

الاسم:
الرقم:

مسابقة في مادة الرياضيات
المدة: ساعة

عدد المسائل: ثلاث

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (5 points)

In a certain store, all the pants are sold at the same price and all the shirts are sold at the same price.

Diala bought 3 pants and 4 shirts for 240 000 LL.

Joudi bought 2 pants and 2 shirts for 140 000 LL.

- 1) Calculate the price of one pant and that of one shirt.
- 2) The store proposes two offers for Diala if she buys 5 shirts and 5 pants:
 - **Offer 1**
10% discount on the price of each pant and 30% discount on the price of each shirt.
 - **Offer 2**
A reduction of 60 000 LL on the total amount.

Which one of the two offers is better for Diala? Justify your answer.

II- (5 points)

80 tourists are travelling on a boat to visit a certain island. These tourists are distributed as shown in the table below:

Age in years	[16 ; 24[[24 ; 32[[32 ; 40[[40 ; 48[[48 ; 56]
Europeans	6	7	12	3	8
Asians	3	13	11	12	5

- 1) Determine the average age of the European tourists on this boat.
- 2) The captain of this boat chose randomly one person from these tourists to be the guest of honor.
Consider the following events:

E: " the chosen tourist is European ".

A: " the chosen tourist is Asian ".

Y: " the chosen tourist is less than 32 years old ".

a. Verify that the probability of Y is $\frac{29}{80}$.

b. Calculate the following probabilities:

$$P(E), P\left(\frac{Y}{A}\right), P(Y \cap A), P(Y \cup E) \text{ and } P\left(\frac{A}{\bar{Y}}\right).$$

III- (10 points)

Let f be the function defined on the interval $I =]-1; +\infty[$ as $f(x) = \frac{x^2 + 3}{x + 1}$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

- 1) Show that $f(x) = x - 1 + \frac{4}{x + 1}$.
- 2) a. Determine $\lim_{\substack{x \rightarrow -1 \\ x > -1}} f(x)$ and deduce an equation of an asymptote to (C) .
 b. Determine $\lim_{x \rightarrow +\infty} f(x)$.
 c. Prove that the line (d) with equation $y = x - 1$ is an asymptote to (C) .
- 3) a. Verify that $f'(x) = \frac{(x - 1)(x + 3)}{(x + 1)^2}$.
 b. Copy and complete the following table of variations of f .

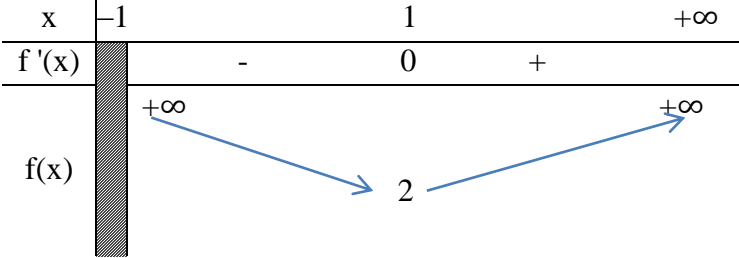
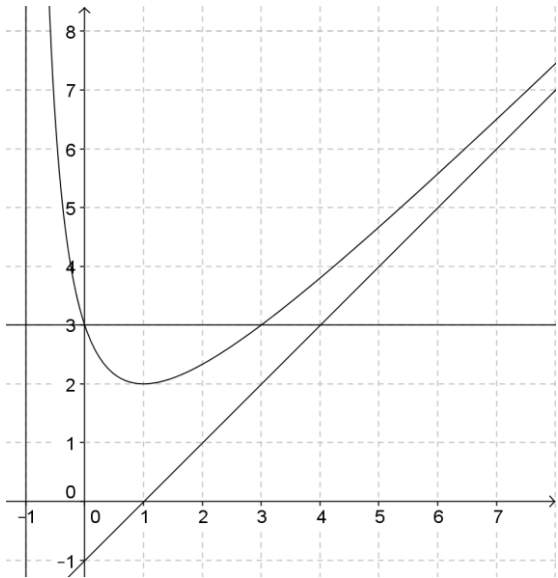
x	-1	1	$+\infty$
$f'(x)$		0	
$f(x)$			

- 4) a. Calculate the coordinates of the points of intersection of (C) and the line with equation $y = 3$.
 b. Find an equation of the tangent to (C) at its point with abscissa 0 .
 c. Draw the curve (C) and its two asymptotes.
- 5) Solve graphically: $2 < f(x) \leq 3$.

QI	Answers	Mark
1	Let x be the price of a pant and y the price of a shirt $\begin{cases} 3x + 4y = 240000 \\ 2x + 2y = 140000 \end{cases}$ $x = 40000 \text{ LL and } y = 30000 \text{ LL}$	2
2	Offer 1: after the discount, the price of a pant is : $40000 \times 0,9 = 36000 \text{ LL}$. And the price of a shirt is : $30000 \times 0,7 = 21000 \text{ L.L}$. $5 \times 36000 + 5 \times 21000 = 285000 \text{ LL}$ Offer 2 : $5 \times 40000 + 5 \times 30000 - 60000 = 290000 \text{ LL}$ Offer 1 is better for Diala.	3

QII	Answers	Mark
1	The average age of the Europeans is: $\frac{20 \times 6 + 7 \times 28 + 12 \times 36 + 3 \times 44 + 8 \times 52}{36} = 36 \text{ years}$	1
2.a	$P(Y) = \frac{9+20}{80} = \frac{29}{80}$	1
2.b	$P(E) = \frac{36}{80}$; $P(Y/A) = \frac{10}{20}$; $P(Y \cap A) = \frac{6}{80}$, $P(Y \cup E) = P(Y) + P(E) - P(Y \cap E) = \frac{52}{80}$ $P(A/\bar{Y}) = \frac{28}{51}$	3

QIII	Answers	Mark
1	$x - 1 + \frac{4}{x+1} = \frac{(x-1)(x+1)+4}{x+1} = \frac{x^2+3}{x+1}$	1
2.a	$\lim_{\substack{x \rightarrow -1 \\ x > -1}} f(x) = \frac{4}{0^+} = +\infty$ $x = -1$ vertical asymptote of (C)	1
2.b	$\lim_{x \rightarrow +\infty} f(x) = +\infty$	1
2.c	$\lim_{x \rightarrow +\infty} [f(x) - (x-1)] = \lim_{x \rightarrow +\infty} \left[\frac{4}{x+1} \right] = 0$, $y = x - 1$ is an asymptote of (C)	1
3.a	$f'(x) = \frac{2x(x+1) - (1)(x^2+1)}{(x+1)^2} = \frac{x^2+2x-3}{(x+1)^2} = \frac{(x-1)(x+3)}{(x+1)^2}$	1

QIII	Answers	Mark								
3.b	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">$+\infty$</td> </tr> <tr> <td style="padding: 5px;">$f'(x)$</td> <td style="padding: 5px;">$-$</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">$+$</td> </tr> </table> 	x	-1	1	$+\infty$	$f'(x)$	$-$	0	$+$	1
x	-1	1	$+\infty$							
$f'(x)$	$-$	0	$+$							
4.a	$\frac{x^2 + 3}{x + 1} = 3$, $x^2 - 3x = 0$, then $x = 0$ or $x = 3$ then the points of intersection are $(0,3)$ and $(3,3)$	1								
4.b	$y - f(0) = f'(0)(x)$ \square $y - 3 = -3x, y = -3x + 3$	1								
4.c		1								
5.	$x \in [0;1[\cup]1,3]$	1								