عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the two points A(0 , 1 , 2) and B(2 , 0 , 2) and the plane (P) with equation $x + 2y - 2 = 0$.

1) Verify that the two points A and B are in plane (P).
2) Show that the plane (Q) containing the line (AB) and perpendicular to plane (P) has an equation $z-2=0$.
3) Let (L): $\begin{cases} x = t+2 \\ y = 2t \\ z = 2 \end{cases}$ $(t \in \mathbf{R})$ be the line perpendicular to plane (P) at B.

   a- Show that (L) lies in plane (Q).

   b- Let E be a point of (L) with $y_E > 0$.
   Determine the coordinates of the point E so that triangle ABE is right isosceles with vertex B.

   c- Let $I\left(\frac{3}{2},\frac{3}{2},2\right)$ be the midpoint of [AE]. Consider in plane (Q) the circle (C) with center I and passing through B. Write a system of parametric equations of the line (T) tangent to (C) at B.

II- (4 points)

The customer service department in a supermarket organizes a game to offer vouchers to its clients. For this purpose, an urn is placed at the entrance of the supermarket. The urn contains:

- three red balls each holding the number 10 000
- two white balls each holding the number 30 000
- one black ball holding the number –10 000.

A client who wants to participate in the game selects, simultaneously and randomly, three balls from the urn.

Consider the following events:

A : " the three selected balls have the same color "
B : " the three selected balls have three different colors "
C : " only two of the three selected balls have the same color "

1) a- Calculate the probabilities P(A) and P(B).

   b- Show that $P(C) = \frac{13}{20}$.

2) A client who participates in the game receives a voucher whose value, in LL, is equal to the sum of the numbers on the three selected balls.
   Let X be the random variable equal to the value of the voucher received by the client.

   a- Verify that the possible values of X are: 10 000 , 30 000 , 50 000 , 70 000.

   b- Show that $P(X = 50\ 000) = \frac{7}{20}$.

   c- Show that $P(X > 35\ 000) = \frac{1}{2}$.

   d- Knowing that a client made purchases with a voucher whose value is greater than 35 000 LL, calculate the probability that exactly one red ball is selected from the urn.

III- (4 points)

The complex plane is referred to a direct orthonormal system $(O;\vec{u},\vec{v})$. For all points M of the plane with affix $z\neq 0$, we associate the point M' with affix z' such that $z'=\dfrac{z-5i}{z}$.

1) Write z in exponential form in the case where $z'=\dfrac{1}{2}-\dfrac{1}{2}i$.
2) Denote by E the point with affix $z_E=1$.

   a- Verify that $z'-1=\dfrac{-5i}{z}$.

   b- Calculate EM' when $OM=5$.
3) Suppose that $z=x+iy$ and $z'=x'+iy'$ with x, y, x' and y' being real numbers.

   a- Show that $x'=\dfrac{x^2+y^2-5y}{x^2+y^2}$ and $y'=\dfrac{-5x}{x^2+y^2}$.

   b- Deduce that when M' moves on the line with equation $y=x$, M moves on a circle whose center and radius are to be determined.

IV- (8 points)

Consider the function f defined on $\mathbb{R}$ as $f(x)=1-2e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$ and calculate $f(-1)$.
2) a- Determine $\lim\limits_{x\to+\infty} f(x)$ and deduce an equation of the asymptote (d) to (C).

   b- Show that (C) is below (d) for all x in $\mathbb{R}$.
3) The curve (C) intersects the x-axis at A and the y-axis at B. Find the coordinates of A and B.
4) a- Calculate $f'(x)$ and set up the table of variations of f.

   b- Draw (C) and (d).
5) a- Show that f has, on $\mathbb{R}$, an inverse function g.

   b- Determine the domain of definition of g.

   c- Verify that $g(x)=\ln(2)-\ln(1-x)$.
6) Let (C') be the representative curve of g and let F be the point of (C') with abscissa 0.

   a- Determine an equation of the tangent (T) to (C') at F.

   b- Draw (C') and (T) in the same system as that of (C).
7) Calculate the area of the region bounded by (C'), the x-axis and the y-axis.

