وزارة التربية والتعليم العالي | امتحانات الشهادة المتوسطة | دورة العام ٢٠١٨ الاستثنائية
المديريّة العامة للتربية | السبت ١١ آب ٢٠١٨
دائرة الامتحانـات الرسمية

عدد المسائل: خمسة | مسابقة في مادة الرياضيات | الاسم:
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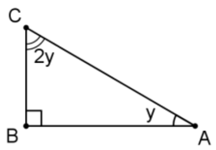
إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I - (3 points)

In the table below, only one of the proposed answers to each question is correct.

Write down the number of the question and give, **with justification**, its corresponding answer.

N°	Questions	Proposed answers		
		a	b	c
1	A car costs 15 000 000 LL. After a reduction of 11%, its price becomes	1 650 000 LL	13 350 000 LL	16 650 000 LL
2	If $\left(\sqrt{2}-1\right)x=1$ then $x=$	$\sqrt{2}$	1	$\sqrt{2}+1$
3	n is a non-zero real number, $\frac{n}{2}-\frac{n}{2}\times 3=$	3	$-n$	0
4	ABC is a right triangle at B such that $\widehat{BAC}=y$ and $\widehat{BCA}=2y$ where y is a real number. The value of $\cos\widehat{BAC}$ is 	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$

II - (3.5 points)

Given $A(x)=2x^2-6x-(x-3)(x-1)$.

1) a. Show that $A(x)=(x+1)(x-3)$.

b. Solve the equation $A(x)=0$.

2) Verify that $A(x)=x^2-2x-3$.

3) The grades of students, in mathematics, are given in the following table. (x *is a natural number*)

Grades	4	9	12	19	Total
Number of students	1	x^2	x	1	x^2+x+2

Calculate x, knowing that the average (mean) of the grades is 10.

III - (3 points)

1) Solve, showing all the steps of calculation, the following system: $\begin{cases} x-2y=0 \\ 3y-x=6. \end{cases}$

2) In a class, the number of boys is double that of girls.

If 2 girls leave the class, the number of boys becomes triple that of the girls.

The teacher confirms that there are 18 students in this class. Is he right? Justify.

IV - (5.5 points)

In an orthonormal system of axes x'Ox and y'Oy, given the points $F(0;4)$ and $B(-2;2)$.
Let (d) be the line with equation $y = x + 4$.

1) Plot the points F and B.

2) Show that F and B are two points on (d), then draw (d).

3) Let H be the point of coordinates $(-1;3)$.

a. Verify that H is the midpoint of [BF].

b. Show that the equation of (d'), the perpendicular bisector of [BF], is $y = -x + 2$.

4) a. Show that (OB) and (d') are parallel.

b. Show that the triangle OBF is right isosceles at B.

5) Let (C) be the circle circumscribed about triangle OBF.

Show that the point $E(0;2)$ is the center of circle (C), and calculate its radius.

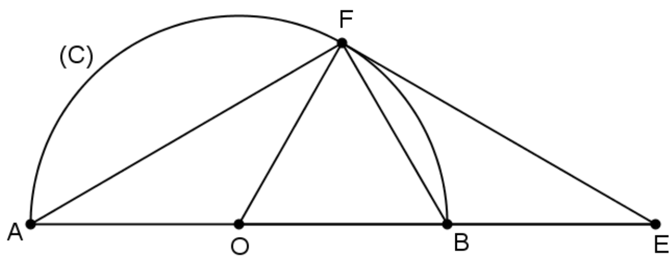
6) Let K be the point of coordinates $(2;2)$ and $L(2;0)$ the point of intersection of (d') and x'Ox.

Show that K is a point of circle (C), and that (LK) is tangent to circle (C).

V - (5 points)

In the adjacent figure:

- (C) is a semicircle with center O, diameter [AB] and radius 2 cm.
- F is a point on (C) so that BF = 2 cm.
- E is the symmetric of O with respect to B.



1) Reproduce the figure.

2) Verify that $AF = 2\sqrt{3}$ cm.

3) Show that (EF) is tangent to (C).

4) Let L be the midpoint of [OB]. Show that (FL) is perpendicular to (OB).

5) T is the point so that $\overrightarrow{FT} = \overrightarrow{LE}$.

The parallel through T to (OF) intersects [EF] at R and [LE] at G.

a. Show that (TG) is perpendicular to (EF).

b. Show that the two triangles FLE and GRE are similar.

c. Deduce that $\frac{EG}{ER} = \frac{2\sqrt{3}}{3}$.

