ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

مسابقة في مادة الرياضيات

المدة: أربع ساعات

(باللغة الإنكليزية)

الإسم : ....

الرقم : ....

I- (2 points)

Prove each of the following propositions.

1) If $\arg(z)=\alpha+2k\pi,(k\in\mathbb{Z})$ and $z'=\dfrac{iz}{\overline{z}}$, where $z\neq 0$, then an argument of $\dfrac{z'}{z}$ is $\dfrac{\pi}{2}+\alpha$.
2) If (u_n) is an arithmetic sequence with common difference d $(d\neq 0)$, and (v_n) is the sequence defined by: $v_n=e^{u_n}$, then (v_n) is a geometric sequence with common ratio e^d.
3) If $z=e^{i\theta}+e^{-i\theta}$ where $\theta\in\left[0;\dfrac{\pi}{2}\right[$, then an argument of z is 0.
4) $\displaystyle\int\frac{(\arctan x)^2}{1+x^2}\,dx=\frac{(\arctan x)^3}{3}+c$

II- (2 points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the two points A(1 , 0 , 1) and B(–1 , 2 , 0) and the two lines (L) and (D) with parametric equations:

$$(L):\begin{cases}x=2t-1\\ y=t-1\\ z=-2t+3\end{cases}\quad (t\in\mathbb{R})\quad \text{and}\quad (D):\begin{cases}x=2\\ y=m-1\\ z=-m\end{cases}\quad (m\in\mathbb{R})$$

1) **Write** a Cartesian equation of the plane (P) passing through the two points A and B and parallel to (D).

2) a- **Verify** that the line (L) lies in plane (P).

   b- **Show** that (L) is perpendicular to (AB) at A.

3) **Find** the coordinates of the point C on (L) with x < 0 so that $AC=6$.
4) Let M(2 , m – 1 , –m) be a point on (D).

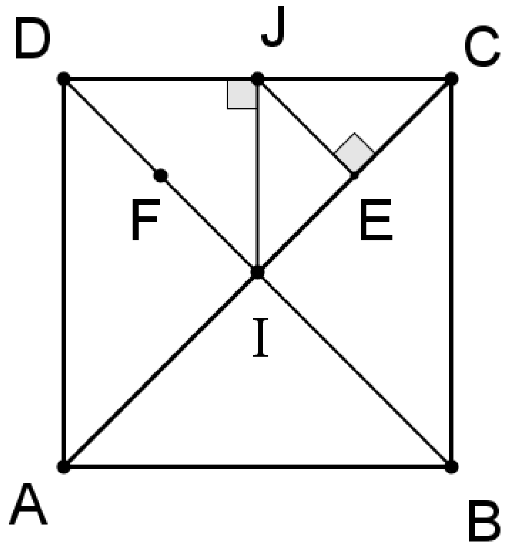
   Show that the volume of the tetrahedron MABC remains constant as M varies on line (D).

III- (3 points)

ABCD is a direct square with side 1 such that $\left(\overrightarrow{AB}, \overrightarrow{AD}\right) = \frac{\pi}{2} \quad [2\pi]$.

Denote by I, J, E and F the midpoints of the segments [AC], [CD], [IC] and [DI] respectively.

Consider the direct plane similitude S that transforms A onto I and C onto J.



1) **Verify** that the ratio k of S is equal to $\frac{\sqrt{2}}{4}$

   find an angle α of S.

2) a- **Show** that S(B) = E.

   b- **Deduce** the image of the square ABCD by S.

3) The plane is referred to the direct orthonormal system $\left(A; \overrightarrow{AB}, \overrightarrow{AD}\right)$.

   a- **Determine** the complex form of S.

   b- **Deduce** the affix of W, the center of S.

4) Let (P) be the parabola with focus A and directrix (BC) and (P') be the image of (P) by S.

   a- **Show** that D is on (P).

   b- **Specify** the tangent to (P') at F.

IV- (3 points)

An urn contains four black balls and 1 white ball.

A game runs in the following manner:

A fair die is rolled;

- If the die shows an odd number, then one white ball is added to the urn.
- If the die shows an even number, then one black ball is added to the urn.

After that, **three** balls are randomly and simultaneously selected from the urn.

Consider the following events:

O: "The die shows an odd number"

B: "The three selected balls are black".