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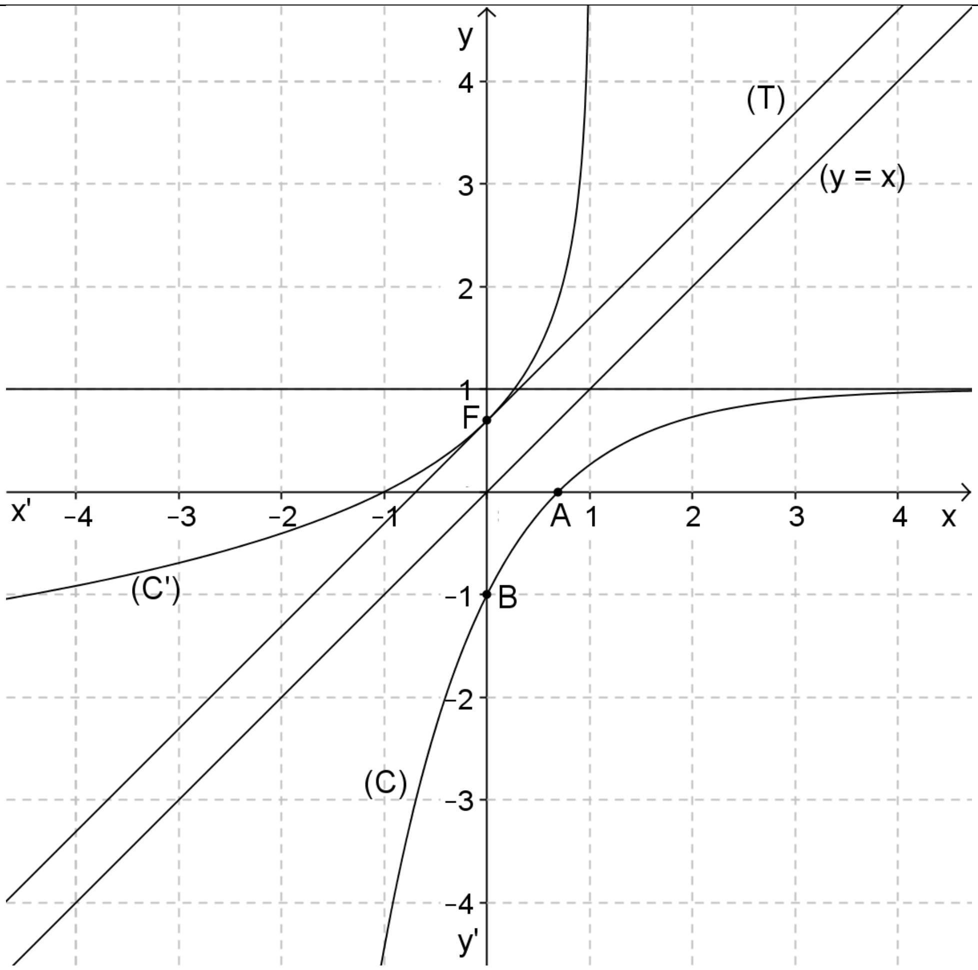
Q.I	Answer key	4 pts
1	$A \in (P) : (x_A) + 2(y_A) - 2 = 0,\ 2(0) + 2(1) - 2 = 0,\ 0 = 0$ Similarly: $B \in (P)$.	0.5
2	$A \in (Q)$ … et $B \in (Q)$ … $\overrightarrow{n_Q}.\overrightarrow{n_P} = (0)(1) + (0)(2) + (1)(0) = 0$.	0.5
3.a	$(L) \subset (Q)$ since $2 - 2 = 0$,	0.5
3.b	$E(t+2 ; 2t ; 2)$, AB = BE then $\sqrt{5} = \sqrt{4t^2 + t^2}$ hence $t = 1$ or $t = -1$, Therefore E (3 ; 2 ; 2) accepted . E (1 ; –2 ; 2) rejected.	1.5
3.c	A direction vector of (T) is : $\overrightarrow{IB} \wedge \overrightarrow{N_Q} = \frac{3}{2}\vec{i} - \frac{1}{2}\vec{j}$. hence (T): $\begin{cases} x = \frac{3}{2}m + 2 \\ y = \frac{1}{2}m \\ z = 2 \end{cases}$ **Another method:** ABE is right isosceles at B, so (BI) is perpendicular to (AE) hence (T) // (AE) and passes in B.	1
Q.II	**Answer key**	**4 pts**
1.a	$P(A) = \frac{C_3^3}{C_6^3} = \frac{1}{20}$, $P(B) = \frac{C_3^1 \times C_2^1 \times C_1^1}{C_6^3} = \frac{3}{10}$	0.5 0.5
1.b	$P(C) = 1 - P(A) - P(B) = \frac{13}{20}$ ou $P(C) = \frac{C_3^2 \times C_3^1 + C_2^2 \times C_4^1}{C_6^3} = \frac{13}{20}$	0.5
2.a	10 000 (RRN); 30 000 (RRR or RBN); 50 000 (RRB or BBN); 70 000 (RBB)	0.5
2.b	$P(X = 50\,000) = P(RRB) + P(BBN) = \frac{C_3^2 \times C_2^1 + C_2^2 \times C_1^1}{C_6^3} = \frac{7}{20}$	0.5
2.c	$P(X > 35\,000) = P(X = 50\,000) + P(X = 70\,000) = \frac{7}{20} + \frac{C_3^1 \times C_2^2}{C_6^3} = \frac{7}{20} + \frac{3}{20} = \frac{1}{2}$	1
2.d	$P(1\text{ red} / x > 35\,000) = \frac{P(RBB)}{P(X>35\,000)} = \frac{\frac{C_3^1 \times C_2^2}{C_6^3}}{\frac{1}{2}} = \frac{3}{10}$	0.5
Q.III	**Answer key**	**4 pts**
1	$z = 5 + 5i$ then exponential form of z is $5\sqrt{2}e^{i\frac{\pi}{4}}$	0.5
2.a	$z' - 1 = \frac{z-5i}{z} - 1 = -\frac{5i}{z}$	0.5
2.b	OM = 5 so $\lvert z \rvert = 5$. $EM' = \lvert z' - 1 \rvert = \left\lvert -\frac{5i}{z} \right\rvert = \frac{5}{\lvert z \rvert} = 1$.	1
3.a	$x' + iy' = \frac{x+iy-5i}{(x+iy)} \times \frac{x-iy}{x-iy} = \frac{x^2+y^2-5y-5ix}{x^2+y^2} = \frac{x^2+y^2-5y}{x^2+y^2} + i\frac{-5x}{x^2+y^2}$	1
3.b	$x' = y'$ then $\frac{x^2+y^2-5y}{x^2+y^2} = \frac{-5x}{x^2+y^2}$ therefore $x^2 + y^2 - 5y + 5x = 0$ hence M varies on a circle with center $I(-\frac{5}{2} ; \frac{5}{2})$ and radius $R = \frac{5\sqrt{2}}{2}$.	1

