interest rate of 6% compounded monthly.

   Calculate the future value at the end of the 5 years of investment.

IV- (8 points)

Part A

Consider the function f defined on $[0,+\infty[$ as $f(x) = 2x + 1 + xe^{-x+2}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

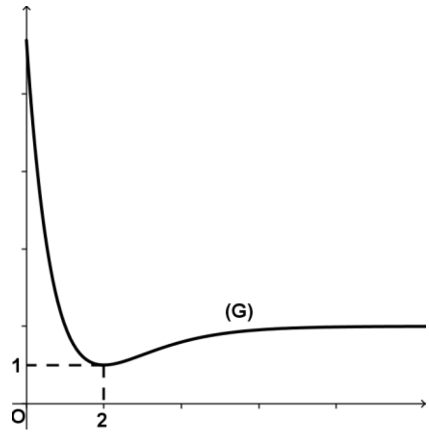
Let (d) be the line with equation y = 2x + 1.

1) Determine $\lim\limits_{x \to +\infty} f(x)$ and calculate f(1).
2) a- Study, according to the values of x, the relative position of (C) and (d) and specify the coordinates of their point of intersection.

   b- Show that (d) is an asymptote to (C).
3) a- Show that $f'(x) = 2 + (1-x)e^{-x+2}$.

   b- The curve (G) of the function f' is shown in the adjacent figure. For all x on $[0,+\infty[$, verify that $f'(x) > 0$.

   c- Set the table of variations of f.
4) The line (D) with equation y = 4x intersects (C) at the point with abscissa α. Show that 1.66 < α < 1.68.
5) Draw (d), (D) and (C).



Part B

In what follows, suppose that α = 1.67.

A factory produces watches. The average cost function $\overline{C}$ is modeled as $\overline{C}(x) = 2 + \frac{1}{x} + e^{-x+2}$.

For all $0 < x \leq 4$, x is the number of produced watches expressed in hundreds.

The total cost, average cost, revenue and profit functions as well as the unit price are all expressed in the same unit which is in millions LL.

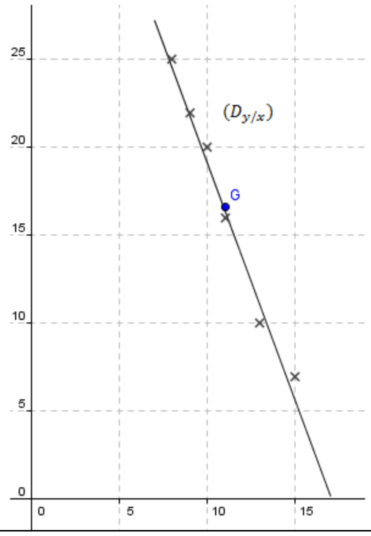
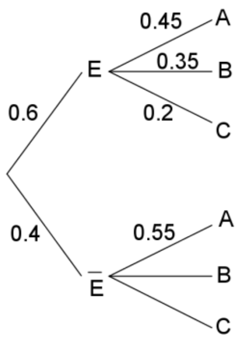
1) Calculate $\overline{C}(3)$. Deduce, in LL, the average cost of producing a watch among the first 300 watches produced.
2) Verify that the total cost function is modeled as: $C_T(x) = f(x) = 2x + 1 + xe^{-x+2}$.
3) Knowing that the whole production is sold, the revenue function R is modeled as R(x) = 4x.

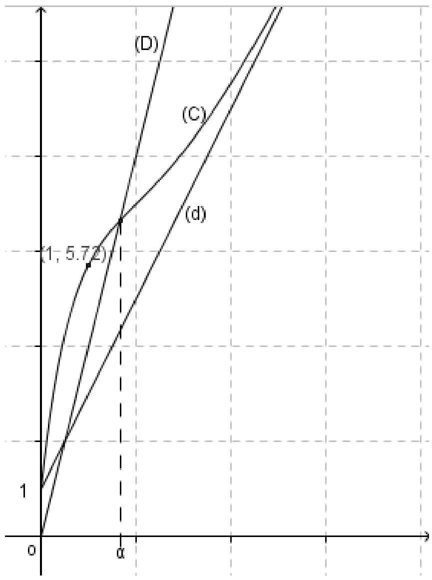
   a- Determine the minimal number of watches for which the factory achieves a profit.

   b- 20% of the watches are defective. Each defective watch is sold for 12000 LL and each non-defective watch is sold for p LL.

   Show that p = 47000.