1) **Calculate** the probabilities $P(B/O)$ and $P(B \cap O)$, then verify that $P(B) = 0.35$.

2) The three selected balls are black. What is the probability that the die shows an even number?

3) Denote by X the random variable equal to the number of white balls selected during this game.

   a- **Show** that $P(X = 1) = 0.55$.

   b- **Determine** the probability distribution of X.

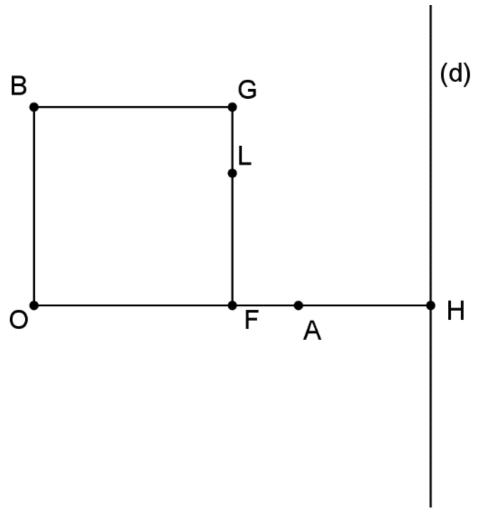
4) Each of Sami and Karim played this game once. Let S be the random variable equal to the total number of white balls obtained by both Sami and Karim. **Calculate** $P(S \geq 1)$.

V- (3 points)

In the figure shown below:

- OFGB is a square with side $\sqrt{2}$,
- F is the midpoint of the segment [OH],
- (d) is the perpendicular to (OF) at H,
- A is the point on [OH] such that $OA = 2$,
- L is the point on [FG] such that FL = 1.

Consider the ellipse (E) with focus F, directrix (d) and passing through B.



Part A

1) **Verify** that the eccentricity of (E) is $e = \frac{\sqrt{2}}{2}$.
2) **Show** that A is a vertex of the ellipse (E).
3) **Verify** that O is the center of the ellipse (E) and that B is a vertex of (E).

Part B

The plane is referred to a direct orthonormal system $(O; \vec{i}, \vec{j})$, where $\vec{i} = \frac{1}{2}\overrightarrow{OA}$ and $F(\sqrt{2}, 0)$.

Consider the point $S(0, -1)$.

1) **Write** an equation of (E).
2) **Verify** that L is a point on the ellipse (E).
3) **Draw** (E).
4) **Show** that the line (LH) is tangent to (E) at L and that the line (SL) is the normal to (E) at L.

VI- (7 points)

Part A

Consider the differential equation $(E): y'' + 2y' + y = x + 2$. Let $y = z + x$.

1) **Form** a differential equation (E_1) satisfied by z.
2) **Solve** (E_1), then deduce the general solution of (E).
3) **Determine** the particular solution of (E) satisfying $y(0) = -1$ and $y'(0) = 3$.

Part B

Let f and g be two functions defined on $\mathbb{R}$ as $f(x) = x + (x-1)e^{-x}$ and $g(x) = 1 + (2-x)e^{-x}$.

Denote by (C) the representative curve of the function f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- **Set up** the table of variations of g . (The limits of g at $-\infty$ and at $+\infty$ are not required).

   b- **Deduce** that $g(x) > 0$ for all x.

2) a- **Determine** $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

   b- **Determine** $\lim_{x \to -\infty} \frac{f(x)}{x}$. Interpret this result graphically.

3) Let (L) be the line with equation: $y = x$.

   a- **Study**, according to the values of x, the relative positions of (L) and (C).

   b- **Show** that the line (L) is an asymptote to (C) at $+\infty$.

4) **Verify** that $f'(x) = g(x)$ and set up the table of variations of f

5) **Determine** the coordinates of the point A on (C) where the tangent to (C) at A is parallel to (L).

6) **Prove** that the equation $f(x)=0$ has a unique root α and verify that

$0.4<\alpha<0.5$

Draw (L) and (C).

7) The function f has an inverse function h. Denote by (C') the representative curve of h.

Draw (C') in the same system as (C).

8) a- **Determine** $\int [x-f(x)]dx$.

b- Consider the points E (0 , –1) on (C) and F(–1 , 0) on (C').

Calculate the area of the region bounded by (C), (C') and the segment [EF].