عدد المسائل: ست	مسابقة في مادة الرياضيات المدة: أربع ساعات	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (2 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, the answer corresponding to it.

N°	Questions	Answers a	b	c
1	If $\arg(z)=\alpha+2k\pi,(k\in\mathbb{Z})$ and $z'=\frac{iz}{\bar{z}}$, where $z\neq 0$, then an argument of $\frac{z'}{z}$ is	$\pi+\alpha$	$\frac{\pi}{2}+\alpha$	$\frac{\pi}{2}-\alpha$
2	If (u_n) is an arithmetic sequence with common difference d $(d\neq 0)$, and (v_n) is the sequence defined by: $v_n=e^{u_n}$, then (v_n) is	a geometric sequence with common ratio e^d	an arithmetic sequence with common difference e^d	a geometric sequence with common ratio d
3	If $z=e^{i\theta}+e^{-i\theta}$ where $\theta\in\left[0;\frac{\pi}{2}\right[$, then an argument of z is	π	$\frac{\pi}{2}$	0
4	$\int\frac{(\arctan x)^2}{1+x^2}\,dx=$	$(\arctan x)^3+c$	$\frac{(\arctan x)^2}{2}+c$	$\frac{(\arctan x)^3}{3}+c$

II- (2 points)

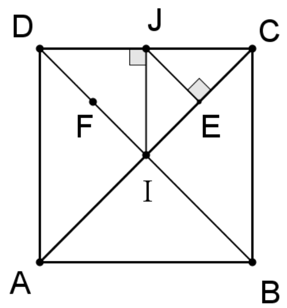
In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the two points $A(1\ ,0\ ,1)$ and $B(-1\ ,2\ ,0)$ and the two lines (L) and (D) with parametric equations:

$$(L):\begin{cases}x=2t-1\\ y=t-1\\ z=-2t+3\end{cases}\quad (t\in\mathbb{R})\quad \text{and}\quad (D):\begin{cases}x=2\\ y=m-1\\ z=-m\end{cases}\quad (m\in\mathbb{R}).$$

1) Write a Cartesian equation of the plane (P) passing through the two points A and B and parallel to (D).
2) a- Verify that the line (L) lies in plane (P).

   b- Show that (L) is perpendicular to (AB) at A.
3) Find the coordinates of the point C on (L) with negative abscissa so that $AC=6$.
4) Let M be a variable point on (D). Show that the volume of the tetrahedron MABC remains constant as M varies on line (D).

III- (3 points)



ABCD is a direct square with side 1 such that $\left(\overrightarrow{AB}\ ,\overrightarrow{AD}\right)=\frac{\pi}{2}\quad[2\pi]$.

Denote by I, J, E and F the midpoints of the segments [AC], [CD], [IC] and [DI] respectively.

Consider the direct plane similitude S that transforms A onto I and C onto J.

1) Verify that the ratio k of S is equal to $\frac{\sqrt{2}}{4}$ and find an angle α of S.
2) a- Show that S(B) = E.

   b- Deduce the image of the square ABCD by S.
3) The plane is referred to the direct orthonormal system $\left(A;\overrightarrow{AB}\ ,\overrightarrow{AD}\right)$.

   a- Determine the complex form of S.

   b- Deduce the affix of W, the center of S.
4) Let (P) be the parabola with focus A and directrix (BC) and (P') be the image of (P) by S.

   a- Show that D is on (P).

   b- Specify the tangent to (P') at F.

IV- (3 points)

An urn contains four black balls and 1 white ball.

A game runs in the following manner:

A fair die is rolled;

- If the die shows an odd number, then one white ball is added to the urn.
- If the die shows an even number, then one black ball is added to the urn.

After that, three balls are randomly and simultaneously selected from the urn.

Consider the following events:

O: "The die shows an odd number"

B: "The three selected balls are black".

1) Calculate the probabilities $P(B/O)$ and $P(B\cap O)$, then verify that $P(B)=0.35$.
2) The three selected balls are black. What is the probability that the die shows an even number?
3) Denote by X the random variable equal to the number of white balls selected during this game.

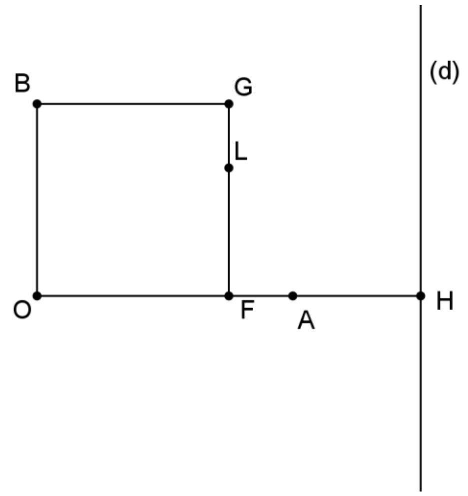
   a- Show that $P(X=1)=0.55$.

   b- Determine the probability distribution of X.
4) Each of Sami and Karim played this game once. Let S be the random variable equal to the total number of white balls obtained by both Sami and Karim.