Q.IV	Answer key	8 pts
1	$\lim\limits_{x\to-\infty} f(x) = -\infty$. $f(-1) = 1 - 2e$.	0.5
2.a	$\lim\limits_{x\to+\infty} f(x) = 1$ so $y = 1$ is a horizontal asymptote to (C).	0.5
2.b	$f(x) - 1 = -2e^{-x} < 0$ therefore (C) is below (d)	0.5
3	A(ln2 ; 0) and B(0 ; –1)	0.5
4.a	$f'(x) = 2e^{-x} > 0$. x: $-\infty$ … $+\infty$ f '(x): + f(x): $-\infty$ ↗ 1	1
4.b	 	1
5.a	f is continuous and strictly increasing over ℝ.	0.5
5.b	$D_g =]-\infty, 1[$	0.5
5.c	$y = f(x) = 1 - 2e^{-x}$, $e^{-x} = \frac{1-y}{2}$, $-x = \ln(\frac{1-y}{2})$, $x = \ln(\frac{2}{1-y}) = \ln 2 - \ln(1-y)$ Then $g(x) = \ln 2 - \ln(1-x)$. Or $f(g(x)) = x$, $1 - 2e^{-g(x)} = x$ so $-g(x) = \ln\left(\frac{1-x}{2}\right)$ therefore $g(x) = \ln\left(\frac{2}{1-x}\right)$	1
6.a	F(0 ; ln2) , $g'(x) = \frac{1}{1-x}$ then $g'(0) = 1$ so : (T) : $y = x + \ln 2$	0.5
6.b	Figure. (C') and (C) are symmetric of each other w.r.t line $y = x$.	0.5
7	$A = -\int_0^{\ln 2} f(x)dx = -[x + 2e^{-x}]_0^{\ln 2} = [\ln 2 + 2e^{\ln 0.5}] + [0 + 2]$ A = 1 – ln2 (sq units).	1