أسس تصحيح مسابقة الرياضيات

Q.I	Answers	7 pts
1.a	$\overline{x} = 11$; $\overline{y} = 16.6$. G(11 ; 16.6)	0.5
1.b		1
1.c	$(D_{y/x})$: y = – 2.7x + 46.1	1
2	% of decrease = $\dfrac{22-10}{22}\times 100 = 54.5$	1
3	x = 15.9 , y = – 2.7(15.9) + 46.1 = 3.2 3 televisions	1.5
4.a	$E(x) = -x\dfrac{y'}{y} = \dfrac{2{,}7x}{-2{,}7x+46{,}1}$	1
4.b	E(11) = 1.8 When the price increases 1% to 1100000 the demand decreases 1.8%	1
Q.II	**Answers**	**7 pts**
		
1	$P(\overline{E}) = 1 - P(E) = 1 - 0.6 = 0.4$	0.5
2.a	$P(C\cap E) = P(E)\times P(C/E) = 0.6\times 0.2 = 0.12$ $P(C\cap \overline{E}) = P(C) - P(C\cap E) = 0.18 - 0.12 = 0.06$	2
2.b	$P(C/\overline{E}) = \dfrac{P(C\cap\overline{E})}{P(\overline{E})} = \dfrac{0.06}{0.4} = 0.15$	1
3.a	$P(X = 30000) = P(C\cap E) = 0.12$ $P(X = 40000) = P(\overline{E}\cap C) + P(E\cap A) = 0.4\times 0.15 + 0.6\times 0.45 = 0.33$ Or P(X = 40000) = 1 – (0.12 + 0.43 + 0.33)	1.5
3.b	E(X) = 30000×0.12 + 40000×0.33 + 50000×0.43 + 60000×0.12 = 45500	1
3.c	Revenue = 45500×100000 = 4550000000 LL.	1

Q.III	Answers	7 pts
1	$U_1 = (1 - 0.12)U_0 + 480 = 3120.$	0.5
2	$U_{n+1} = U_n - 0.12\,U_n = 0.88\,U_n + 480$	1
3.a	$V_{n+1} = U_{n+1} - 4000 = 0.88\,U_n - 3520$ $\dfrac{V_{n+1}}{V_n} = \dfrac{0.88\,Un - 3520}{U_n - 4000} = \dfrac{0.88(Un - 4000)}{U_n - 4000} = 0.88$ Then (V_n) is a geometric sequence of common ratio is 0.88 and first term $V_0 = -1000$.	1.5
3.b	$V_n = -1000(0.88)^n$ then $U_n = V_n + 4000 = -1000(0.88)^n + 4000$	1
3.c	$n = 6$, $U_n = -1000(0.88)^6 + 4000 = 3535.5$ the number of students in 2017 was 3535	1
4	Acquired value = $\dfrac{10}{100} \times 3535000000(1+\dfrac{0.06}{12})^{5\times12} = 476818528.9$LL	2
Q.IV	**Answers**	**14 pts**
A.1	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}(2x+1+xe^{-x+2}) = \lim\limits_{x\to+\infty}(2x+1+xe^{-x}e^{-2}) = +\infty + 1 + 0 = +\infty.$ $f(1) = 3 + e \approx 5{,}7$ **OR** $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}(2x+1+\frac{x}{e^x}e^2) = +\infty + 1 + 0 = +\infty.$	1
A2a	$f(x) - (2x+1) = xe^{-x+2}$; If $x = 0$, then (C) cuts (d) at I(0 ; 1); If $x > 0$, then (C) is above (d).	1.5
A2b	$\lim\limits_{x\to+\infty}(f(x) - y) = \lim\limits_{x\to+\infty}(xe^{-x+2}) = \lim\limits_{x\to+\infty}(xe^{-x}e^{-2}) = 0$ then (d) is an oblique asymptote to (C)	1
A3a	$f'(x) = 2 + e^{-x+2} - xe^{-x+2} = 2 + (1-x)e^{-x+2}$	0.5
A3b	The curve (G) is above the axis of abscissas then $f'(x) > 0$ for every $x \in [0,+\infty[$ **OR** 1 is the minimal value of $f'(x)$ then $f'(x) > 0$	1
A3c	x: 0 … $+\infty$ f '(x): + f(x): 1 ↗ $+\infty$	1
A4	$f(1.66) = 6.65 > 4(1.66) = 6.64$ and $f(1.68) = 6.67 < 4(1.68) = 6.72$ thn $1.66 < \alpha < 1.68$ **Or** $f(1.66) - 4(1.66) = 0.01 > 0$ and $f(1.68) - 4(1.68) = -0.04 < 0$ then $1.66 < \alpha < 1.68$	1
B1	$C_M(3) = 2.701213$ The average cost of producing a watch among the first 300 watches produced is $2.701213 \times \frac{1000000}{100} = 27012.13$ LL	2
B2	$C_T(x) = xC_M(x) = 2x + 1 + xe^{-x+2}$	0.5
B.3a	The enterprise realizes a gain if $R(x) > C_T(x)$; $4x > f(x)$. But according to part A5) the line (D) which represents the revenue function is above the curve (C) which represents the cost function if $1.67 \le x \le 4$. Then breakeven level is realized at x = 1.67 in hundreds of watches. Thus 167 watches. Then the minimum number of watches to realized a profit is 168 watches.	1.5
A5		1.5
B3b	$R(x) = 4x = \frac{(12000)(100)}{1000000}(x)\left(\frac{20}{100}\right) + \frac{(p)(100)}{1000000}(x)\left(\frac{80}{100}\right)$; $4 = 0.24 + 0.00008p$; $p = 47000$.	1.5