   Calculate $P(S\geq 1)$.

V- (3 points)

In the adjacent figure:



- OFGB is a square with side $\sqrt{2}$,
- F is the midpoint of the segment [OH],
- (d) is the perpendicular to (OF) at H,
- A is the point on [OH] such that $OA=2$,
- L is the point on [FG] such that $FL=1$.

Consider the ellipse (E) with focus F, directrix (d) and passing through B.

Part A

1) Verify that the eccentricity of (E) is $e=\frac{\sqrt{2}}{2}$.
2) Show that A is a vertex of the ellipse (E).
3) Verify that O is the center of the ellipse (E) and that B is a vertex of (E).

Part B

The plane is referred to a direct orthonormal system $(O;\vec{i},\vec{j})$, where $\vec{i}=\frac{1}{2}\overrightarrow{OA}$ and $F(\sqrt{2},0)$.

Consider the point $S(0,-1)$.

1) Write an equation of (E).
2) Verify that L is a point on the ellipse (E).
3) Draw (E).
4) Show that the line (LH) is tangent to (E) at L and that the line (SL) is the normal to (E) at L.

VI- (7 points)

Part A

Consider the differential equation (E): $y'' + 2y' + y = x + 2$.

Let $y = z + x$.

1) Form a differential equation (E_1) satisfied by z.
2) Solve (E_1), then deduce the general solution of (E).
3) Determine the particular solution of (E) satisfying $y(0) = -1$ and $y'(0) = 3$.

Part B

Let f and g be two functions defined on $\mathbb{R}$ as $f(x) = x + (x-1)e^{-x}$ and $g(x) = 1 + (2-x)e^{-x}$.

Denote by (C) the representative curve of the function f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Set up the table of variations of g . (The limits of g at $-\infty$ and at $+\infty$ are not required).

   b- Deduce that $g(x) > 0$ for all x.

2) a- Determine $\lim\limits_{x \to +\infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$.

   b- Determine $\lim\limits_{x \to -\infty} \frac{f(x)}{x}$. Interpret this result graphically.

3) Let (L) be the line with equation: $y = x$.

   a- Study, according to the values of x, the relative positions of (L) and (C).

   b- Show that the line (L) is an asymptote to (C) at $+\infty$.

4) Verify that $f'(x) = g(x)$ and set up the table of variations of f.
5) Determine the coordinates of the point A on (C) where the tangent to (C) at A is parallel to (L).
6) Prove that the equation $f(x) = 0$ has a unique root α and verify that $0.4 < \alpha < 0.5$.
7) Draw (L) and (C).
8) The function f has an inverse function h. Denote by (C') the representative curve of h.

   Draw (C') in the same system as (C).

9) a- Determine $\int [x - f(x)]dx$.

   b- Consider the points E (0 , –1) on (C) and F(–1 , 0) on (C').

   Calculate the area of the region bounded by (C), (C') and the segment [EF].

عدد المسائل: ست

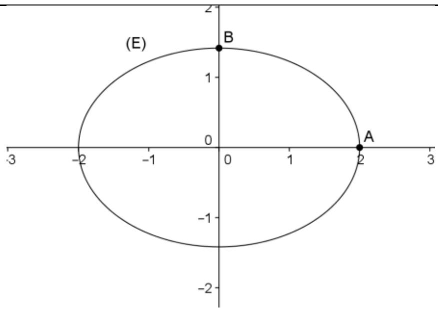
أسس تصحيح مسابقة الرياضيات

	Question I (4 points)	Mark
1	$\arg\left(\frac{z'}{z}\right)=\arg\left(\frac{i}{\bar z}\right)=\arg(i)-\arg\left(\bar z\right)=\frac{\pi}{2}+\theta\ [2\pi]$ **b**	**1**
2	$V_{n+1}=e^{u_{n+1}}=e^{u_n+d}=e^{u_n}.e^{d}=v_n e^{d}$. (v_n) is a geometric sequence with common ratio $r = e^{d}$ **a**	**1**
3	$\arg(z) = \arg(e^{i\theta}+e^{-i\theta})\ [2\pi] = \arg(2\cos\theta) = 0\ [2\pi]$ **c**	**1**
4	$\int\frac{(\arctan x)^2}{1+x^2}dx=\int\frac{(\arctan x)^2}{1+x^2}dx=\int(\arctan x)^2(\arctan x)'dx=\frac{(\arctan x)^3}{3}+c$ **c**	**1**