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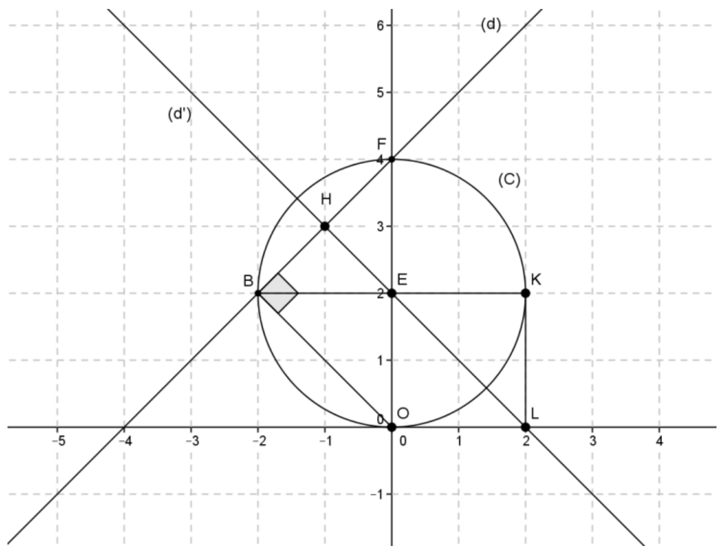
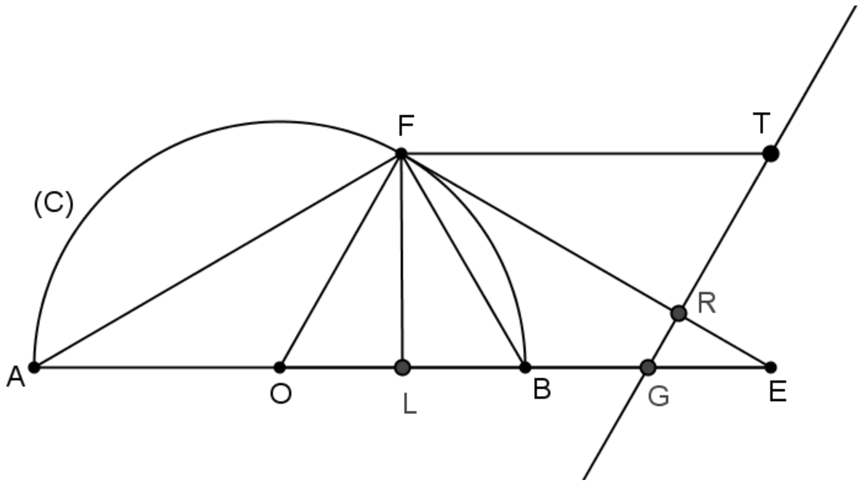
عدد المسائل: خمسة	مسابقة في مادة الرياضيات	مشروع اسس التصحيح - انكليزي

Parts of the Q.	Answers		Grades
	Question I		
1	$15\,000\,000 \times 0.89 = 13\,350\,000$ (b)	**0.5 + 0.25**	**0.75**
2	$x = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \sqrt{2}+1$ or replacing. (c)	**0.5 + 0.25**	**0.75**
3	$\frac{n}{2} - \frac{3n}{2} = \frac{-2n}{2} = -n$ (b)	**0.5 + 0.25**	**0.75**
4	$2y+y=90^\circ$ then $y = 30^\circ, cos30 = \frac{\sqrt{3}}{2}$ (c)	**0.5 + 0.25**	**0.75**
	Question II		
1a	$A(x) = 2x^2 - 6x - (x-3)(x-1)$ $A(x) = 2x(x-3) - (x-3)(x-1)$ $A(x) = (x-3)(2x - x + 1)$ $A(x) = (x-3)(x+1)$	 **0.25** **0.5** **0.25**	**1**
1b	x = 3 or x = -1	**0.25 + 0.25**	**0.5**
2	$A(x) = (x-3)(x+1) = x^2 + x - 3x - 3 = x^2 - 2x - 3$	**0.25 + 0.25**	**0.5**
3	$\frac{4+9x^2+12x+19}{x^2+x+2} = 10$ then $4 + 9x^2 + 12x + 19 = 10x^2 + 10x + 20$ $so\ x^2 - 2x - 3 = 0$ $A(x) = 0$ $so\ x = 3\ (accepted) or\ x = -1 (rejected)$	**0.5** **0.25** **0.25** **0.25** **0.25**	**1.5**
	Question III		
1	$\begin{cases} x - 2y = 0 \\ -x + 3y = 6 \end{cases}$ gives : $x = 12$; $y = 6$	**0.75 + 0.75**	**1.5**
2	Let x be the number of boys and y be the number of girls. $\begin{cases} x = 2y & (0.25) \\ x = 3(y-2) & (0.25) \end{cases}$ so $\begin{cases} x - 2y = 0 \\ 3y - x = 6 \end{cases}$ therefore x = 12 and y = 6 **(0.25)** so the number of students is 12 + 6 = 18 **(0.25)**	**0.25** **0.25**	**1.5**

