عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

مسابقة في مادة الرياضيات

المدة: ساعتان

(باللغة الإنكليزية)

الاسم: ....

الرقم: ....

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the two points

A(0 , 1 , 2) and B(2 , 0 , 2) and the plane (P) with equation $x + 2y - 2 = 0$.

1) **Verify** that the two points A and B are in plane (P).

2) Let (Q) be the plane with equation $z-2=0$.

   - **Show that** (AB) is contained in (Q).

   - **Show that** (P) is perpendicular to (Q).

3) Let (L): $\begin{cases} x = t+2 \\ y = 2t \\ z = 2 \end{cases}$ $(t \in \mathbf{R})$ be the line perpendicular to plane (P) at B.

   a- **Show that** (L) lies in plane (Q).

   b- Let E be a point of (L) with $y_E > 0$.

   Determine the coordinates of the point E so that triangle ABE is right isosceles with vertex B.

   c- Let $I\left(\frac{3}{2},\frac{3}{2},2\right)$ be the midpoint of [AE]. Consider in plane (Q) the circle (C) with center I and passing through B.

   Write a system of parametric equations of the line (T) tangent to (C) at B.

II- (4 points)

The customer service department in a supermarket organizes a game to offer vouchers to its clients. For this purpose, an urn is placed at the entrance of the supermarket. The urn contains:

- three red balls each holding the number 10 000
- two white balls each holding the number 30 000
- one black ball holding the number –10 000.

A client who wants to participate in the game selects, simultaneously and randomly, three balls from the urn.

Consider the following events:

A : " the three selected balls have the same color "

B : " the three selected balls have three different colors "

C : " only two of the three selected balls have the same color "

1) a- **Calculate** the probability P(A) .

   Calculate the probability P(B).

   b- **Show that** $P(C) = \frac{13}{20}$.

2) A client who participates in the game receives a voucher whose value, in LL, is equal to the sum of the numbers on the three selected balls.

   Let X be the random variable equal to the value of the voucher received by the client.

   a- **Verify** that the possible values of X are: 10 000 , 30 000 , 50 000 , 70 000.

   b- **Show that** $P(X = 50\ 000) = \frac{7}{20}$.

   c- **Show that** $P(X > 35\ 000) = \frac{1}{2}$.

   d- Knowing that a client made purchases with a voucher whose value is greater than 35 000 LL,

   calculate the probability that exactly one red ball is selected from the urn.

III- (4 points)

The complex plane is referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$. For all points M of the plane with affix $z \neq 0$, we associate the point M' with affix z' such that $z' = \frac{z - 5i}{z}$.

1) **Write** z in exponential form in the case where $z' = \frac{1}{2} - \frac{1}{2}i$.

2) Denote by E the point with affix $z_E = 1$.

   a- **Verify that** $z' - 1 = \frac{-5i}{z}$.

   b- **Calculate** EM' when $OM = 5$.

3) Suppose that $z = x + iy$ and $z' = x' + iy'$ with x, y, x' and y' being real numbers.

   a- **Show that** $x' = \frac{x^2 + y^2 - 5y}{x^2 + y^2}$ and $y' = \frac{-5x}{x^2 + y^2}$.

   b- **Deduce** that when M' moves on the line with equation $y = x$, M moves on a circle whose center and radius are to be determined.

IV- (8 points)

Consider the function f defined on $\mathbb{R}$ as $f(x)=1-2e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- **Determine** $\lim\limits_{x\to-\infty} f(x)$

   b- **Calculate** $f(-1)$.

2) a- **Determine** $\lim\limits_{x\to+\infty} f(x)$.

   b- **Deduce** an equation of the asymptote (d) to (C).

   c- **Show that** (C) is below (d) for all x in $\mathbb{R}$.

3) The curve (C) intersects the x-axis at A and the y-axis at B.

   Find the coordinates of A and B.

4) a- **Calculate** $f'(x)$

   b- **Set up** the table of variations of f.

   c- **Draw** (C) and (d).

5) a- **Show that** f has, on $\mathbb{R}$, an inverse function g.

   b- **Determine** the domain of definition of g.

   c- **Verify that** $g(x)=\ln(2)-\ln(1-x)$.

6) Let (C′) be the representative curve of g and let F be the point of (C′) with abscissa 0.

   a- **Determine** an equation of the tangent (T) to (C′) at F.

   b- **Draw** (C′) and (T) in the same system as that of (C).

7) **Calculate** the area of the region **bounded** by **(C′)**, the **x-axis** and the **y-axis**.