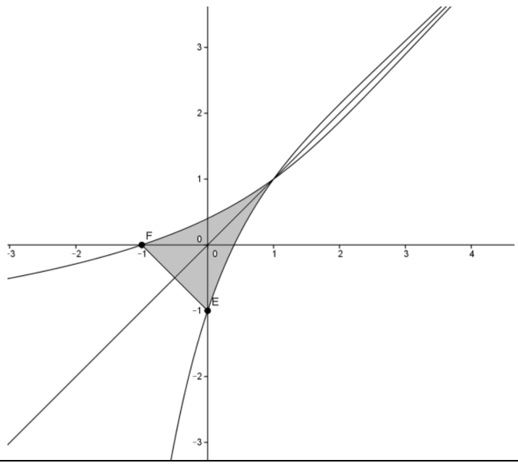
	Question II (4 points)	Mark
1	$\vec{AM}.(\vec{AB}\wedge\vec{v})=0$, then x + 2y + 2z – 3 = 0	**1**
2.a	(L) ⊂ (P).	**0.5**
2.b	$\vec{V}_L\cdot\vec{AB}=0$ and A ∈ (L).	**0.5**
3)	$AC^2 = 36$, gives $9(t-1)^2 = 36$, then t = 3 or t = –1 For t = 3 then $x_c = 5 > 0$ rejeted For t = –1 then $x_c = -1$ therefore C(–3 , –2 , 5)	**1**
4)	(D) / / (P) and M belongs to (D), then d(M, (P)) is constant A, B and C are fixed, then the area of ABC is constant Therefore, the volume $V=\frac{1}{3}\times d(M,(P))\times Area_{ABC}$ is constant **<u>OR</u>** Let M(2, m – 1, –m) ∈ (D). $\vec{AM}.\left(\vec{AB}\wedge\vec{AC}\right)=-18$, then V = 3 units of volume which is constant.	**1**

	Question III (6 points)	Mark
1)	$k=\frac{JI}{AC}=\frac{\frac{1}{2}AD}{AC}=\frac{\frac{1}{2}AD}{AD\sqrt{2}}=\frac{\sqrt{2}}{4}$ and $\alpha=\left(\vec{AC};\vec{IJ}\right)=\left(\vec{IC};\vec{IJ}\right)=\frac{\pi}{4}$.	**1**
2.a	$\frac{IE}{AB}=k$ and $\left(\vec{AB},\vec{IE}\right)=\alpha$. But S(A) = I, then S(B) = E **<u>OR</u>** ABC is direct and right isosceles at B. IEJ is also direct right isosceles at E. But S(A) = I and S(C) = J, then S(B) = E	**1**
2.b	S(A) = I, S(B) = E, S(C) = J, then the image of the square ABCD is another square which is IEJF.	**0.5**
3a	z' = az + b; a= $ke^{i\theta}=\frac{\sqrt{2}}{4}e^{i\frac{\pi}{4}}=\frac{1}{4}+\frac{1}{4}i$; S(A) = I and $z_I=\frac{1}{2}+\frac{1}{2}i$ therefore $b=\frac{1}{2}+\frac{1}{2}i$. Thus $z'=\left(\frac{1}{4}+\frac{1}{4}i\right)z+\frac{1}{2}+\frac{1}{2}i$	**1.5**
3b	$Z_w=\frac{b}{1-a}=\frac{2}{5}+\frac{4}{5}i$	**0.5**
4a	DA = DC, then D is on (P)	**0.5**
4b	D ∈ (P), then (DB) is the tangent to (P) at D since (DB) is the bisector of angle $\widehat{ADC}$. S(DB) = (EF), then (EF) is the tangent to (P') at F.	**1**

Question IV (6 points)		Mark
1)	$P(B/O) = \frac{C_4^3}{C_6^3} = 0.2$ and $P(B \cap O) = 0.2 \times \frac{1}{2} = 0.1$ $P(B \cap \overline{O}) = 0.5 \times \frac{1}{2} = 0.25$ $P(B) = 0.1 + 0.25 = 0.35$	1.5
2)	$P(\overline{O}/B) = \frac{P(\overline{O} \cap B)}{P(B)} = \frac{P(B) - P(B \cap O)}{P(B)} = \frac{5}{7}$	1
3.a	$P(X=1) = \frac{1}{2} \times \frac{C_4^2 \times C_2^1}{C_6^3} + \frac{1}{2} \times \frac{C_5^2 \times C_1^1}{C_6^3} = 0.55$	1
3.b	The possible values of X are 0, 1, and 2. $P(X = 0) = 0.35$; $P(X = 1) = 0.55$; $P(X = 2) = 1 - (P(X = 0) + P(X = 1)) = 0.1$	1.5
4	$P(S \geq 1) = 1 - P(S < 1) = 1 - [P(x = 0)]^2 = 1 - (0.35)^2 = 0.8775$	1