Question IV		
1	 **0.25 + 0.25**	**0.5**
2	$F \in (d)$ $since$ $y_F = x_F + 4 = 0 + 4 = 4$ **0.25** $B \in (d)$ $since$ $y_B = x_B + 4 = -2 + 4 = 2$ **0.25** **Draw (d)** **0.25**	**0.75**
3a	$x_H = \frac{x_F + x_B}{2}$ $y_H = \frac{y_F + y_B}{2}$ $-1 = \frac{-2+0}{2}$ $3 = \frac{2+4}{2}$ $-1 = -1$ **0.25** $3 = 3$ so H is the midpoint of [FB]. **0.25**	**0.5**
3b	$(d') \perp (d)$ since $a_{(d)} \times a_{(d')} = -1$ **0.25** and $H \in (d')$ since $y_H = -x_H + 2 = 1 + 2 = 3$ **0.25** Therefore (d') is the perpendicular bisector of [FB] since (d') is perpendicular at H midpoint of [BF].	**0.5**
4a	$a_{(OB)} = \frac{y_B}{x_B} = -1 = a_{(d')}$ therefore (d') // (OB) **0.25 + 0.25**	**0.5**
4b	$(OB) \perp (d)$ $since$ $a_{(OB)} \times a_{(d)} = -1$ and $OB = 2\sqrt{2}$ = BF then OBF is a right isosceles triangle at B. **or …..** **0.25 + 0.5**	**0.75**
5	E is the midpoint of the hypotenuse [OF] therefore $x_E = \frac{x_F + x_O}{2} = 0$ **0.25** $y_E = \frac{y_F + y_0}{2} = \frac{4}{2} = 2$ then E (0;2) **0.25** Radius = OE = 2 **0.25**	**0.75**
6	EK= 2= radius then K $\in$ (C). **0.5** (LK): x=2 // y'oy **0.25** and (EK): y = 2 **0.25** therefore (LK) $\perp$ (EK) at K then (LK) is tangent to (C) at K. **0.25** Or : EK = 2 ; KL = 2 ; EL = $2\sqrt{2}$ $EL^2 = EK^2 + KL^2$ therefore EKL is a right triangle at K (Converse of Pythagorean theorem)	**1.25**

	Question V	
1		0.5
2	AFB is a right triangle at F since it is an inscribed triangle in a semicircle of diameter [AB] **0.25** $AF^2 = AB^2 - FB^2 = 16 - 4 = 12$ (Pythagorean) **0.25** $AF = \sqrt{12} = 2\sqrt{3}$ cm . **0.25** Or : Semi – equilateral triangle.	**0.75**
3	FB=BE=OB=2 then OEF is a right triangle since the median [FB] relative to [OE] measures its half. Therefore is tangent to (C) at F. Or: $\widehat{AFB} = 90^0$ (Calculation of angles) or ……	**0.75**
4	OFB is an equilateral triangle and L midpoint of [OB] then [FL] median and at the same time a height, then (FL) ⊥ (OB)	**0.5**
5a	(TG) // (OF) (Given) and (EF) ⊥ (OF) (Definition of tangent) then (EF) ⊥ (TG)	**0.5**
5b	$\hat{E}$(common angle) $\widehat{GRE} = \widehat{FLE} = 90°$ then the two triangles FLE and GRE are similar **0.5 + 0.5**	**1**
5c	<u>Ratio of similitude :</u> $\frac{FL}{GR} = \frac{LE}{RE} = \frac{FE}{GE}$ then: $\frac{LE}{RE} = \frac{FE}{GE}$ gives: $LE \times GE = RE \times FE$ **0.25 + 0.25** $3 \times GE = RE \times 2\sqrt{3}$ **0.25** $\frac{GE}{RE} = \frac{2\sqrt{3}}{3}$ **0.25**	**1**