Question V (6 points)		Mark
A.1	$e = \frac{BF}{d(B;(d))} = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$	0.5
A.2	$\frac{AF}{d(A;(d))} = \frac{2-\sqrt{2}}{2\sqrt{2}-2} = \frac{\sqrt{2}}{2} = e$ and A belongs to the focal axis (FH), then A is a vertex	1
A.3	O belongs to the focal axis; $\frac{OF}{OA} = \frac{\sqrt{2}}{2} = e = \frac{c}{a}$, and **O, F and A are collinear in this sense** then O is the centre of (E). (OB) is perpendicular to (OA), then B belongs to the non focal axis; but B ∈ (E), then B is a vertex of (E). **OR** $e = \frac{c}{a}$ and $AF = a - c = 2 - \sqrt{2}$ give $c = \sqrt{2} = OF$ where O is a point on the focal axis thus O is the center of (E)	1
B.1	$\frac{x^2}{4} + \frac{y^2}{2} = 1$	0.5
B.2	$L(\sqrt{2}; 1) \in (E)$	0.5
B.3		1
B.4	An equation of the tangent to (E) at L is $y = \frac{-\sqrt{2}}{2}x + 2$; An equation of line (LH) is $y = \frac{-\sqrt{2}}{2}x + 2$. Thus, (LH) is tangent to (E) at L	1.5

	$L \in (LS)$ and $a_{(LS)} \times a_{(LH)} = -1$, then (SL) is the normal to (E) at L.	

Question VI (14 points)		Mark
A.1	$y' = z' + 1$; $y'' = z''$; $z'' + 2z' + z = 0$ (E_1)	**0.5**
A.2	The characteristic equation $r^2 + 2r + 1 = 0$; $r_1 = r_2 = -1$; Hence $z = (c_1 + c_2x)e^{-x}$ is the general solution of (E_1); $z = (c_1 + c_2x)e^{-x}$ is the general solution of (E).	**1.5**
A.3	$y(0) = -1$ gives $c_1 = -1$; $y'(0) = 3$ gives $c_2 = 1$	**0.5**
B.1a	$g(x) = 1 + (2 - x)e^{-x}$ $g'(x) = (x - 3)e^{-x}$ $\begin{array}{c\|ccccc} x & -\infty & & 3 & & +\infty \\ \hline g'(x) & & - & 0 & + & \\ \hline g(x) & & \searrow & 1-e^{-3} & \nearrow & \end{array}$	**1**
B.1b	The minimum value of g(x) is > 0 then g(x) > 0 for all x	**0.5**
B.2a	$\lim_{x\to-\infty} f(x) = (-\infty) + (-\infty)(+\infty) = -\infty$; $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} (x + \frac{x}{e^x} - \frac{1}{e^x}) = +\infty$	**1**
B.2b	$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} (1 + \frac{x+1}{x}e^{-x}) = 1 + (1)(+\infty) = +\infty$, Asymptotic direction according to y-axis	**1**
B3.a	$\delta(x) = f(x) - x = (x - 1)e^{-x}$ $\delta(x) = 0$; (L) cuts (C) at point (1 ; 1) $\delta(x) > 0$; $x > 1$ and (C) is above (L) $\delta(x) < 0$; $x < 1$ and (C) is below (L)	**1**
B.3b	$\lim_{x\to+\infty} (f(x) - x) = \lim_{x\to+\infty} (\frac{x}{e^x} - \frac{1}{e^x}) = 0$; (L) is an asymptote to C at $(+\infty)$.	**0.5**
B.4	$f'(x) = 1 + e^{-x} - e^{-x}(x - 1) = 1 + e^{-x}(1 - x + 1) = 1 + (2 - x)e^{-x} = g(x) > 0$ $\begin{array}{c\|ccc} x & -\infty & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & -\infty & \nearrow & +\infty \end{array}$	**1**
B.5	$f'(x_A) = 1$ gives $x_A = 2$; hence, $A(2; 2 + \frac{1}{e^2})$	**0.5**
B.6	Over IR, f is defined, continuous and strictly increasing, and f(x) changes signs (– to +), then the equation f(x) = 0 admits a unique solution α. f(0.4) = -0.002 < 0, f(0.5) = 0.196 > 0, then 0.4 < α < 0.5	**1**
B.7		**1**
B.8	(C) and (C′) are symmetric with respect to y = x; see the figure	**1**
B.9a	$\int (x - f(x))dx = \int (1 - x)e^{-x}dx = xe^{-x} + c$	**1**
B.9b	$A = \frac{1\times1}{2} + 2\int_0^1 (x - f(x))dx = \frac{1}{2} + \frac{2}{e}u^2$	**1**